Theoretical Analysis as a Function of the Total Q Factor Loudspeaker Characteristics

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Abstract – **This work considers researching of the following characteristics of a directly radiating electrodynamic loudspeaker as a function of the total Q factor: frequency response of the amplitude and phase of the sound pressure created, of the group time delay, of the module and the complex impedance.**

For the purposes of the theoretical analysis of the above loudspeaker characteristics depending on the responses of the total Q factor, three regions are considered, where equations describing the output signal at input excitation - Heaviside function - are offered for each of them.

Keywords – **Loudspeaker, frequency response, step response.**

The transitional function of a loudspeaker and a closed-box loudspeaker system [1÷7] can be described by the following equation (1):

$$W(s,Q)=\frac{\dfrac{s^2}{\varpi_s{}^2}}{\dfrac{s^2}{\varpi_s{}^2}+\dfrac{s}{\omega_s}\cdot\dfrac{1}{Q}+1} \tag{1}$$

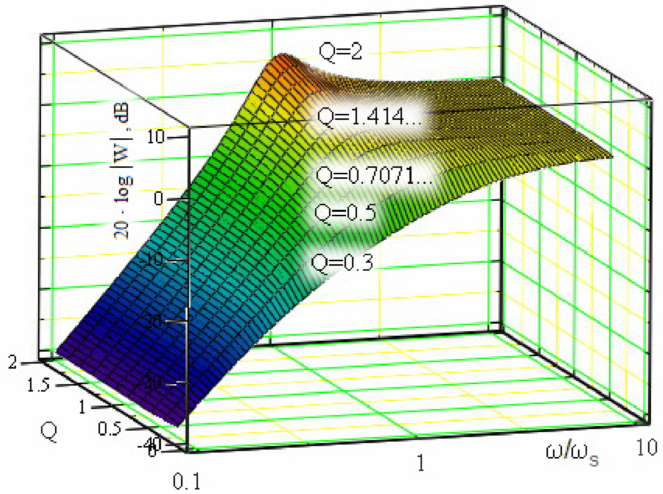

Fig. 1. Normalized Amplitude-Frequency Response of the Sound Pressure Created by the Loudspeaker (Q=0.3÷2)

This function is analogical to a second-order high-pass filter (40 dB/dec. cutoff).

Where: $s=\sigma+i.\varpi$ is the complex frequency variable,

f_s is the resonance frequency ($\varpi_s=2.\pi.f_s$) of the loudspeaker.

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Fig. 1 shows the normalized characteristic of the sound pressure created by the loudspeaker $20.\log\left|W\left(\dfrac{i.\varpi}{\varpi_s},Q\right)\right|$ as a function of the frequency normalized to $\varpi_s=2.\pi.f_s$ - in a logarithmic scale - and the total Q factor (Q=0.3÷2).

These responses for Q greater than $1/\sqrt{2}$ i.e. 0.7071 are second-order Chebyshev equal-ripple alignments [1].

For $Q=0.7071$ (i.e. $1/\sqrt{2}$) is a second-order Butterworth maximally-flat alignment.

For $Q=0.5$ is a second-order Linkwits-Riley alignment

The responses for $Q=0.3\div0.7$ are second-order alignment for real world loudspeakers.

Table 1 shows the DYNAUDIO loudspeakers parameters [16].

Loud-speaker	fs [Hz]	Qts	1w/1m dB	W to fs dB
T-330 D	750	0.2	92	-10
D-21 AF	1300	0.41	91	-4
D-28/2	880	0.41	89	-5
D-260	1000	0.48	90	-4
M-560D	325	0.35	91	-5
D-52 AF	350	0.4	88	-6
D-54 AF	325	0.3	92	-7
D-76 AF	350	0.9	89	0
17 W-75XL	42	0.44	89	BOX
24 W-75	32	0.35	90	BOX
30 W-54	22	0.36	92	BOX

Table 1. DYNAUDIO Loudspeaker parameters [16]

Where:

fs [Hz] is the loudspeaker resonance frequency,

Qts – total Q factor,

1w/1m – sensitivity - i.e. the sound pressure created - in dB – along the axis of radiation at a distance normalized to 1m and 1W power supplied to the loudspeaker.

W to fs – loudspeaker sensitivity for the resonance frequent-cy, fs, in dB.

The first four lines of Table 1 apply to high-frequency (tweeter) dome loudspeakers (T-330 D, D-21 AF, D-28/2 and D-260), and then follow the parameters of middle-frequency dome loudspeakers (M-560D, D-52 AF, D-54 AF AND D-54 AF). They are designed as closed-box systems and their Producer's brochures list the so called Thiele/Small parameters and characteristics [16].

