Input-output linearization control of induction motors with load torque compensation

Stanislav E. Enev

Abstract: **An input-output linearizing control for the third order induction motor model described in the fixed stator frame, along with a simple scheme for identifying and compensating the load torque is presented in the paper. The load torque identification algorithm is designed based on rotor speed information. A simulation study of the proposed control algorithm is presented.**

Keywords: **induction motor, input-output linearization control.**

I. Introduction

The induction motor is probably the most widely used electric machine in industrial applications due to its reliability, ruggedness and relatively low cost. Its control however presents an extreme challenge because of the highly complicated nonlinear dynamics of the machine. These two reasons make it very attractive for control researchers and practitioners. A lot of approaches to this problem can be found in the literature. The first solutions gave the so-called field-oriented control [5],[8], which consists of rewriting the equations of the motor through a nonlinear transformation in order to decouple the rotor flux and the rotor speed. The disadvantage of this method is that the decoupling is valid only after the flux is constant.

The control algorithm investigated here is based on the feedback linearization of the induction motor. A good introduction to the exact linearization by means of a state feedback can be found in [10],[11],[12]. A lot works based on this approach can be found in the literature. In [3] and [4], dynamic feedback linearizing transformations are presented, enabling the full state linearization of the induction motor. However the most commonly targeted for control variables i.e. the rotor speed and the rotor flux magnitude are well "hidden" in the new transformed states, so that the advantage of the linear behavior of the system is reduced. Input-output linearizing transformations for the induction motor, with system outputs the rotor speed and the rotor flux magnitude, are found in [1], [2], [5], [9]. Using this approach, the stability of the so-called zero dynamics (in the motor dynamics remains a nonlinear part, made unobservable by the introduced feedback) must be guaranteed. The advantage of the input-output linearization approach over the field-oriented control is the fact that, by applying the linearizing transformation, a complete decoupling of the rotor speed and flux is achieved, which enables the optimization of the power efficiency of the motor without degradation of the speed regulation. However, the exact cancellation of the nonlinear terms is possible only when perfect knowledge of the motor parameters and the load torque is available. Generalized algorithms for adaptive control of feedback linearizable systems are proposed in [11]. In [9], an adaptive input-output linearizing control is designed for a fifth-order model of the motor (voltage-command mode), including algorithms for identification of the load torque and the rotor resistance, which are assumed to be constant.

Here, an input-output linearizing control for the third order (current-fed) induction motor model described in the fixed stator frame, along with a simple scheme for identifying and tracking the load torque, which enters the system as a disturbance is presented and investigated.

II. Dynamic modeling of the induction motor

The induction motor considered here is a three-phase stator, three-phase short circuited rotor machine. Since a squirrel-cage rotor can be represented as a three-phase short-circuited one by means of a simple transformation, the following considerations are valid for this case too. The common assumptions are adopted i.e. symmetrical construction, linearity of the magnetic circuits, sinusoidal distribution of the field in the air-gap. After a series of transformations, the following two-phase equivalent model with all state variables, i.e. the stator currents $I_{S\alpha}$, $I_{S\beta}$, the rotor fluxes $\Psi_{R\alpha}$ $\Psi_{R\beta}$, expressed in the fixed α-β stator frame is obtained:

$$\begin{aligned}
\dot{\omega} &= \mu(\Psi_{R\alpha} I_{S\beta} - \Psi_{R\beta} I_{S\alpha}) - c/J\omega - \tau_L / J \\
\dot{I}_{S\alpha} &= -\gamma I_{S\alpha} + \eta\zeta\Psi_{R\alpha} + \zeta n_p \omega \Psi_{R\beta} + V_{S\alpha}/(\sigma \bar{l}_S) \\
\dot{I}_{S\beta} &= -\gamma I_{S\beta} + \eta\zeta\Psi_{R\beta} - \zeta n_p \omega \Psi_{R\alpha} + V_{S\beta}/(\sigma \bar{l}_S) \\
\dot{\Psi}_{R\alpha} &= -\eta\Psi_{R\alpha} - n_p \omega \Psi_{R\beta} + \eta m I_{S\alpha} \\
\dot{\Psi}_{R\beta} &= -\eta\Psi_{R\beta} + n_p \omega \Psi_{R\alpha} + \eta m I_{S\beta}
\end{aligned} \quad (1)$$

with

$$\mu = n_p m/(\bar{l}_R J),\ \eta = r_R/\bar{l}_R,\ \zeta = m/(\sigma \bar{l}_R \bar{l}_S),$$
$$\sigma = (\bar{l}_R \bar{l}_S - m^2)/\bar{l}_R \bar{l}_S,\ \gamma = (\bar{l}_R^2 r_S + m^2 r_R)/(\sigma \bar{l}_R^2 \bar{l}_S).$$

where: $l_{S(R)}$ - the stator(rotor) windings inductances, $r_{S(R)}$ - the stator(rotor) windings resistances, $m_{S(R)}$ - mutual inductances between the stator (rotor) windings, m_0 - mutual inductance between stator and rotor windings, $m = 3/2m_0$,

