An Influence of the Wavelet Packet Decomposition on the Noise Reduction in Ultrasound Images

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Abstract – **The wavelet packet method is a generalization of wavelet decomposition that offers a richer image analyses. It's made a trial in the paper to analyze the influence of different wavelet packet decompositions over the reduction of specific noise in medical ultrasound images. The optimal decomposition is found with the respect to the entropy-based criteria. Some results of the experiments are presented, which are made by computer simulation in MATLAB environment.**

Keywords – **Wavelet packet analyses, optimal decomposition, entropy-based criteria, noise reduction and medical ultrasound images.**

I. Introduction

The conventional methods of filtration basic on the processing in spatial domain [1], can not give a high quality for the ultrasound images, which needed for precise diagnostic. In noise reduction with wavelet transforms is used the method of the limitation the level of circumstantially coefficients. Giving the determinate threshold of their level and breaking on level the circumstantially coefficients, can be reduced the level of the noise, too. But the most interesting aspect of this problem is that the level of limitation can be determinate separately for every one coefficient. This permits to make adaptive changes of the image.

The wavelet packet methods for denoising give more complete analyses,that provides increased flexibility. They have the following important properties:

- Give a richer presentation of the image, basic on functions with wavelet forms, which consist of 3 parameters: position, scale and frequency of the fluctuations around a given position.
- Propose numerous decompositions of the image, that allows estimate the noise reduction of different levels of its decomposition.
- Allow adaptive denoising on each level of the decomposition by choice of optimal decomposition tree and optimal thresholds parameters.

In the paper is presented an analysis of the influence of the decomposition, based of entropy-criteria, on the specific noise reduction in ultrasound images. This noise is a product of specific phenomena in the human tissues impact of the ultrasound and physiological state of the patient, which can be presented as additive Gaussian white noise [2]. It exists in the diagnostic signals as "grit" in the light as in the dark regions.

The optimal decomposition can be chosen on the base of the best level and minimum of the entropy criteria.

Some results of the experiments are presented, which are made by computer simulation in MATLAB environment.

II. Problem Formulation

The wavelet packet analysis is a generalization of wavelet decomposition that offers a richer image analysis.

The wavelet packet atoms are waveforms [3]. Their mathematical description is given in Eq.1.

$$W_{j,n,k}(x) = 2^{-j/2} W_n(2^{-j}x - k), \quad (1)$$

where

$n \in N$ and $(j,k) \in Z^2$, j is a scale parameter, k – time-localization, n – frequency.

The basic idea of the wavelet packets is that for fixed values of j and k, $W_{j,n,k}$ analyzes the fluctuations of the signal roughly around the position $2^j.k$, at the scale 2^j and at various frequencies for the different admissible values of the parameter n.

The set of functions $W_{j,n} = W_{j,n,k}(x), k \in Z$ is (j,n) packet. For the positive values of integers j and n, wavelet packet are organized in trees. For each scale j, the possible values of parameter n are: $0,1,.....2^j - 1$, where j definite the level of decomposition and n - the position on the tree.

For the given orthogonal wavelet functions exists library of bases, called wavelet packet bases. Each of these bases offers a particular way of coding images, preserving global energy, and reconstructing exact features.

Based on the organization of the wavelet packet library, it is determinated the decomposition issued from a given orthogonal wavelets.

A signal of length $N = 2^L$ can be expand in α different ways, where α is the number of binary subtrees of a complete binary tree of a depth L.The result is $\alpha \geq 2^{N/2}$ [4]. As this number may be very large, it is interesting to find an optimal decomposition with respect to a conventional criterion. The classical entropy-based criterion is a common concept. It is looking for a minimum of the criterion. In case of denoising the 2D joint entropy of the wavelet co-occurrence matrix is used as the cost function to determine the optimal threshold. In this case 2D Discrete Wavelet Transform (DWT) is used to compose the noisy image into wavelet coefficients [5].

