Overview of COST 273 Part II: Parabolic equation method application

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***Abstract* – This paper summarizes the application of the parabolic equation method in COST 273 activities. The parabolic equation method has been applied to solve microwave propagation prediction problems over irregular terrain and in forest environment.**

***Keywords* – Microwave propagation prediction, parabolic equation, Digital Terrain Elevation Data, forest.**

I. INTRODUCTION

The Parabolic Equation (PE) method is one of the deterministic propagation methods used in COST 273 activities. It is based on a paraxial approximation to the wave equation. The PE is a full-wave method accounting simultaneously and accurately for the wave diffraction, refraction and scattering propagation mechanisms. The advantage of the PE in comparison to the wave equation is that, combined with a numerical technique, it can be easily marched in range provided the field is known on an initial plane and adequate boundary conditions on the scattering objects and at the outer boundaries of the integration domain are given. This has turned the PE method into one of the most widely used propagation prediction techniques for large classes of wave propagation problems.

In fulfilment of the IE-BAS commitment to the COST 273 initiative, that was the investigation of microwave propagation in strongly varying environment, the 2D version of this method has been applied firstly to the study of the influence of tropospheric ducting phenomenon on microwave propagation, [1], [2]. Indeed, ducting layers of different nature are present significant percentage of the time all over the world, especially in coastal zones, and hence being able to compute the propagation in those circumstances is important. The following investigations using PE method have been made: study of the surface-based ducts influence in short ranges (up to 3 km) [1], study of the path loss changes provoked by changes in duct parameters including range dependent ducting (corresponding to the refractivity profiles along a mixed land-sea path) [2], assessment of the combined effect of terrain and ducting on path losses at UMTS frequencies [3]. These Temporary Documents (TDs) have been summarized in [4].

This paper is based essentially on TDs [5] and [6]. The first TD reports results on the combination of the PE method with digital terrain data used when path loss is calculated in an open area. The second deals with wave propagation over a hilly terrain containing a forest edge in its end. In the next Section these TDs PE method is very briefly summarized.

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II. PE METHOD DESCRIPTION

The standard 2D narrow-angle forward propagating parabolic equation is given by Eq. (1), [7]:

$$\frac{\partial u(x,z)}{\partial x} = \frac{i}{2k}\frac{\partial^2 u(x,z)}{\partial z^2} + \frac{ik}{2}\left(n^2(x,z)-1\right)u(x,z), \qquad (1)$$

where azimuthal symmetry is assumed, k is the free-space wave number, n is the refractive index of the troposphere, $u(x,z)$ is the reduced function related to an electromagnetic field component Ψ as: $\Psi(x,z) = u(x,z)exp(ikx)/(kx)^{1/2}$. To solve Eq. (1) different numerical techniques are applied. For elevated terrain study in [5] we made essentially use of the split-step Fourier technique, [7]. This technique searches the solution of the Fourier transform of Eq. (1) where the transform is performed while treating the term $(n^2(x,z)-1)$ as constant:

$$U(x,p) = \Im\{u(x,z)\} \equiv \int_{-\infty}^{\infty} u(x,z)e^{-ipz}dz, \qquad (2)$$

$$U(x+\Delta x,p) = U(x,p)e^{i(-p^2/2k+kM10^{-6})\Delta x}, \qquad (3)$$

where Eq. (2) defines the transform with the transform variable p referred to as the vertical wave number and the solution of the transformed equation at $x+\Delta x$ in terms of the solution at x is given by Eq. (3). Then the inverse transform is applied to Eq. (3) to obtain $u(x+\Delta x,z)$:

$$u(x+\Delta x,z) = e^{ikM(z)10^{-6}\Delta x}\Im^{-1}\left\{e^{-j(p^2/2k)\Delta x}\Im\{u(x,z)\}\right\}. \qquad (4)$$

Equations (3)-(4) have been obtained under the assumption that the refractive index varies only with height and under the Earth-flattening concept, [8], that is, $n(z)$ in Eq. (1) is replaced by the modified refractive index $m(z)$ ($m(z)=n(z)+z/a_e$, with a_e - the Earth radius; $M=(m-1)10^6$ is the modified refractivity).