Stanislav E. Enev, Ph.D. Student, French Language Department of Electrical Engineering,Technical University of Sofia, 8 Kl. Ohridski Str., 1000 Sofia, BULGARIA, e-mail: sta_enev@yahoo.com

$\bar{l}_S = l_S - m_S$, $\bar{l}_R = l_R - m_R$, J - rotor moment of inertia, τ_L - a load torque, c - friction coefficient, n_p - number of pole-pairs. The complete derivation of the model can be found in [2],[5],[6],[7].

By using fast control loops (generally implemented with high-gain PI controllers), the currents are forced to follow a reference trajectory. Thus, if the tracking is fast enough, the current dynamics equations can be neglected and one can achieve current command of the motor, with $I_{S\alpha}$ and $I_{S\beta}$ the new inputs. The system can be put in the general form $\dot{x} = f(x) + g_1(x)u_1 + g_2(x)u_2$:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -c/Jx_1 - \tau_L/J \\ -\eta x_2 - n_p x_1 x_3 \\ -\eta x_3 + n_p x_1 x_2 \end{bmatrix} + \begin{bmatrix} -\mu x_3 \\ \eta m \\ 0 \end{bmatrix} .u_1 + \begin{bmatrix} \mu x_2 \\ 0 \\ \eta m \end{bmatrix} .u_2 \quad (2)$$

with

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \omega \\ \Psi_{R\alpha} \\ \Psi_{R\beta} \end{bmatrix}, \quad u_1 = I_{S\alpha}, \quad u_2 = I_{S\beta}.$$

III. Input-output linearization

For a 2-by-2 system of the form:

$$\begin{aligned} &\dot{x} = f(x) + g_1(x)u_1 + g_2(x)u_2 \\ &y_1 = h_1(x) \\ &y_1 = h_2(x) \end{aligned} \quad , \quad (3)$$

with y_1 and y_2 the outputs, the feedback linearizing control law is given by:

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = A(x)^{-1} \begin{bmatrix} v_1 - L_f^{\gamma_1} h_1 \\ v_2 - L_f^{\gamma_2} h_2 \end{bmatrix}, \quad (4)$$

where $A(x) = \begin{bmatrix} L_{g_1} L_f^{\gamma_1 - 1} h_1 & L_{g_2} L_f^{\gamma_1 - 1} h_1 \\ L_{g_1} L_f^{y_2 - 1} h_2 & L_{g_2} L_f^{\gamma_2 - 1} h_2 \end{bmatrix}$ - decoupling matrix.

$L_* h$ stands for the Lie derivative of the scalar function h with respect to the corresponding vector field.

As long as the decoupling matrix is non-singular, the feedback law transforms the system into two decoupled differential equations representing two chains of integrators, v_1 and v_2 being the new inputs

$$y_1^{(\gamma_1)} = v_1 \text{ and } y_2^{(\gamma_2)} = v_2. \quad (5)$$

The zero dynamics is given by:

$$\dot{x} = f(x) - \begin{bmatrix} g_1 & g_2 \end{bmatrix} A(x)^{-1} \begin{bmatrix} L_f^{\gamma_1} h_1 & L_f^{\gamma_2} h_2 \end{bmatrix}^T. \quad (6)$$

For the case of the third order induction motor model, we define the outputs as follows:

$$h_1 = x_1, \; h_2 = x_2^2 + x_3^2. \quad (7)$$

We have the following:

$$L_f h_1 = -c/Jx_1 - \tau_L/J, \; L_{g_1} h_1 = -\mu x_3, \; L_{g_2} h_1 = \mu x_2$$
$$L_f h_2 = -2\eta(x_2^2 + x_3^2), \; L_{g_1} h_2 = 2\eta m x_2, \; L_{g_2} h_2 = 2\eta m x_3.$$

The decoupling matrix is given by:

$$A = \begin{bmatrix} -\mu x_3 & \mu x_2 \\ 2\eta m x_2 & 2\eta m x_3 \end{bmatrix}. \quad (8)$$

Since $\det(A) = -1/(2\eta m \mu(x_2^2 + x_3^2))$, the matrix is nonsingular as long as the rotor flux is not zero.