In the paper is presented on other approach. It is looking for the optimal from three different entropy criteria [6]. They are the followings:

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- The norm entropy with $1 \le p$, where P is parameter, given in Eq.2.

$$E_1(S) = \sum_i \sum_j |s_{ij}|^p \,, \qquad (2)$$

where S is the image, s_{ij} are the coefficients of S in an orthonormal basis.

- The (none normalized) Shanon entropy. It is given by the Eq. 3.

$$E_2(S) = -\sum_i \sum_j s_{ij}^2 \log(s_{ij}^2) \qquad (3)$$

- The logarithm of the "energy" entropy, given in Eq. 4.

$$E_3(S) = \sum_i \sum_j \log(s_{ij}^2) \qquad (4)$$

By looking for optimal decomposition to noise reduction, on this basis we can formulate the following problems:

- To choose the level for the best tree decomposition.
- To select the optimal entropy, by the minimum of criteria.

To solve these problems, two important conditions must be realized together. The conditions are given in Eq. 5 and Eq. 6:

$$E_K(S) = \min, \; for K = 1,2,3,...n \,, \qquad (5)$$

where E_K is the entropy in the level K for the best tree decomposition

$$s_{ij} \le t \,, \qquad (6)$$

where t is the threshold of the coefficients.

By determination of the threshold it is used the strategy of Birge-Massart [7]. This strategy is flexibility and allows to determinate the threshold in three directions: horizontal, vertical and diagonally. In addition the threshold can be hard or soft [4].

III. EXPERIMENTAL PART

The formulated problems are solved by computer simulation in MATLAB, version 6.5 environment with using the WAVELET TOOLBOX.

In analysis are used 20 ultrasound images from cardiology with different sizes. The original images are in different file formats: tiff, jpeg, bmp, but all of them are converted in bmp. All images are 24 bpp, in RGB system.

The simulation is made with additive Gaussian white noise with normal distribution and covariance 0.03.

The experiments are associated with assignment of some basic elements:

- The choice the level of the best decomposition

From the implemented experiments with image of size 640x480, the optimal results are obtained by the second level of the decomposition. In the lower levels of the decomposition the noise reduction is greater but the images are visual with badly quality.

- The assignment of the optimal decomposition, by using the entropy criteria.

The potentiality of the noise reduction it is tested by using of three different entropy criteria, given in Eq. 2,Eq. 3 and Eq. 4. The corresponding optimal decompositions in node (2, 0) are shown in Fig. 1 (Optimal decomposition by Shanon entropy criteria), Fig. 2 (Optimal decomposition by norm entropy criteria) and Fig. 3 (Optimal decomposition by log energy criteria).

The best results are obtained by the (none normalized) Shanon entropy.