The narrow-angle PE given by Eq. (1) is very accurate at propagation angles within $\pm 15^0$ of the preferred direction of propagation, [9]. Equation (1) holds for both (horizontal and vertical) polarizations, the difference between them being contained in the boundary conditions at the Earth surface. Perfect conductivity of the ground has been assumed in this paper. The initial field required to start the calculation

procedure is provided by horizontally polarized Gaussian beam source with pattern factor:

$$F(\theta) = \exp\left[\frac{\ln(0.707)(\theta - \theta_s)^2}{(\theta_0/2)^2}\right], \qquad (5)$$

where θ_0 and θ_s are the half power beamwidth and the antenna elevation angle.

To model the vertical profile $M(z)$ of the modified refractivity the standard troposphere is used. The standard troposphere is modeled by a modified refractivity gradient that increases monotonically at a rate of 0.118 M-units per m. The results are presented in the form of path loss (PL, in dB):

$$PL = 20\log\left(\frac{4\pi r}{\lambda}\right) - PF, \qquad (6)$$

where λ is the free-space wavelength, r is the distance between the corresponding points and PF is the pattern propagation factor as defined in [8].

The authors of TD [6] have applied the finite difference (FD) numerical technique to solve Eq. (1). This technique may be found in [9] and will not be given here.

III. Results and Discussion

Relying on the accuracy of the PE method and its ability to provide quantitative field assessment, TD [5] compares results of path loss obtained using PE in combination with two Digital Terrain Elevation Data (DTED) collections for terrain description. The examples are for a specific region in the plateau of Sofia. The two terrain data collections we made use of are those of the DTED Level 0 provided by USA National Imagery and Mapping Agency (NIMA). These are standardized digital datasets, the one providing the minimal heights, the other containing the maximal heights, taken in 30 arc second square areas (nominally one kilometre). These data are publicly available and give near world wide coverage. The used polarization is horizontal. Antenna half power beamwidth θ_0=2^0 is used with different antenna tilt. The other parameters of the problem concerning path loss calculation are: frequency f=2 GHz, transmitter height h_t=30 m and receiver situated in the first two meters above the ground.

Fig. 1 shows the region of Sofia plateau with the surrounding mountains, the red line indicating one of the studied paths. Below are seen the terrain elevation profiles for this path as obtained from the DTED Level 0 maximal and minimal heights datasets, respectively, with the two Fresnel zones indicated on the upper profile. The difference between the two elevation profiles is not only in the absolute height values but, especially, in the shadowed regions (the red parts of the curves). Figs. 2-4 show path loss for θ_s=-3^0, θ_s=-2^0 and θ_s=-1^0, respectively, obtained from the two DTED datasets. As it is seen from these figures, in the vicinity of the path loss shadow peaks obtained from the DTED maximal heights dataset there is drastic difference between the two path loss

