Designing of scalar quantizer based on the hybrid model for the Laplacian source

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Abstract – **In this paper we suggest a hybrid model for scalar quantizer construction in order to achieve the quantization performances arbitrarily close to those of the optimal scalar quantization. This model is based on the combination of two quantization techniques. One of them is the companding technique and the other one is the Lloyd-Max's procedure for designing scalar quantizers. The suggested mixed technique has low implementtation complexity. Furthermore, it has a little bit greater complexity than companding technique, but enables better quantizer's performance, very close to those of optimal Lloyd-Max's scalar quantizers. The recommended hybrid model presents the general model of the scalar quantizer considered in [1].**

Keywords – **Hybrid model, Laplacian source, scalar quantizer**

I. INTRODUCTION

One of the important issues from the engineer's point of view is the design and implementation of quantizers to meet the performance objectives. Lloyd [2] and Max [3] respectively proposed an algorithm to compute optimum quantizers using mean-square error distortion measure. Namely, they gave the nonlinear quantization procedure in order to minimize the quantization noise. The primary goal when designing an optimal Lloyd-Max's quantizer is to select the representation levels and the decision thresholds so as to provide the minimum possible average distortion for a fixed number of quantization levels N. Particularly, Lloyd-Max's algorithm is an iterively algorithm, which performs in each iteration calculation of all representation levels and decision thresholds of the N levels scalar quantizer. The size of necessary calculation is the deficiency of this algorithm, especially when designing scalar quantizers with a large number of quantization levels. However, a nonuniform quantization can also be achieved by compressing the input signal, than quantizing it with a uniform quantizer and expanding the quantized version of the compressed signal using a nonuniform transfer characteristic inverse to that of the compressor. The described quantization technique is called the companding technique. Bad approximation of the input signal in the region of high amplitudes is the deficiency of the companding technique which is the consequence of the extensive outermost cells. In order to reduce the size of the necessary calculation in comparision to that for Lloyd-Max's algorithm, as well as to improve the deficiency introduced by using the companding technique, we suggest one model, denoted here as the *hybrid model*. This model is based on the combination of the companding technique and the Lloyd-Max's procedure of designing scalar quantizers. Namely, applying the companding technique to N-2L inner cells and Lloyd-Max's procedure to 2L outer cells it is possible to design N-levels scalar quantizer. The suggested model is very simple for analysis and it provides an almost optimal design of scalar quantizers. Therefore, in this paper we continue the research in the field of finding as simple as possible method for designing optimal scalar quantizers. We perform an exact and complete analysis of the hybrid model considering the Laplacian input signals. Furthermore, we derive the expression for determining the decision thresholds and the representation levels of the considered scalar quantizer. Thus, knowing the decision threshold t_{2K-1} it is possible to determine the support region of the observed scalar quantizer, ranging $(-t_{2K-1}, t_{2K-1})$. Optimal determining of the support region has been considered by a lot of researchers [1], [4], [5]. The problem of determining the support region was considered by Sangsin Na and David Neuhoff [1]. They applied the companding technique when they calculated the values of the representation levels and the decision thresholds of N-2 inner cells and Lloyd-Max's procedure when calculated the values of the first and the last representation levels. Namely, they considered the special case for L=1, of the hybrid model suggested in this paper. At the end of this paper we are considering the performances (relative distortion error) of the quantizers designed by using the hybrid model. We show that these performances are arbitrarily close to those of optimal scalar quantizers.

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II. HYBRID MODEL

Let us consider an N-level nonuniform scalar quantizer Q for the Laplacian input signals. Scalar quantizer Q is defined with Q: $R \rightarrow C$, as a functional mapping of the set of real numbers R onto the set of the output representation. The set of the output representation constitutes the code book:

$$C \equiv \{y_1, y_2, y_3, \cdots y_N\} \subset R \qquad (1)$$

that has the size $|C|=N$. The output values, y_j, are called the representation levels. The nonuniform scalar quantizer Q is defined with the set of the output values and with the partition of the input range of values onto N cells i.e. intervals α_j, $j=1,2,...,N$. Cells α_j are defined with the decision thresholds $\{t_0, t_1,..., t_N\}$, such that $\alpha_j=(t_{j-1},t_j]$, $j=1,2,...,N$. A quantized signal has value y_j when the original signal belongs to the quantization cell α_j. Hence, N-level scalar quantizer is defined as a functional mapping of an input value x onto an output representation, such as:

$$Q(x) = y_j, \quad x \in \alpha_j. \qquad (2)$$

The negative thresholds and the representation levels are symmetric to their nonnegative counterparts. Hence, the considered quantizer can be depicted by using positive values of the decision thresholds $0=t_{N/2}<t_{N/2+1}<\ldots<t_{N-1}<t_N=\infty$ and representation levels $y_{N/2+1}<y_{N/2+2}<\ldots<y_N$. Let us denote the distances from the representative levels to the nether decision thresholds, i.e. reconstruction offsets δ_j $j=1,\ldots,N$, that are necessary when calculating the parameters of the scalar quantizers.

