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Noise Characteristics for Amplifier Model with a Thevenin Source

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Abstract - **Accurate noise analysis is a prerequisite for low-noise circuit design. In this paper, noise analysis for V_n–I_n amplifier model with a Thevenin source is presented. Noise characteristics, such as signal – to – noise ratio, noise factor, noise figure and noise temperature, as well as, the optimum source impedance, which minimizes the noise factor, are obtained. MATLAB simulation results are presented.**

Keywords – **amplifier noise model, noise factor, noise figure, noise temperature, signal – to – noise ratio.**

I. INTRODUCTION

Noise is one of the most important factors affecting the operations of communications circuits. That's why the main objective of noise analysis is to design low-noise circuits.

When designing an amplifier for a specific application, there are many characteristics to be met and decisions to be made. They include gain, bandwidth, impedance levels, feedback, stability, dc power, cost and signal - to - noise ratio (SNR) requirements. Usually requirements low - noise design contradict to the other important circuit parameters such as bandwidth, gain, input/output characteristics, etc. The low-nose design is different for each specific case, but amplifier designers can elect one of two paths [1]:

- The wrong approach is to worry about the gain and bandwidth first and later in the design power they check for the noise.
- Alternatively, one can design the system with initial emphasis on noise performance

Although there are many low-noise devices available they do not perform equally for all signal sources.

To obtain the optimum noise performance, it is necessary to select the proper amplifying device (BJT, FET, or IC) and operating point for the specific input source.

In order to predict the noise behavior of an amplifier correctly, accurate noise models are required. Without them, the design and optimization of an amplifier's noise characteristics cannot be successful.

If it is necessary to study a circuit's noise in terms of the physical phenomena, the Van der Ziel models [2] which are well accepted models and have a solid physical basic, can be used. But for amplifier design purposes, both amplifier noise models, with a Thevenin or with a Norton input source, are more appropriate.

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II. NOISE CHARACTERISTICS FOR AMPLIFIER MODEL WITH A THEVENIN SOURCE

The amplifier model with a Thevenin noise input source, described in [3], is shown in Fig.1, where the control voltage Vi is the voltage across Zi.

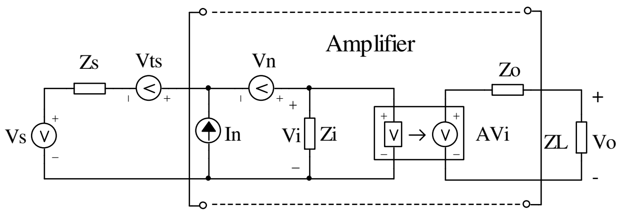

Fig. 1. Vn – In amplifier model with Thevenin source.

A. *Signal – to – Noise Ratio*

If the amplifier is noiseless, the signal – to - noise ratio is given by $SNR = v_s^2 / v_{ts}^2$, where v_s^2 is the mean-square source voltage and v_{ts}^2 is the mean-square thermal noise voltage generated by the source impedance. When the amplifier noise is included, the signal – to - noise ratio is $SNR = v_s^2 / v_{ni}^2$. The total mean-square noise voltage v_{ni}^2 at the input of the amplifier can be written as

$$v_{ni}^2 = 4kTR_S\Delta f + v_n^2 + 2v_n i_n \,\mathrm{Re}(cZ_S^*) + i_n^2 |Z_S|^2 \quad (1)$$

where $Z_S = R_S + jX_S$ is the source impedance and $c = c_r + jc_i$ is the correlation coefficient between noise sources V_n and I_n.

Thus, the signal – to – noise ratio can be determined as

$$SNR = \frac{v_s^2}{v_{ni}^2} =$$

$$= \frac{v_s^2}{4kTR_S\Delta f + v_n^2 + 2v_n i_n \,\mathrm{Re}(cZ_S^*) + i_n^2 |Z_S|^2}. \quad (2)$$