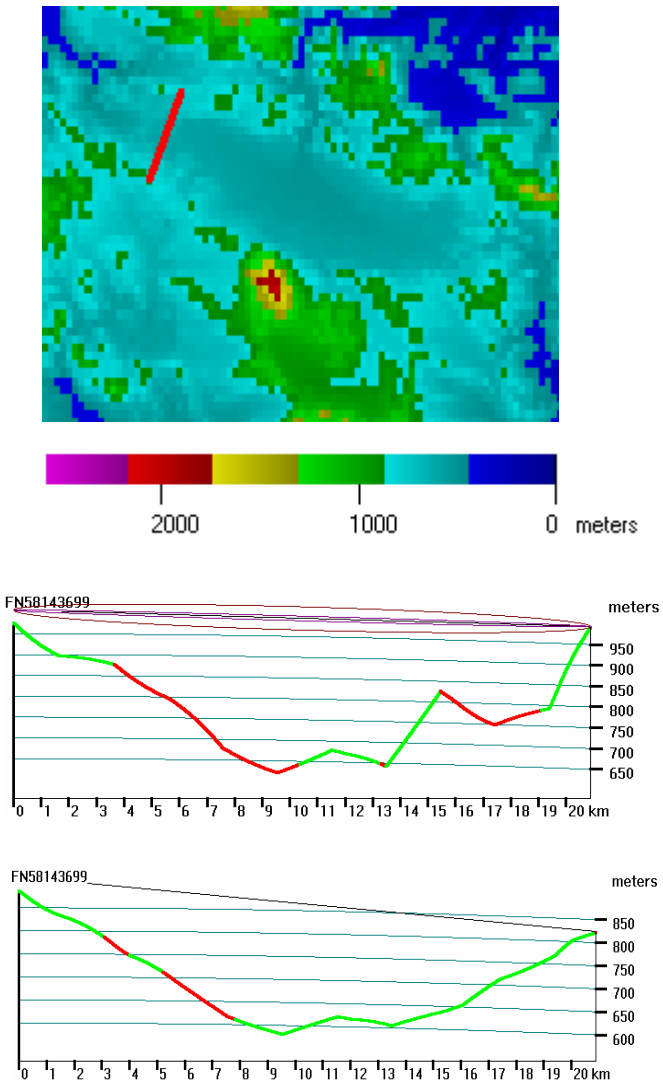

Fig. 1. Region of Sofia with the studied path loss indicated and the two terrain profiles for it obtained from DTED maximal (upper) and minimal (lower) heights dataset