Nonuniform quantization can be achieved by the following procedure: compress the signal x using a nonlinear compressor characteristic $c(\cdot)$, quantize the compressed signal $c(x)$ with a uniform quantizer, expand the quantized version of the compressed signal using a nonlinear transfer characteristic $c^{-1}(\cdot)$ inverse to that of the compressor. The corresponding structure of a nonuniform quantizer consisting of a compressor, a uniform quantizer, and expandor in cascade is called the compandor. Let us define the compressor function $c(x)$ similarly as in [6]:

$$c(t_j) = -1 + 2\frac{\int_{-\infty}^{t_j} p^{1/3}(x)dx}{\int_{-\infty}^{+\infty} p^{1/3}(x)dx}. \qquad (3)$$

Also, the following equation is valid for the compressor function $c(x)$ [7]:

$$c(t_j) = -1 + \frac{2j}{N}. \qquad (4)$$

When the values of the input signal x are within the $(-\infty,\infty)$ range, the values of $c(t_j)$ are copied into the $[-1,1]$ range by using thus defined compressor function. Decision thresholds t_j, $j=1,2,\ldots,N-1$ can be determined by equating the last two equations:

$$t_j = \frac{3}{\sqrt{2}}\ln\left(\frac{N}{2(N-j)}\right) \qquad j=1,\ldots,N-1. \qquad (5)$$

In order to simplify the equations that are necessary for the scalar quantizer's parameters calculation we can introduce the relation $K=N/2$. Thresholds t_{2K-L}, $L \ll K$, can be determined by using the following expression:

$$t_{2K-L} = \frac{3}{\sqrt{2}}\ln\left(\frac{K}{L}\right). \qquad (6)$$

In this paper we are considering the hybrid model based on the combination of two quantization techniques. One of them is the companding technique and the other one is the Lloyd-Max's procedure for designing scalar quantizers. Namely, applying the companding technique to the range $(-t_{2K-L}, t_{2K-L})$ (inner region), i.e. to N-$2L$ inner cells $\alpha_{L+1},\ldots,\alpha_{2K-L}$, and Lloyd-Max's procedure to union of ranges $(t_0, -t_{2K-L})$ and (t_{2K-L}, t_{2K}) (outer region), i.e. $2L$ outer cells $\alpha_1,\ldots,\alpha_L$ and $\alpha_{2K-L+1},\ldots,\alpha_{2K}$ it is possible to design the N-levels scalar quantizer. The widths of the outer cells $\alpha_1,\ldots,\alpha_L$ and $\alpha_{2K-L+1},\ldots,\alpha_{2K}$ are constant and independent of the number of quantization levels N. Considering that fact the performances of the designed quantizer will be better than those of the quantizer realized by using the companding technique. Therefore, by using the compressor function $c(x)$, defined with Eqs. (3) and (4), we can use Eq. (6) to calculate the edge of the inner region t_{2K-L}. In order to calculate the decision thresholds and the representation levels of the outer region we use the well known values of the reconstruction offsets $\delta_{2K-L+1},\ldots,\delta_{2K}$ that are calculated in case of optimal Lloyd-Max's scalar quantizers [7]:

$$y_{2K-i+1} = t_{2K-i} + \delta_{2K-i+1}, \qquad i=1,\ldots,L \qquad (7)$$

$$t_{2K-i+1} = y_{2K-i+1} + \delta_{2K-i+2}, \qquad i=2,\ldots,L \qquad (8)$$

The values of the decision thresholds and representation levels of scalar quantizers realized by using the companding technique are not optimal. The goal of the suggested hybrid model of quantizers is to make, as much as possible, the decision thresholds and the representation levels to be optimal. Also, when designing N-level Lloyd-Max's scalar quantizer it is necessary to know all the values of the decision thresholds and the representation levels. Hence, in such a case $4K$ values should be memorized. However, when designing N-level scalar quantizer, based on the hybrid model, it is required to know the edge of the inner region t_{2K-L} and the set of L values of the reconstruction offsets $\delta_{2K-L+1},\ldots,\delta_{2K}$, i.e. the set of $L+1$ values. Thus, sparing the memory space simpler solution of hardware can be achieved. This is particularly of interest when designing scalar quantizers with large number of quantization levels $N=2K$. Hence, the compromise between the design complexity and the distances from the optimal solution of the scalar quantizer designing problem should be obtained. Furthermore, this model is the generalized model which for $L=K$ presents the model of the Lloyd-Max's quantizer while in case of $L=0$ presents the model of the quantizer realized by using the companding technique. Choosing the values of L it is possible to arbitrarily approach the optimal solution of the scalar quantizer construction problem.