The signal – to – noise ratio is maximized by minimizing v_{ni}^2. The source impedance which minimizes v_{ni}^2 can be obtained by setting $\partial v_{ni}^2 / \partial R_S = 0$ and $\partial v_{ni}^2 / \partial X_S = 0$ and solving for R_S and X_S. The solution for R_S is

$$Rs = -\frac{2kT\Delta f + v_n i_n}{i_n^2} \; . \tag{3}$$

It is negative. Because this is no realizable, $R_S = 0$ is the realizable solution for the least noise. The solution for X_S is

$$X_S = -c_i \frac{v_n}{i_n} \; . \tag{4}$$

Therefore, the source impedance which minimizes the total noise input voltage v_{ni}^2, is given as

$$Z_S = R_S + jX_S = 0 - jc_i \frac{v_n}{i_n} \; . \tag{5}$$

Because minimum noise occurs for $R_S = 0$, it can be concluded that a resistor should never be connected in series with a source at the amplifier input if noise performance is a design criterion. If a series resistor is required, e.g. for stability, it should be much smaller than R_S. Although the output impedance of a source is usually fixed, the signal – to – noise ratio can be improved by adding a reactance in series with the source which makes the total series reactance equal to the imaginary part of Z_S in Eq. (5). When this is the case, after substituting Eq. (5) into Eq. (1), the equivalent noise input voltage can be determined by

$$v_{ni}^2 = 4kTR_S\Delta f + v_n^2\left(1-c_i^2\right) + 2c_r R_S v_n i_n + i_n^2 R_S^2 \; . \tag{6}$$

B. Noise Factor and Noise Figure

Dividing the noiseless amplifier signal – to – noise ratio by noise amplifier signal – to – noise ratio gives the noise factor as

$$F = \frac{v_s^2 / v_{ts}^2}{v_s^2 / v_{ni}^2} = \frac{v_{ni}^2}{v_{ts}^2} =$$

$$= 1 + \frac{v_n^2 + 2v_n i_n \operatorname{Re}(cZ_S^*) + i_n^2 |Z_S|^2}{4kT_0 R_S \Delta f} \tag{7}$$

where T_0 is the standard temperature.

It follows from this expression that a noiseless amplifier has the noise factor $F = 1$. A common way of presenting the noise factor is the so-called noise figure N.

$$N = 10\log(F) \; . \tag{8}$$

Eqs. (7) and (8) give a first hint of why a low noise figure is considered important, and why a high gain is often associated with a low noise figure. Noise figure is, however, often overstated in its importance in a low noise system design. It is the correct balance between noise figure and gain that is important rather than a low noise figure itself. It may not be obvious, but an amplifier with a noise figure (in dB) numerically greater than its gain (in dB) is not useful. Indeed, it can be proven that there exists a combination of passive components which would perform better than an amplifier with lower gain than noise factor.

Often, it is convenient to express F in terms of the amplifier noise resistance R_n, noise conductance G_n and the correlation impedance Z_c. These are related to v_n^2, i_n^2, and c by [4], [5]:

$$R_n = \frac{v_n^2}{4kT_0\Delta f} \, , \tag{9}$$

$$G_n = \frac{i_n^2}{4kT_0\Delta f} \; , \tag{10}$$

$$Z_c = R_c + jX_c = c\frac{v_n}{i_n} = \left(c_r + jc_i\right)\frac{v_n}{i_n} \, . \tag{11}$$

R_n, and G_n, respectively, represent normalized values of v_n^2 and i_n^2, where the normalization factor is $4kT_0\Delta f$.

If the amplifier noise parameters are expressed in terms of R_n, G_n and c, the noise factor is given by

$$F = \frac{v_{ni}^2}{v_{ts}^2} = 1 + \frac{v_n^2 + 2v_n i_n \operatorname{Re}\left(cZ_S^*\right) + i_n^2 |Z_S|^2}{4kT_0 R_S \Delta f} =$$

$$= 1 + \frac{R_n + G_n\left[2\left(R_S R_c + X_S X_c\right) + \left(R_S^2 + X_S^2\right)\right]}{R_S} \, . \tag{12}$$

The optimum source impedance which minimizes the noise factor F is obtained by setting $\partial F / \partial R_S = 0$ and $\partial F / \partial X_S = 0$ and solving for R_S and X_S. It follows from $\partial F / \partial R_S = 0$ that

$$G_n R_S^2 - 2G_n X_S X_c - G_n X_S^2 - R_n = 0 \, . \tag{13}$$

Solving Eq. (13) for R_S gives

$$R_{Sopt} = \sqrt{1 - c_i^2}\,\frac{v_n}{i_n} \, . \tag{14}$$

From $\partial F / \partial X_S = 0$ it follows that

$$2G_n X_c + 2G_n X_S = 0 \, , \text{ i.e.,}$$

$$X_{Sopt} = -X_c \, . \tag{15}$$

Therefore, the optimum source impedance is obtained as

$$Z_{opt} = R_{opt} + jX_{opt} = \left[\sqrt{1 - c_i^2} - jc_i\right]\frac{v_n}{i_n} =$$