The parameters of the low-frequency (woofer) loudspeakers (17 W-75XL, 24 W-75 and 30 W-54) determine the selection of the BOX: closed-box, variovented box, bass-reflex, transmission line, band pass, etc. The selected box and the actual loudspeaker, they both determine the parameters and the characteristics in the low-frequency region [1÷7].

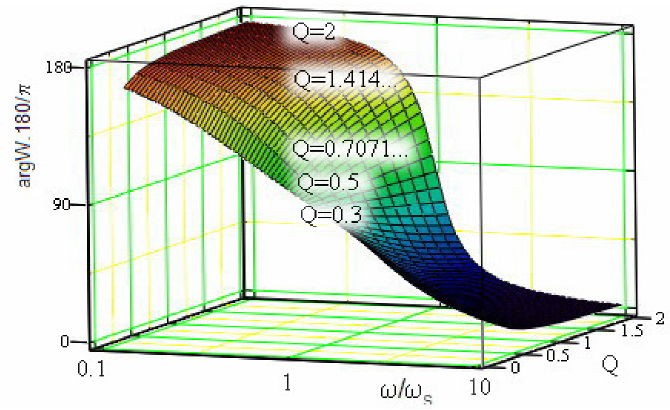

Fig. 2. Normalized frequency response of the phase of the sound pressure created by the loudspeaker where Q factor is the parameter.

The phase of the sound pressure created by the loudspeaker, as a function of the total Q factor, is plotted in Fig. 2.

$$\phi(s,Q)=\arg(W(s,Q)).\frac{180}{\pi} \tag{2}$$

The displacement of the loudspeaker voice-coil, as defined by formula (3) is analogical to a function of a low-pass second-order filter.

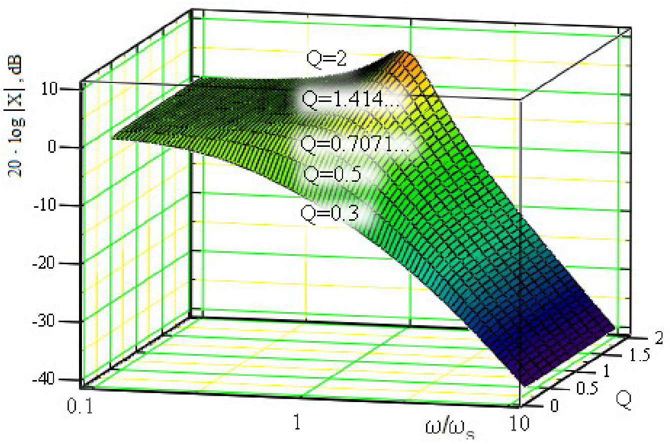

Fig. 3. Normalized voice-coil displacement of the loudspeaker as a function of normalized frequency response for parameter total Q factor.

Fig. 3 shows the normalized voice-coil displacement magnitude $20.\log\left|X\left(\frac{i.\varpi}{\varpi_s},Q\right)\right|$ with frequency normalized to ϖ_s as a function of the total Q factor.

$$X(s,Q)=\frac{1}{\frac{s^2}{\varpi_s{}^2}+\frac{s}{\omega_s}.\frac{1}{Q}+1} \tag{3}$$

The group time delay can be defined as:

$$G(\frac{i.\varpi}{\varpi_s},Q)=-\frac{d}{d\varpi}\left(\phi(\frac{i.\varpi}{\varpi_s},Q)\right) \tag{4}$$

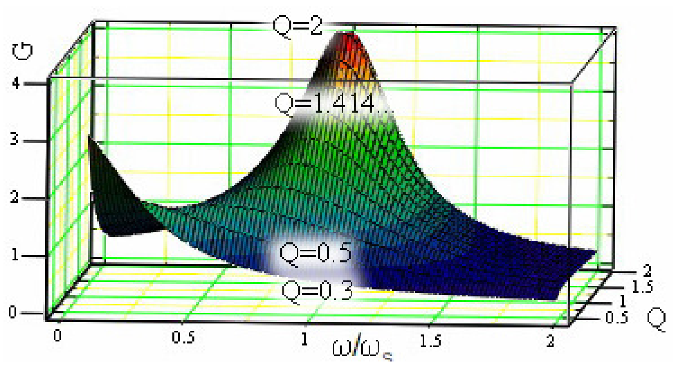

Fig. 4. Frequency response of the group time delay of a loudspeaker, where parameter is the Q factor.