III. THE QUANTIZER PERFORMANCES

The performance of a quantizer is often specified in terms of $SNRQ$ (signal to quantization noise ratio), given by [8]:

$$SNRQ = 10\log_{10}\left(\frac{\sigma^2}{D}\right) \qquad (9)$$

measured in decibels, with σ^2 denoting the variance of x. Here we assume the unit variance input signal, therefore $SNRQ$ can be given by:

$$SNRQ = 10\log_{10}\left(\frac{1}{D}\right). \qquad (10)$$

Let us define the relative distortion error δ such as:

$$\delta = \frac{D - D^{opt}}{D^{opt}} \qquad (11)$$

where D^{opt} is the optimal distortion value. Also, let us denote the optimal value of $SNRQ$ with $SNRQ^{opt}$. Introducing the relation:

$$\Delta SNRQ = SNRQ - SNRQ^{opt} \tag{12}$$

the Eq. (11) becomes:

$$\delta = 10^{-\frac{\Delta SNRQ}{10}} - 1 . \tag{13}$$

In analyzing the behavior of the quantizer, it is preferable to use relative quantities, like signal to quantization noise ratio and relative distortion error instead of absolute quantities, such as distortion. Relative parameters portray the behavior of the quantizer in a way that is independent of the signal level and hence is more general. Good distortion approximation of quantizers based on the companding technique can be achieved by using Bennett's integral [9], [10] ranging $[-t_{2K-L},+t_{2K-L}]$:

$$D = \frac{1}{12(2K-2L)^2}\left(\int_{-t_{2K-L}}^{t_{2K-L}} p^{1/3}(x)dx\right)^3 + 2\sum_{j=2K-L}^{2K-1}\int_{t_j}^{t_{j+1}}(x-y_{j+1})^2 p(x)dx. \tag{14}$$

III. Numerical results

Table I provides numerical values of the the relative distortion error δ, calculated for L=0,1,2,4, when the number of quantization levels varies (N=32, 64, 128). Namely, one confirmation of the hybrid model validity is given by Table I. It is apparent that for L=8, for large enough number of quantization levels N, numerical values of the relative distortion error δ are below 0.005 [11], whereby the one of stopping criterion that allows interruption of the Lloyd-Max's algorithm is satisfied. Hence, it is obvious that for L=8, recommended the hybrid model enables optimal scalar quantizer designing. Summary of the numerical values for *SNRQ*, calculated for L=0,1,2,4,8, when the number of quantization levels are N=32, 64, 128, is given by Table II. Assimilating the appropriate values from Table II and Table III, one can notice that when the value of L grows it is approximately possible to approach to optimal values of the *SNRQ*.

Table I

Numerical values of the relative distortion error δ, calculated for L=0,1,2,4, 8, when the number of quantization levels varies (N=32, 64, 128).

δ	N=32	N=64	N=128
L=0	0.0713	0.0348	0.0181
L=1	0.0341	0.0168	0.0093
L=2	0.0213	0.0106	0.0062
L=4	0.0110	0.0057	0.0038
L=8	0.0041	0.0023	0.0021

Table II

Numerical values of the *SNRQ*, calculated for L=0,1,2,4, 8, when the number of quantization levels varies (N=32, 64, 128).

SNRQ	N=32	N=64	N=128
L=0	23.5709	29.5915	35.6121
L=1	23.7244	29.6675	35.6499
L=2	23.7785	29.6941	35.6631
L=4	23.8226	29.7155	35.6736
L=8	23.8522	29.7299	35.6807

Table III

Optimal reference of the *SNRQ*, ($SNRQ^{OPT}$), when the number of quantization levels varies (N=32, 64, 128).

	N=32	N=64	N=128
$SNRQ^{opt}$	23.87	29.74	35.69

III. Conclusion

The suggested hybrid model for scalar quantizer construction enables sophisticated relation, i.e. compromise between design complexity and the distances from the optimal solution of the scalar quantizer designing problem. The depicted model is based on the quantization technique that is very simple and convenient for use. It is very important to point out that for fixed L, by using the suggested hybrid model, when the number of quantization levels N varies, the size of the necessary calculation of scalar quantizers' parameters values is constant. Furthermore, the required memory space remains constant. However, the size of the necessary calculation and the capacity of memory space for Lloyd-Max's quantizers grow with the number of quantization levels N. In such a way, comparing Lloyd-Max's quantizers considerable contribution can be achieved by using the proposed hybrid model. Also, by using the suggested hybrid model for designing scalar quantizers with large enough number of quantization levels N, it is possible to achieve nearly optimal values of *SNRQ*, i.e. the order of difference is 10^{-2}. This is yet another proof of the hybrid model validity. Accordingly, the analyses shown here is of a practical importance because it can be of great help to engineers.

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