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Comparative Analysis of Cooperative Diversity and Classical Spatial Diversity

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Abstract – **In this paper is presented comparative analysis of systems in which spatial diversity is achieved differently. The performances of systems with classical spatial diversity are compared with cooperative diversity systems. For cooperative diversity systems, performance are evaluated in both symmetric and asymmetric interuser-channel cases.**

Keywords **– Spatial diversity, cooperative diversity.**

I. Introduction

In wireless networks, signal fading arising from multipath propagation is a particularly severe form of interference that can be mitigated through the use of diversity techniques. Space, or multi-antenna, diversity techniques are particularly attractive for implementation. Cooperative diversity is a method that enables single antenna mobiles in a multi-user environment to share their antennas and generate a virtual multiple-antenna transmitter that allows them to achieve spatial diversity.

II. Spatial diversity

Mobile users' data rate and quality of service are limited by the fact that they experience severe variations in signal attenuation, thereby necessitating the use of some type of diversity. Spatial diversity relies on the principle that signals transmitted from geographically separated transmitters, and/or to geographically separated receivers, experience fading that is independent. In this paper is analyzed system with DPSK modulation and two diversity branches.

Bit error probability (BEP) for DPSK modulated signal at output of a receiver with L branches is given by [1]:

$$P_b = \frac{1}{2^{2L-1}(L-1)!(1+\gamma_d)^L}\sum_{i=0}^{L-1} b_i (L-1+i)!\left(\frac{\gamma_d}{\gamma_d+1}\right)^i \qquad (1)$$

where γ_d is signal-to-noise ratio (SNR), and:

$$b_i = \frac{1}{i!}\sum_{n=0}^{L-1-i}\binom{2L-1}{n}.$$

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III. Cooperative diversity

Classical multiple-input multiple-output systems exploit spatial diversity through multiple transmit and/or receive antennas. However, due to the size of a mobile and carrier frequency constraints, achieving spatial diversity through multiple antennas may not be possible. Cooperative diversity is an alternative way to achieve diversity with a single antenna at the mobile. As there are many users in the wireless system, cooperative diversity allows sharing the resources among cooperative users and each user can serve as a relay for the transmissions of other users' information. Diversity is achieved by the relay transmissions by other users in the system.

In this paper is presented the two-user cooperative diversity system, which uses a differential modulation scheme. Both users communicate to the same destination (e.g., base station, BS). Both users send information in the same frequency band, therefore, multiple access is done by time-division multiple access (TDMA). The transmission is half-duplex. The cyclic redundancy check (CRC) bits are used to detect whether the information has been received correctly.

A. System model

The paper is analyzing system in which two cooperative users are transmitting their information to the BS. Each user and the receiver at BS has single antenna. Both users share the same frequency band and the mobile of each user cannot transmit and receive a signal at the same time. The TDMA scheme is shown in Fig. 1. A total time frame is divided into three time intervals. During the first time interval, first user sends its information. The second time interval is reserved for the second user information. The third time interval is used by both users for relaying each other's message to the BS.

1. user informations	2. user informations	Relay informations 1. i 2. user	BS
Sends informations	Receives informations	Sends relay informations	1. user
Receives informations	Sends informations	Sends relay informations	2. user
N_k	N_k	N_k	

Fig. 1. TDMA scheme for analyzed system.

The channels between users (interuser channels) and user-BS channels are mutually independent, with frequency flat feding. One time frame consists of N symbol intervals, and each user, in one time interval, sends information in $N_k = N/3-1$ symbol intervals.

In the first time interval, the first user mobile transmits its own message and the BS receiver and the second user mobile receive the signal. At the BS receiver, the received signal corresponding to the direct transmission $y_{1d}[n]$ is given as

$$y_{1d}[n] = a_{1d}x_1[n] + w_d[n] \quad (2)$$

where $x_1[n]$ is a symbol transmitted from the first user at the nth symbol interval, $n = 0, 1, \ldots, (N/3) - 1$. During this time interval, at the second user mobile, the received signal $y_{12}[n]$ is written as

$$y_{12}[n] = a_{12}x_1[n] + w_2[n]. \quad (3)$$

In the second time interval, the roles of the first and the second user are switched. Therefore, at the BS receiver and at the first user mobile, the respective received signals $y_{2d}[n]$ and $y_{21}[n]$ are

$$y_{2d}[n] = a_{2d}x_2[n] + w_d[n],$$

$$y_{21}[n] = a_{21}x_2[n] + w_1[n]. \quad (4)$$

where $x_2[n]$ is a symbol transmitted from the second user at the nth symbol interval, $n = N/3, (N/3) + 1, \ldots, (2N/3) - 1$.