The equation [1÷7], which defines the input impedance, includes the sum of the voice-coil electrical impedance $R_e+i.\omega.L_e$ and the inserted impedance of the loudspeaker $Z_{вн}$:

$$Z_{LS}=R_e+i.\omega.L_e+\frac{(B.L)^2}{r+\frac{1}{i.\omega.c}+i.\omega.m} \tag{5}$$

where: R_e is the voice-coil DC resistance,
L_e - voice-coil inductance,
$B.L$ - force factor magnet system,
r - mech. resistance,
m - moving mass,
c - suspension compliance.

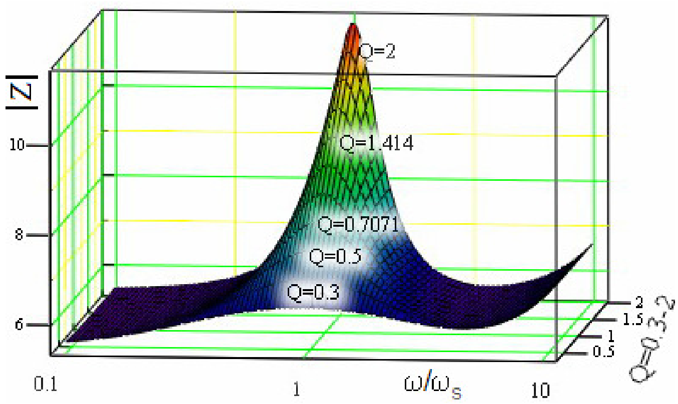

Fig. 5. Magnitude of the loudspeaker impedance depending on the normalized frequency as a function of the Q factor.

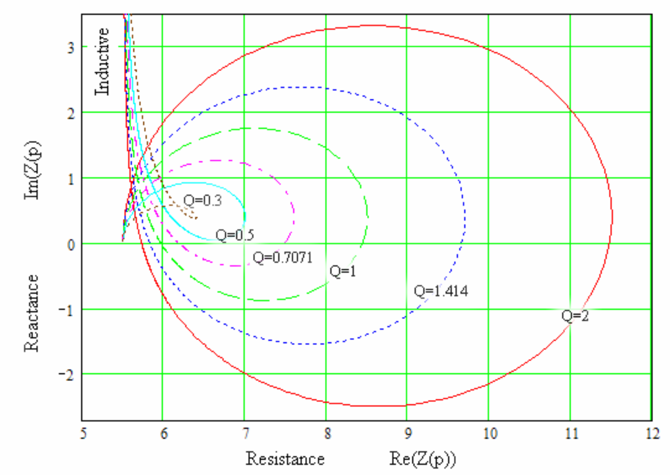

Fig. 6. Complex Impedance (Nyquist plots)

Loudspeaker Step Response

The input step function is:

$$\Phi(t) := if\,(t < 0{,}0{,}1) \tag{6}$$

The excitation is the Heaviside function for which the Laplace transform is $1/s$ [8].

$$\Phi(\mathrm{tn})\ \mathrm{laplace}, \mathrm{tn}\ \rightarrow \frac{1}{\mathrm{s}}$$

With the program MathCad symbolic transform [13,14], *invlaplace*, from the step function (1) we find an equation which describes the sound pressure created by the loudspeaker as a function of time with single input excitation and Q factor - a parameter:

$$\frac{1}{s}.W(s)\mathit{invlaplace}, s \rightarrow \tag{7}$$

The following three spaces can be defined depending on the values of the total Q factor:

1. Damped oscillation - at Q>0.5

$$Out1(t) = \left\{e^{-\alpha_1.t}.\left[\cos(\alpha_2.t) - \alpha_3.\sin(\alpha_2.t)\right]\right\}\Phi(t) \tag{8}$$

$$\alpha_1 = \frac{1}{2.Q}, \ \alpha_2 = \frac{1}{2}.\sqrt{4 - \frac{1}{Q^2}} \ \text{и} \ \alpha_3 = \frac{1}{Q.\sqrt{4 - \frac{1}{Q^2}}}$$

2. Critical value - at Q=0.5

$$Out2(t) = \left\{e^{-t}.[1 - t]\right\}\Phi(t) \tag{9}$$

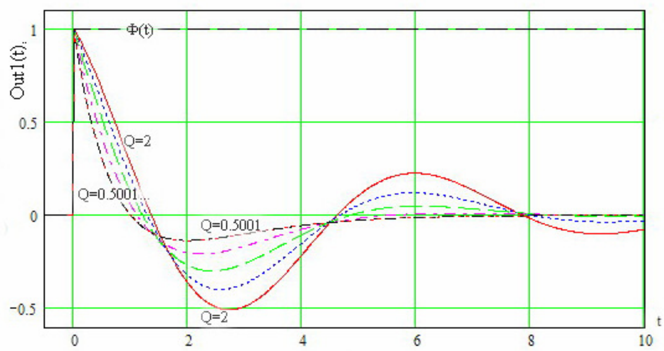

Fig. 7. Normalized step response of a loudspeaker at Q=0.501, 0.7071, 1, 1.414 and 2, according to Eq. (8).