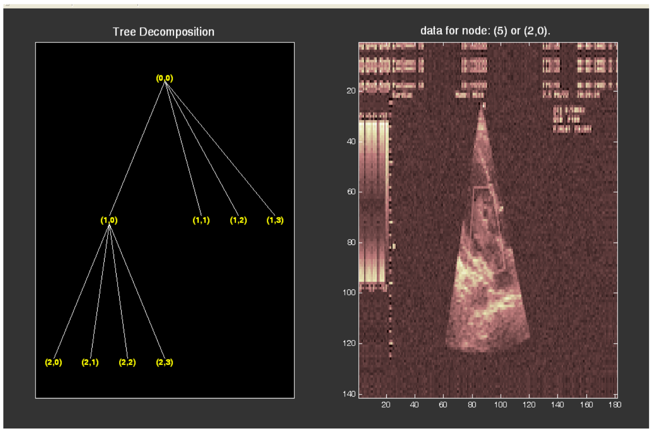

Fig. 1. Optimal decomposition by Shanon entropy criteria

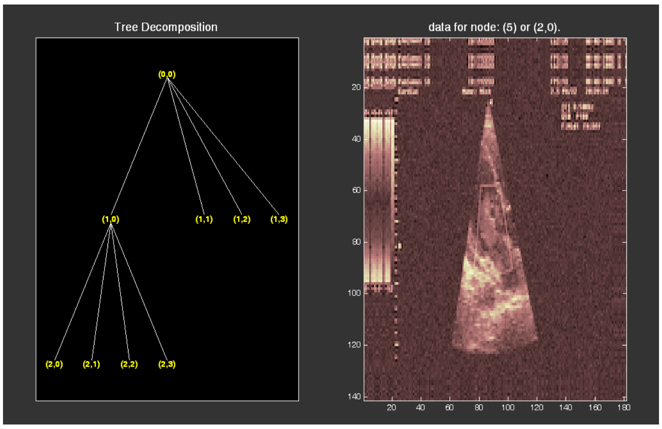

Fig. 2. Optimal decomposition by norm entropy criteria

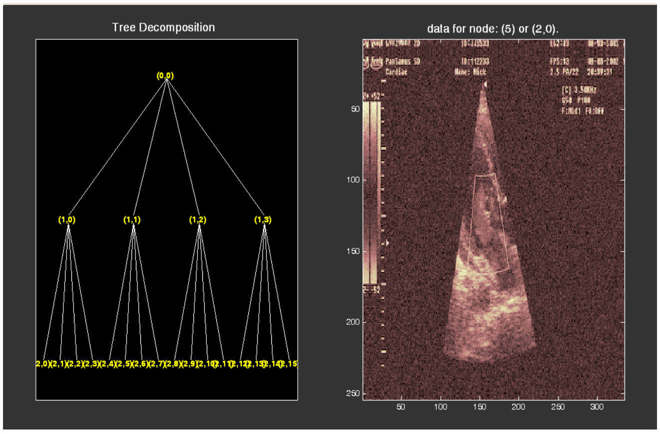

Fig. 3. Optimal decomposition by log energy criteria

The following quantitative estimations are used:

- Coefficient of noise reduction (CNR), given in Eq. 5:

$$K=\frac{\sum_{i=1}^{N}\sum_{j=1}^{M}\left[x_F(i,j)-x(i,j)\right]^2}{\sum_{i=1}^{N}\sum_{j=1}^{M}\left[y(i,j)-x(i,j)\right]^2}\,, \qquad (5)$$

where $x(i,j)$ is a pixel from the original image; $x_F(i,j)$ is a pixel from the filtrated image; $y(i,j)$ is a pixel from the noised image.

- Signal to noise ratio SNR_Y [dB], given in Eq. 6:

$$SNR_y=10\lg\frac{\sum_{i=1}^{N}\sum_{j=1}^{M}\left[x(i,j)\right]^2}{\sum_{i=1}^{N}\sum_{j=1}^{M}\left[y(i,j)-x(i,j)\right]^2} \qquad (6)$$

- Signal to noise ratio SNR_F [dB], given in Eq. 7:

$$SNR_F=10\lg\frac{\sum_{i=1}^{N}\sum_{j=1}^{M}\left[x_F(i,j)\right]^2}{\sum_{i=1}^{N}\sum_{j=1}^{M}\left[y(i,j)-x(i,j)\right]^2} \qquad (7)$$

- Effectiveness of filtration E_{FF} [dB], given in Eq. 8:

$$E_{FF}=SNR_y-SNR_F \qquad (8)$$

The results for the quantitative estimations of the noise reduction are obtained by N=640 and M=480, for each component of the system YUV. The results from the simulation of noise reduction are presented in Table 1.