$$= \sqrt{\frac{R_n}{G_n} - X_c^2} - jX_c \, . \tag{16}$$

It follows from Eq. (16) that the imaginary part of Z_{opt} is equal to the imaginary part of Z_S in Eq. (5), which minimizes the signal – to – noise ratio. The corresponding minimum value of the noise factor can be obtained by replacing R_S and X_S in Eq. (12) with R_{Sopt} and X_{Sopt}, respectively. In this case the optimum noise factor is given by

$$F_{\min} = 1 + \frac{v_n i_n}{2kT_0 \Delta f}\left(c_r + \sqrt{1-c_i^2}\right) = = 1 + 2G_n\left(R_c + R_{opt}\right). \quad (17)$$

The noise factor can also be expressed in terms of $F_{\min}$ and Z_{opt}. It follows from Eqs. (12) and (17) that the difference $F - F_{\min}$ is given by

$$F - F_{\min} = \frac{R_n + 2G_n\left(R_c R_S + X_c X_S\right) + G_n\left(R_S^2 + X_S^2\right)}{R_S} - 2G_n\left(R_c + R_{opt}\right) = = \frac{R_n - 2G_n\left(R_{opt} R_S + X_{opt} X_S\right) + G_n\left|Z_S\right|^2}{R_S} \quad (18)$$

where $X_c = -X_{opt}$ has been used. Adding and subtracting the term $G_n\left(R_{opt}^2 + X_{opt}^2\right) = G_n\left|Z_{opt}\right|^2$ can complete the square in the nominator of this expression. This leads to the equation

$$F_{\min} = \frac{R_n + G_n\left[\left(R_S - R_{opt}\right)^2 + \left(X_S - X_{opt}\right)^2\right]}{R_S} - - \frac{G_n\left|Z_{opt}\right|^2}{R_S}. \quad (19)$$

By substituting for $\left|Z_{opt}\right|^2$ as

$$\left|Z_{opt}\right|^2 = \frac{R_n}{G_n} \quad (20)$$

in Eq. (19), the noise factor is obtained as

$$F = F_{\min} + \frac{G_n}{R_S}\left[\left(R_S - R_{opt}\right)^2 + \left(X_S - X_{opt}\right)^2\right] = = F_{\min} + \frac{G_n}{R_S}\left|Z_S - Z_{opt}\right|^2. \quad (21)$$

It is clear from Eq. (21) that the amplifier's noise behaviour is completely characterised by the optimum source impedance Z_{opt} which results to a minimum noise factor $F_{\min}$, and the noise conductance G_n describing how rapidly the noise factor increases for a deviation from the optimum source admittance.

C. The Noise Factor Fallacy

The noise factor can be misleading characteristic. If an attempt is made to minimize F by adding resistor in series with the source at the input of an amplifier, the signal – to – noise ratio is always decreased. This is referred to as the noise factor fallacy or the noise figure fallacy. Potential confusion can be avoided if low - noise amplifiers are designed to maximize the signal – to – noise ratio. This is accomplished by minimizing the equivalent noise input voltage.

D. Noise Temperature

The internal noise generated by an amplifier can be expressed as an equivalent input – termination noise temperature. When the source is represented by a Thevenin equivalent circuit, as is shown in Fig. 1, the noise temperature T_n is the temperature of the source resistance that generates a thermal noise voltage equal to the internal noise generated in the amplifier when referred to its input. For the Thevenin source, the noise temperature is defined by

$$4kT_n R_S \Delta f = v_n^2 + 2v_n i_n \operatorname{Re}\left(cZ_S^*\right) + i_n^2\left|Z_S\right|^2 \quad (22)$$

where $R_S = \operatorname{Re}\left(Z_S\right)$. It follows that the noise temperature is

$$T_n = \frac{v_n^2 + 2v_n i_n \operatorname{Re}\left(cZ_S^2\right) + i_n^2\left|Z_S\right|^2}{4kR_S \Delta f}. \quad (23)$$

The noise temperature is related to the noise factor by

$$T_n = \left(F - 1\right)T_0. \quad (24)$$

III. EXAMPLE

A few simulations for the amplifier noise model, shown in Fig. 1, have been performed in MATLAB. Some of noise characteristics, obtained with $v_n = 2nV$, $i_n = 10pA$, and $c = 0{,}1$, are presented in Figs. 2 and 3.