Applying the state feedback control law:

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} -\mu x_3 & \mu x_2 \\ 2\eta m x_2 & 2\eta m x_3 \end{bmatrix}^{-1} \begin{bmatrix} v_1 + c/Jx_1 + \tau_L/J \\ v_2 + 2\eta(x_2^2 + x_3^2) \end{bmatrix} \quad (8)$$

the system is transformed into the following two equations:

$$\frac{dx_1}{dt} = v_1, \frac{d}{dt}(x_2^2 + x_3^2) = v_2. \quad (9)$$

The zero dynamics is stable.

IV. Load torque identification

In the case, when the value of the load torque is not exactly known, in the presence of the linearizing control law, the rotor speed equation is given by the following:

$$\frac{dx_1}{dt} = \dot{y}_1 = \frac{(\tau_{LR} - \tau_L)}{J} + v_1, \quad (10)$$

where τ_{LR} - the load torque value fed to the linearizing controller. We can define the linearizing controller load torque as:

$$\tau_{LR} = k_L(y_{1ref} - y_1), \quad (11)$$

where y_{1ref} is the output of an ideal integrator given by $\dot{y}_{1ref} = v_1$. Substituting the expression for τ_{LR} in the output equation and using the Laplace transform, one can obtain the following expression:

$$y_1(s) = \frac{v_1(s)}{s} - \frac{1}{k_L}\frac{\tau_L(s)}{T_L s + 1}\ , \qquad (12)$$

where $T_L = \frac{J}{k_L}$. It can be seen that in the case of a load torque entering the system as a step function, the behavior of the first output is "returning" to that of an ideal integrator with a time constant T_L, i.e. the load torque is tracked asymptotically with the same time constant.

V. SIMULATION RESULTS

The control scheme is presented in Fig. 1. Speed measurement is assumed. A conventional open-loop observer, representing a simulation of the motor equations, is used to generate the flux signals.

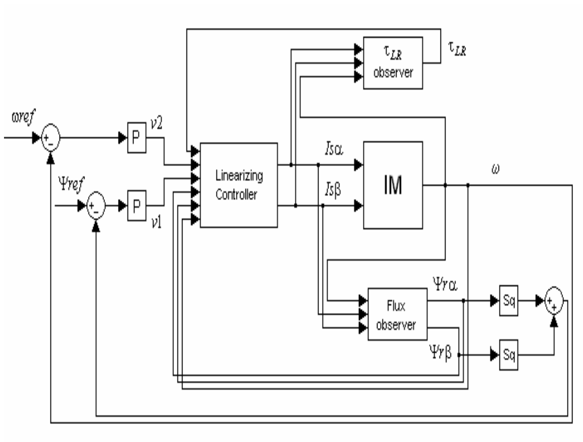

Fig. 1. Control scheme

For the simulation, the same motor as in [7] is selected. The motor parameters are as follows:

$m = 0.0813H$, $\bar{l}_S = 0.084H$, $\bar{l}_R = 0.0852H$, $r_R = 0.842\Omega$, $r_S = 0.687\Omega$, $J = 0.03kgm^2$, $n_p = 2$, $c = 0.0014kgm^2s^{-1}$.

In the outer control loops, P controllers are used for both control subsystems, as seen from Fig. 1. The gain values are set to 60 for the speed control loop, and 40 for the flux control loop. The reference signals represent the responses of a simple lag for the speed loop and a critically damped second-order system for the flux loop to step inputs. This is done to avoid jumps in the current values. In Fig. 2 are given some transient responses showing the evolution of the two outputs, the rotor fluxes and the stator currents for τ_L changing as in Fig.3 and when the exact values of the motor parameters are known. The response of the speed subsystem to load torque changes entering the system as step functions and the torque tracking are shown in Fig. 3. It can be easily found that the transfer function from the load torque τ_L to the output y_1 in the presence of the outer P control loop is given by:

$$\frac{y_1(s)}{\tau_L(s)} = -\frac{1}{kk_L}\frac{s}{(Ts+1)(T_L s+1)}, \qquad (13)$$

where k is the gain of the P controller (equal to 60 in this case) and $T = 1/k$. Thus, the effect of the disturbance (the load torque) is asymptotically reduced to zero, assuming that it is a step function. The speed of response can be tuned by the choice of k and k_L. The value of k_L is set to 5 for the simulation.