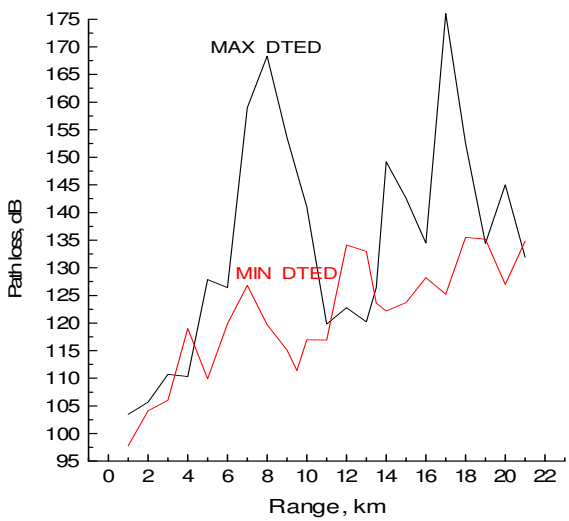

Fig. 2. Antenna tilt $\theta_s = -3^0$

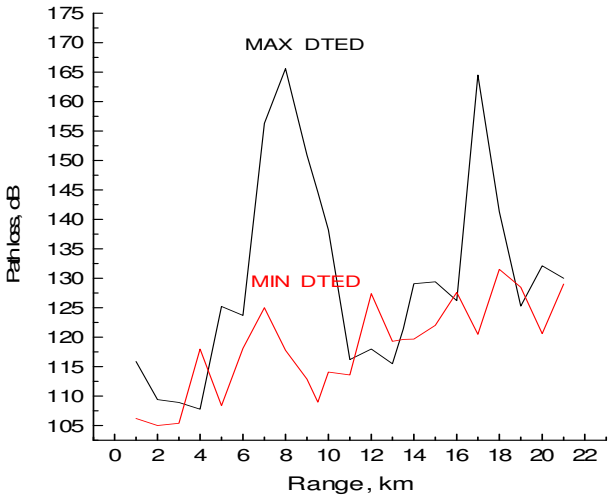

Fig. 3. Antenna tilt $\theta_s = -2^0$

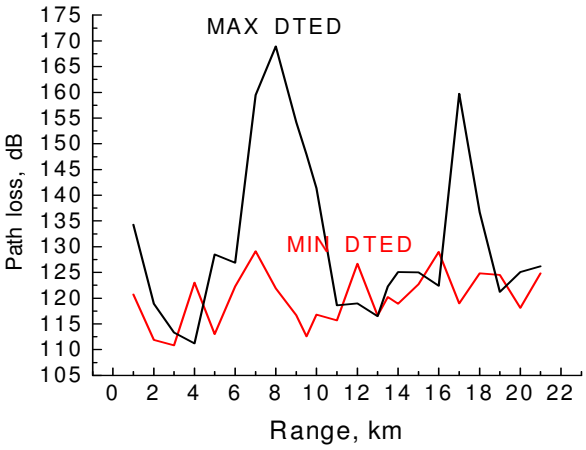

Fig. 4. Antenna tilt $\theta_s = -1^0$

curves. This difference can not be overcome with changes in the antenna tilt. It is not possible to obtain such quantitative results for path loss in the shadowed regions with empirical or simple deterministic propagation methods. The PE method provides the possibility to make an accurate assessment/prediction of the field strength in an early stage, when the coverage is calculated, thus giving information of power needed and possible interference.

In [6] the 2D PE method combined with FD numerical scheme is applied to model the propagation over irregular terrain partly covered by fir forest. The PE based results are compared to the Geometric Theory of Diffraction (GTD) technique and to measurements. The geometry and parameters of the problem are shown on Figs. 5 and 6. Measurements were made with the antennas located in the open area at three different distances behind the forest: 9 m, 51 m, and 109 m, see Fig. 5. For each of these antenna positions, the antenna heights were varied between 6 and 25 m in steps of about 2 m. When applying PE method, Leontovitch boundary condition for the ground was used and the forest was modeled as a dielectric slab characterized by height (18 m) and complex dielectric constant, i.e., relative permittivity ε_r (1.004) and conductivity σ (180 μS/m.). For the GTD model, the forest is modeled as two wedges with the interior angle 90°, see also Fig. 6. Both the wedges and the ground are modeled as perfectly conducting. This means that, for the PE model, some waves will penetrate through the forest while for the GTD model, the forest is a nonpenetrable object. These differences in applying the two models may partially explain the differences in transmission loss obtained through them and shown on Figs. 7 and 8.

The results for horizontally polarised links at 1.3555 and 1.5995 GHz obtained in TD [6] (two examples of them reported on Figs. 7 and 8) show that transmission loss obtained by GTD approximation is in general up to 20 dB worse that the measured cases for low receiver heights. The 2D PE solution agrees in general within ± 5 dB with the measured values. These results indicate also that the main contribution to the field near the forest is transmitted through the forest rather than diffracted over it. However, for larger distances from the forest edge the diffracted field gives the main contribution and the PE method and GTD give similar results.

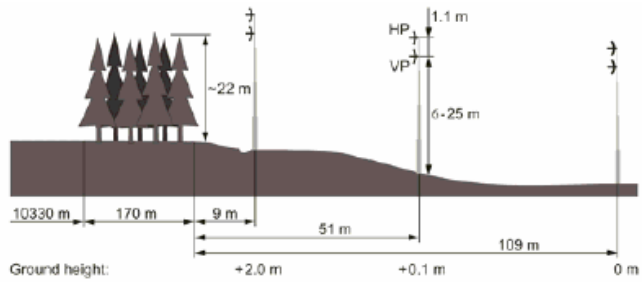

Fig. 5. The receiver antenna locations. (Fig. taken from [6])