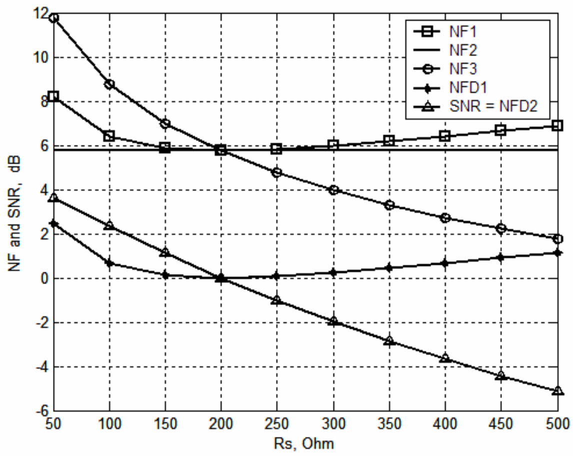

Fig. 2. Variations in SNR and NF over the source resistance range from $R_S = 50\Omega$ to $R_S = 500\Omega$

In Fig. 2 the noise figure $NF1$ for a given source resistance R_S is $NF1 = 10\log(v_{ni}^2(R_S)/v_{ts}^2(R_S))$. If a resistance is added in series with the source impedance to minimize the noise factor, the new noise figure can be expressed as $NF2 = 10\log(v_{ni}^2(R_{Sopt})/v_{ts}^2(R_{Sopt}))$ with R_{Sopt} given by Eq. (14) with $c_i = 0$. The decrease in the noise figure can be found as $NFD1 = NF1 - NF2$, and the dB decrease in SNR as $SNR = 10\log(v_{ni}^2(R_{Sopt})/v_{ni}^2(R_S))$.

The simulation results in Fig. 2 illustrate how the noise figure appears to be decreased by adding resistance in series with an amplifier input. However, the SNR is lowered. The fallacy comes from treating the added resistance as part of the source rather than part of the amplifier. In reality, the added resistor increases the amplifier noise, but the source noise remains constant. Because of that, the correct way to determine the noise figure $NF3$ with the added resistor is $NF3 = 10\log(v_{ni}^2(R_{Sopt})/v_{ts}(R_S))$. In this case the noise figure decreases by $NFD2 = NF3 - NF1$. Thus, this is the same decrease as the dB decrease in the SNR.

In Fig. 3 the noise temperature waveforms, that follow the noise factors $F1$, $F2$ and $F3$ changes, are presented.

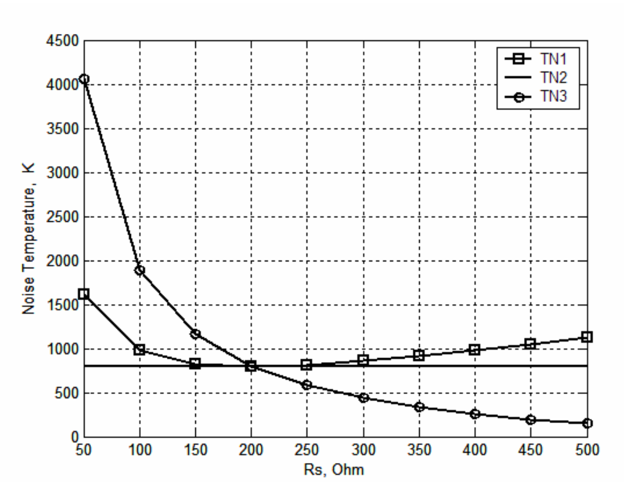

Fig. 3. Variations in noise temperature over the source resistance range from $R_S = 50\Omega$ to $R_S = 500\Omega$

The results in Figs. 2 and 3 show that the minimum noise factor (noise figure) do not correspond to the maximum signal – to – noise ratio. It can also be concluded that if a series resistor must be included at the amplifier input, its value should be much smaller than the source impedance.

IV. CONCLUSION

A simple but effective approach to determine the noise characteristics for amplifier model with a Thevenin source is developed. Expressions for signal – to– noise ratio, for noise factor (noise figure) and noise temperature are derived. The possibilities to improve the noise characteristics are studied. The effect of series resistor, added at the amplifier input, on the noise characteristics is shown. The noise factor (noise figure) fallacy due to the added resistor is commented. The correct way to determine the noise characteristics for this case is proposed. The approach for analysis can be used for low - noise amplifiers design.

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