In the third time interval, both user mobiles act as relays and transmit the relay signals at the same time. The signal arriving at the BS is a linear combination of signals from both transmission paths, which can be written as

$$y_d[n] = a_{1d}x_{1r}[n] + a_{2d}x_{2r}[n] + w_d[n] \quad (5)$$

where $x_{1r}[n]$, $x_{2r}[n]$ are relayed symbols transmitted from the first and the second user mobile, respectively, at the nth symbol interval $n = 2N/3, (2N/3) + 1, \ldots, N - 1$.

In Eq. (2)-(5), the coefficients, a_{ij}, $i,j \in \{1, 2, d\}$, are associated with each transmission path from i to j. They represent channel gains, which take into account fading and shadowing, and incorporate transmission signal energies. They are modeled as independent zero-mean complex Gaussian random variables. The subscripts “1” and “2” represent the first and the second user mobiles and the subscript “d” represents the BS. Since the channels are fixed during the multiple-access time frame, the time variable n is omitted in $a_{ij}[n]$. The $w_i[n]$, $i \in \{1, 2, d\}$ representing additive white noise and interference, are modeled as independent zero-mean complex Gaussian random variables with variances N_i and $i \in \{1, 2, d\}$, respectively. The signal-to-noise ratios (SNR) associated with each link shown in Fig. 2 are defined as

$$\gamma_d = E\left[|a_{1d}|^2\right]/N_d = E\left[|a_{2d}|^2\right]/N_d, \quad (6a)$$

$$\gamma_{k_1} = E\left[|a_{12}|^2\right]/N_2, \quad \gamma_{k_2} = E\left[|a_{21}|^2\right]/N_1. \quad (6b)$$

The γ_{k_1}, γ_{k_2} are referred to as interuser SNRs. For simplicity in the comparison with the no-cooperation case, both SNRs in the user-BS links are set to be equal to γ_d.

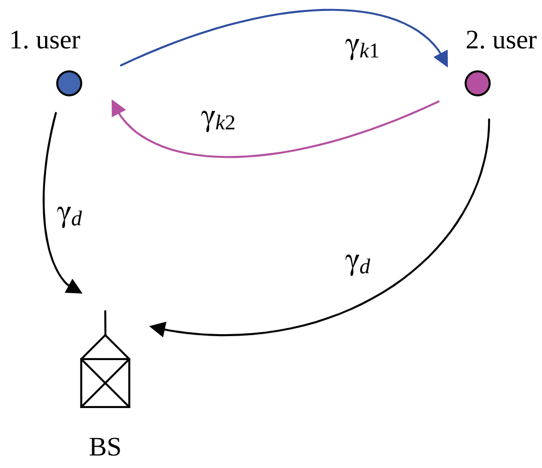

Fig. 2 System with cooperative diversity.

If the quality of interuser channels is low, several relay symbols will be in error. These relay symbols in error result in performance degradation even when the direct transmission channels are good. Therefore, it turns out that it is better not to transmit anything from the relay mobile when the interuser channels are not good. The decision to transmit the relay signals or not depends on the amplitudes of the fading gains between the source mobile and the relay mobile. If these values fall below a certain threshold, the relay mobile is not allowed to transmit the relay signal but instead transmits its own information signal. The CRC bits are added before differential encoding at the source mobile. Then, at the relay mobile, the receiver performs a CRC check after differential decoding to see if the whole frame has been received correctly. If the frame is received correctly, the relay mobile is allowed to perform differential re-encoding and forward the relay signal, otherwise, it does nothing. For the purpose of this paper, 16-bit CRC is applied and its error detection is perfect.

IV. PERFORMANCE ANALYSIS

As a performance measure in this paper is used BEP. The analysis here includes both the symmetric and asymmetric interuser cases.

When the cooperative users have perfect information about each other's transmission sequence, there will be no misleading relay information transmitted. This ideal case of the perfect interuser information represents the limiting performance of cooperative diversity. In the perfect interuser case, the BEP does not depend on the interuser channel. It depends on how the destination decodes the signals.

For the analyzed system, it might happen that one of the mobiles stops its relay transmission (one user does not have perfect information of the other). Therefore, the computation of the BEP is divided into two cases.