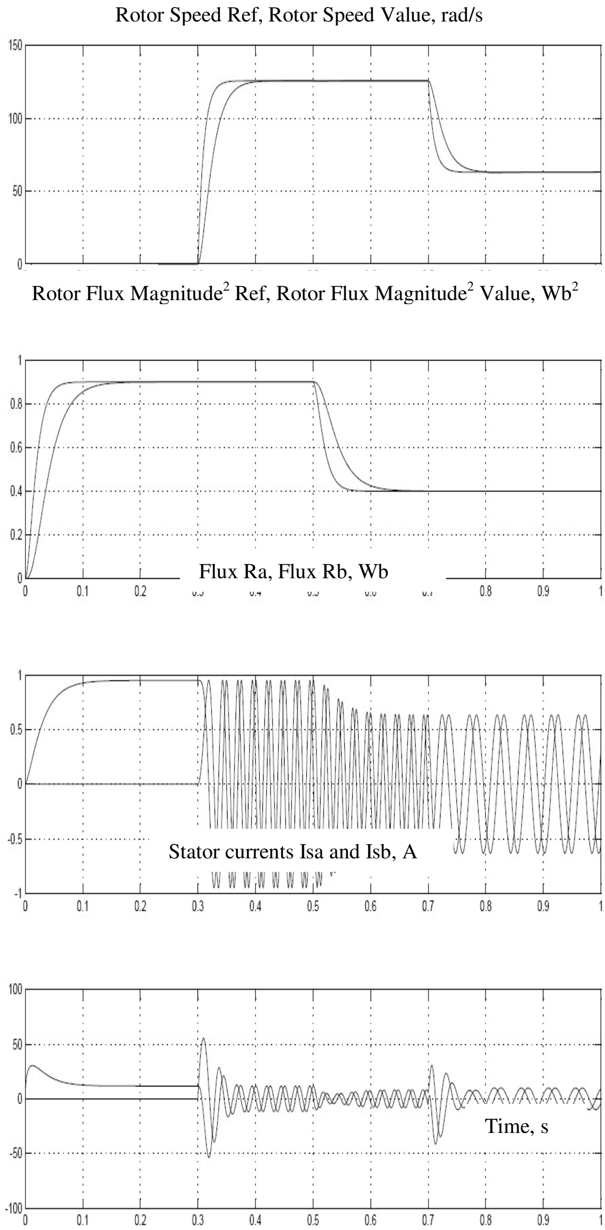

Fig. 2. Some transient responses

The rotor flux dynamics isn't affected by the load torque uncertainty as seen from the motor equations. This remains also in the presence of the linearizing control law.

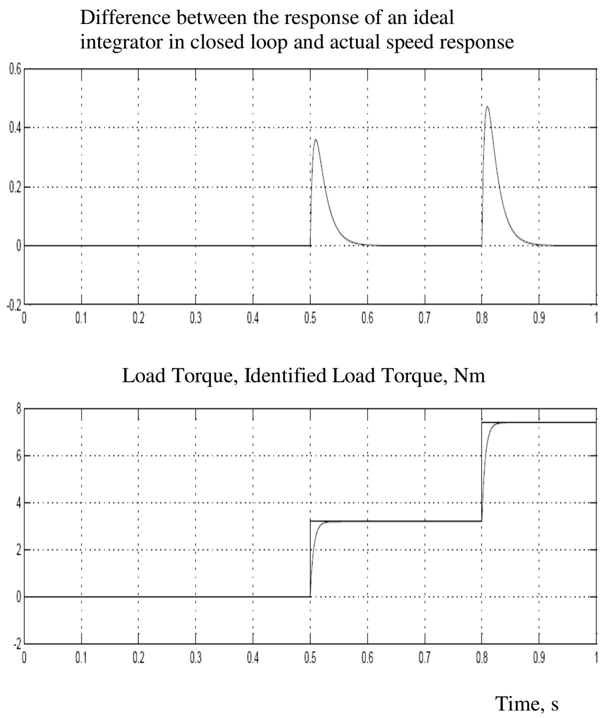

Fig. 3. Speed response to step load torque

VI. Conclusions

An input-output linearizing control for the third order (current-fed) induction motor model described in the fixed stator frame is presented in the paper. Assuming exact knowledge of the values of the motor parameters and the load torque, the use of this control scheme enables the complete decoupling of the dynamics of the system outputs i.e. the rotor speed and the square of the rotor flux. In order to compensate for changes in the load torque, a simple scheme for identification of the load torque is introduced in the control algorithm. The load torque identification algorithm is designed based on rotor speed information. The analysis of the identifier shows that the load torque is tracked asymptotically with time constant T_L in the case its changes take place as step functions. This enables a compensation for these changes in the linearizing control law, and thus the behavior of the speed output can "return" to that of an ideal integrator with the same time constant .

An algorithm for identification of the rotor resistance value, which may change significantly due to heating, can be introduced in the control scheme, in order to ensure the proper work of the conventional open-loop observer.

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