3. Aperiodic damping at Q<0.5

$$Out3(t) = \left\{e^{-\alpha_1.t}.\left[\cosh(\alpha_4.t) - \alpha_5.\sinh(\alpha_4.t)\right]\right\}\Phi(t) \tag{10}$$

$$\alpha_1 = \frac{1}{2.Q}, \ \alpha_4 = \frac{1}{2}.\sqrt{\frac{1}{Q^2} - 4} \ \text{и} \ \alpha_3 = \frac{1}{Q.\sqrt{\frac{1}{Q^2} - 4}}$$

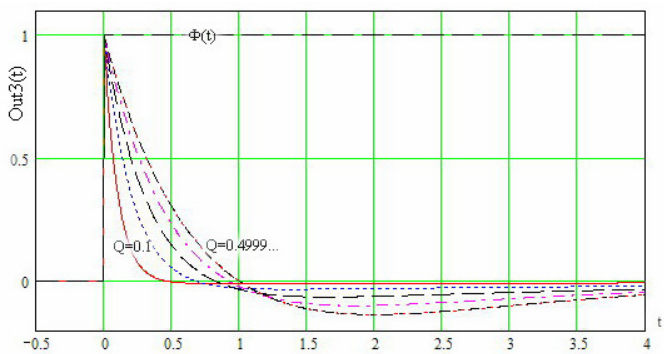

Fig. 8. Normalized step response of a loudspeaker at Q=0.1, 0.2, 0.3, 0.4 and 0.499... according to Eq. (10)

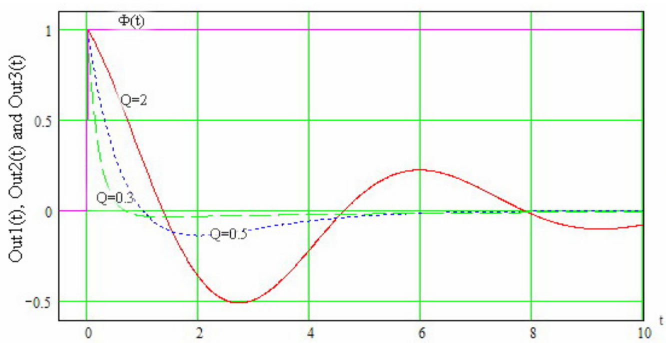

Fig. 9. Normalized step response of a loudspeaker at Q=0.3, 0.5 and 2, according to Eqs. (8), (9) and (10).

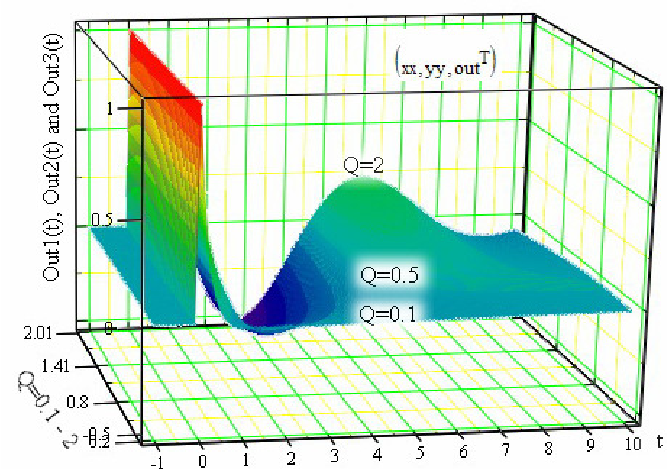

Fig. 10. 3D Normalized step response of a loudspeaker at Q=0.1÷2, according to Eqs. (8), (9) and (10).

CONCLUSION

The value of the total Q factor Q=0.5 appears to be critical for the analysis of the loudspeakers step response. The sound pressure created can be described with equation (9), and Fig. 9 shows the normalized step response.

The loudspeaker step response at Q>0.5 is with specific damped oscillations with the frequency of the mechanical resonance. Real world high-quality loudspeakers with a total Q factor of (0.5<Q<2) are hardly ever offered by producers.

Values of the Q factor within 0.1<Q<0.5 are typical for the real world high- and middle-frequency dome and low-frequency loudspeakers (see Table 1). Their step response can be described with Eq. (10), and the normalized step responses in Fig. 8 and Fig. 10 are marked with aperiodic damping of the signal.

The results obtained in this work can be used for theoretical analysis, design and production of loudspeakers, closed-box systems, etc.

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