TABLE I
RESULT FROM SIMULATION

Entropy criteria	**YUV system**	**Threshold**	**CNR**	**SNR_Y [dB]**	**SNR_F [dB]**	**E_{FF}**
1	**2**	**4**	**5**	**6**	**7**	**8**
Norm	Y	0.2294	0.3967	5.5595	5.7739	0.2144
	U	0.1844	0.3931	5.6078	5.7893	0.1815
	V	0.2729	0.3899	5.8553	6.0174	0.1621
Shanon	Y	0.2270	0.6351	7.7001	7.9847	0.2846
	U	0.1839	0.0328	-4.5401	-4.7753	0.2352
	V	0.2729	0.0362	-4.4538	-4.6234	0.1694
Log energy	Y	0.2297	0.3999	5.5595	5.5308	0.0287
	U	0.1834	0.4023	5.6078	5.5529	0.0549
	V	0.2734	0.4010	5.8553	5.7981	0.0572

The best results from the simulation of noise reduction are obtained by using the Shanon entropy criteria. By using of the log energy and norm criteria the effectiveness of the filtration is smaller.

In Fig. 4 is presented the noised image and in Fig. 5 we can see the denoised image by using the optimal decomposition on the second level, by using (none normalized) Shanon entropy criteria. The noise reduction is realized by hard threshold.

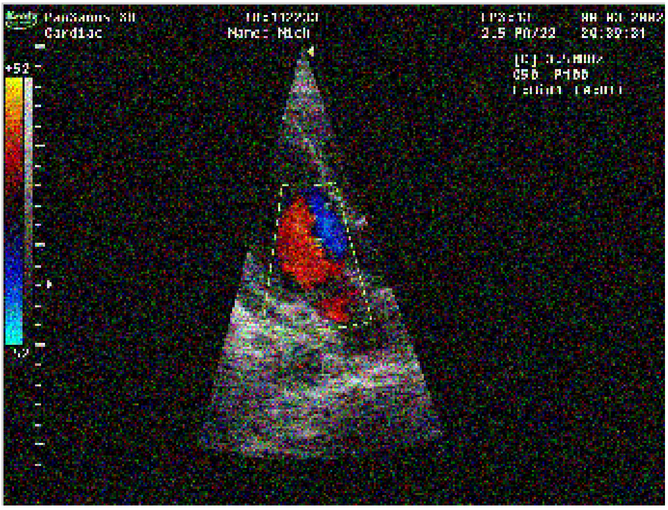

Fig. 4. Noised image

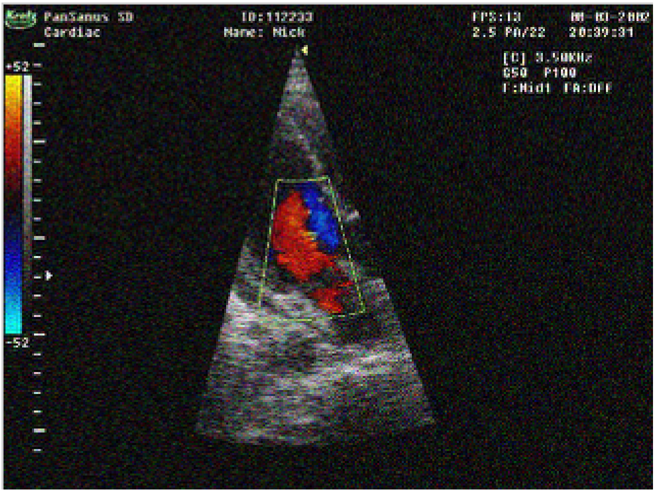

Fig. 5. Denoised image

IV. Conclusion

The problem of the optimal decomposition is important in noise reduction of ultrasound images. In the paper is presented this influence, by changing the level of decomposition and the entropy criteria. On this base is improved the advanced approach for specific additive noise reduction, using wavelet packet transformations [8]. The thoroughly advantages are:

- It is possible to reduce the specific noise in medical ultrasound images by preservation the high quality of the restored images.
- It allows a selective approach by noise reduction on each level of decomposition by choice of optimal parameters of the using wavelet packet transformations.
- It can be used by medical image processing, archiving and making of data basis.

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