1) Full Relaying: Full relaying occurs when both user mobiles transmit their relay signals. For a single receive antenna at the destination, the BEP is given by [3]:

$$P_{b,\ full\ releying} = \frac{1}{2}\left[1-\sqrt{\frac{\gamma_d}{\gamma_d+6}\left(\frac{\gamma_d+9}{\gamma_d+6}\right)}\right]. \quad (7)$$

γ_d is defined in Eq. (6a).

2) Single Relaying: When one of the user mobiles stops its relay transmission, the destination receives only the relay signal from the other user. For a single receive antenna at the destination, the BEP is given by [3]:

$$P_{b,\ si\ ngle\ releying} = \frac{1}{2}\left[1-\sqrt{\frac{\gamma_d}{\gamma_d+2}\left(\frac{\gamma_d+3}{\gamma_d+2}\right)}\right]. \quad (8)$$

γ_d is defined in Eq. (6a).

A user mobile transmits the relay signal when the whole frame has been received correctly. This implies that no symbols in error will be transmitted in the relay signal. The probability of a user mobile transmitting the relay signal is (1 – P_{DPSK,k_i}), where P_{DPSK,k_i} is the frame error probability of DPSK belonging to the *i*th user mobile. The frame error probability of DPSK is given by [3]:

$$P_{\mathrm{DPSK},k_i} = 1-\left(\frac{1}{2}\right)^N\left[1+\sum_{k=1}^{N}\prod_{l=1}^{k}\frac{N+1-l}{l+\frac{l}{\gamma_{k_i}}}\right]. \quad (9)$$

The BEP for the first user transmission can be derived considering the following four situations.

1) *No users transmit a relay signal*: The probability of this case is $P_{\mathrm{DPSK},k_1}\cdot P_{\mathrm{DPSK},k_2}$. Bit errors occurring at the destination are from DPSK decoding of the direct transmission signal belonging to the first user.
2) *Only the second user transmits a relay signal*: The probability of this case is $(1-P_{\mathrm{DPSK},k_1})\cdot P_{\mathrm{DPSK},k_2}$. Bit errors occurring at the destination are from differential decoding with single relaying.
3) *Only the first user transmits a relay signal:* The probability of this case is $P_{\mathrm{DPSK},k_1}\cdot(1-P_{\mathrm{DPSK},k_2})$. Bit errors occurring at the destination are from DPSK decoding similar to the first case.
4) *Both users transmit a relay signal:* The probability of this case is $(1-P_{\mathrm{DPSK},k_1})\cdot(1-P_{\mathrm{DPSK},k_2})$. Bit errors occurring at the destination are from differential decoding with full relaying.

The BEP is found by averaging BEPs of the above four cases. This can be written as

$$\begin{aligned} P_b &= P_{\mathrm{DPSK},k_1}\cdot P_{\mathrm{DPSK},k_2}\cdot P_{b,\mathrm{DPSK}} \\ &+(1-P_{\mathrm{DPSK},k_1})\cdot P_{\mathrm{DPSK},k_2}\cdot P_{b,\ si\ ngle\ releying} \\ &+P_{\mathrm{DPSK},k_1}\cdot(1-P_{\mathrm{DPSK},k_2})\cdot P_{b,\mathrm{DPSK}} \\ &+(1-P_{\mathrm{DPSK},k_1})\cdot(1-P_{\mathrm{DPSK},k_2}) \\ &\cdot P_{b,\ full\ relaying} \end{aligned} \quad (10)$$

where $P_{b,\ \mathrm{DPSK}} = 1/(1+2\gamma_d)$ is a conventional BEP of DPSK under Rayleigh-fading channels. When the $\gamma_{k_1}=\gamma_{k_2}=\gamma_k$, (10) reduces to

$$\begin{aligned} P_b &= P_{\mathrm{DPSK},k}\cdot P_{b,\mathrm{DPSK}} + (1-P_{\mathrm{DPSK},k})\cdot P_{\mathrm{DPSK},k} \\ &\cdot P_{b,\ si\ ngle\ releying} + (1-P_{\mathrm{DPSK},k})^2\cdot P_{b,\ full\ relaying} \end{aligned} \quad (11)$$

In order to compare cooperative diversity and classical spatial diversity with a different rate loss due to a reference symbol and CRC bits, let us define the pure information bit SNR as

$$\mathrm{SNR} = \frac{\gamma_d}{R}\cdot\frac{N+1}{N+1-\alpha} \quad (12)$$

where R = 1, α = 0 for DPSK and R = 2/3, α = 1 + no. of CRC bits for the cooperative diversity. The frame length (N + 1) is 131 symbol intervals which accommodate 130 information symbols.