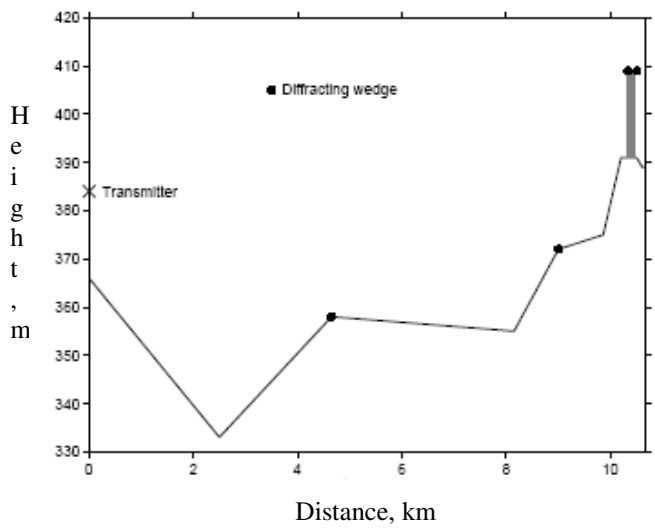

Fig. 6. Terrain height profile used for the PE and GTD calculations. The dots show the choice of the diffracting wedges in the GTD model. The forest is the gray area at the end of the path. (Fig. taken from [6])

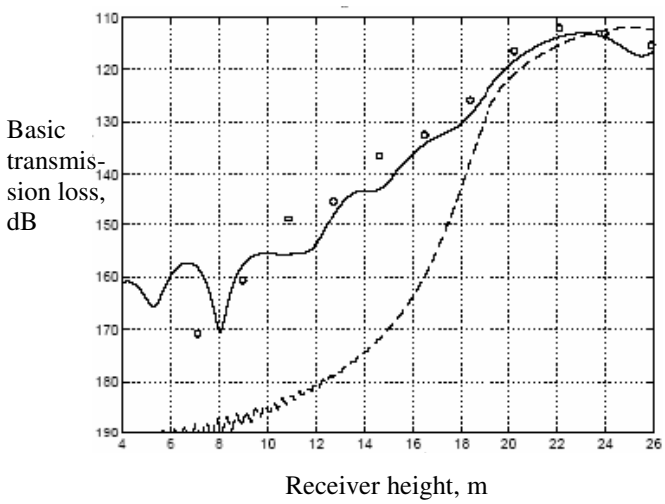

Fig. 7. Basic transmission loss vs receiver height for the frequency 1355.5 MHz and at the distance 9 m from the forest edge. The solid line shows calculations with the PE method, the dashed line is for GTD, the measurements are marked with o. (Fig. taken from [6])

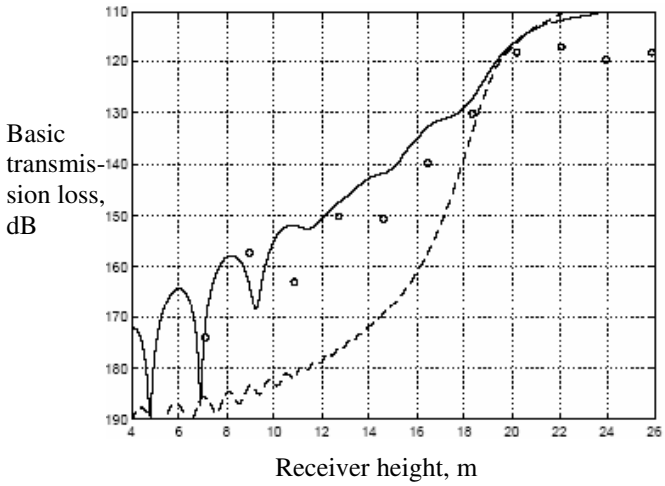

Fig. 8. The same as in Fig. 7 except that the frequency is 1599.5 MHz. (Fig. taken from [6])

It is to note also that another participant in COST 273, the AWE Communications GmbH, which is a spin off from the Stuttgart University, has already implemented the PE method among the other widely used propagation prediction methods in its planning tool, WinProp, [10]. The PE method is used especially for accurate propagation assessment in rural and suburban areas.

IV. Conclusion

This paper presents results obtained in COST 273 with the parabolic equation method applied to the prediction of the microwave propagation in complicated environments. Demonstrated is the ability of the PE method to provide accurate quantitative field assessment and its applicability to some difficult to solve problems as propagation through forest.

Acknowledgements

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