V. Numerical Results

A. Symmetrical interuser channel

The performances of systems with spatial diversity and cooperative diversity are shown in Fig. 3.

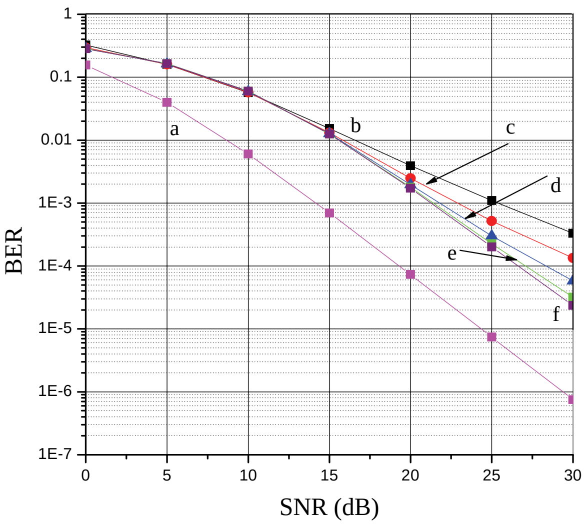

Fig. 3. Performances of systems with spatial diversity and cooperative diversity under symmetric interuser channels. a) system with spatial diversity, cooperative diversity: b) γ_k = 10 dB, c) γ_k = 15 dB, d) γ_k = 20 dB, e) γ_k = 25 dB and f) γ_k = 30 dB.

For cooperative diversity is considered case of symmetrical interuser channels, when performances of both users are identical. In Fig. 3, line (a) represents performance of system with spatial diversity with two branches. Lines (b), (c), (d), (e) and (f) defines performance of cooperative diversity for different interuser channel.

A. Asymmetrical interuser channel

Fig. 4 shows performance of systems with spatial diversity and cooperative diversity, when the interuser channels are asymmetric and in that case each user has different performance. This situation might occur when one user experiences local interference, while the other does not. For example, one user might be surrounded by local interfering devices or mobiles which do not affect the other user since the distance is too far. The asymmetric interuser case can also occur when power control is used and due to different distances from the base station, the users' transmit powers could be different.

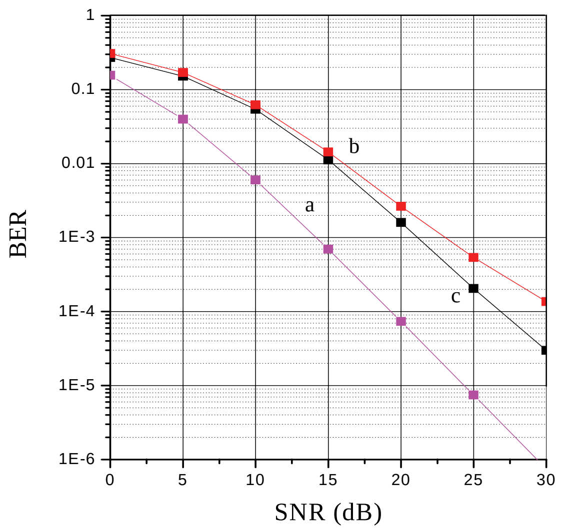

Fig. 4. Performances of systems with spatial diversity and cooperative diversity under asymmetric interuser channels. a) system with spatial diversity, cooperative diversity γ_{k1} = 25 dB and γ_{k2} = 15 dB: b) second user, c) first user.

In Fig. 4, line (a) represents performance of system with spatial diversity with two branches. The lines (b) and (c) defines performance of second and first user, respective, which are cooperating.

As seen on Fig. 3-4, system with classical spatial diversity shows better performance in whole area of interest. Although, in situations where exists some limitations concerning number of available antennas, cooperative diversity offers solution for minimizing effects of fading.

VI. Conclusion

In this paper are presented two ways of achieving spatial diversity. Classical spatial diversity considerable improves performances, but in systems where classical implementation of spatial diversity is not applicable due to various limitations, cooperative diversity provides an effective way for a collection of wireless users to relay signals for one another in order to exploit benefits of spatial diversity in the channel.

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