Analysis of a Matching Network Effect on Noise

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***Abstract* – Effect of a matching network on noise is analyzed. Analysis is applied to an amplifier model with a Thévenin source. Noise characteristics for amplifier model without and with two classes of lossless matching networks between the source and the amplifier input are studied. Some numerical simulations for a matching network effect on noise characteristics demonstrating the effectiveness of the proposed analysis are presented.**

***Keywords* – Amplifier, matching network, noise, Thévenin source, simulation.**

I. INTRODUCTION

Noise behavior is very important characteristic of electronic circuits, such as amplifiers, filters etc, as it usually determines the fundamental limit of the performance of circuits [1]. When designing amplifiers for specific applications, there are many conflicting requirements, to be met and decisions to be made [2] - [4].

As we know, the noise factor for the amplifier is defined as

$$F = 1 + \frac{P_{na}}{GP_{ni}} > 1 \tag{1}$$

where P_{na} is the noise power added by the amplifier, P_{ni} is the input signal power, and G is the gain of the amplifier.

A common way of presenting the noise factor is the noise figure, i.e.

$$NF = 10\,log\,F \tag{2}$$

These equations give a hint of why a low noise figure is considered important, and why a high gain is often associated with a low noise figure. Noise figure is, however, often overstated in its importance in a low noise design system. It is the correct balance between noise figure and gain that is important rather than a low noise figure itself. Because of the conflicting requirements, some compromises are often made in radio frequency amplifier design between lowest noise and largest gain. Indeed, it can be proven [4], that there exists a combination of components which could match the signal source impedance to the amplifier input. To study how the addition of a matching network between the source and the amplifier effects on noise characteristics, an amplifier without and with a matching network is analyzed.

II. MATCHING NETWORK EFFECT ON NOISE

The amplifier noise model with a Thévenin source and a lossless input matching network is shown in Fig. 1.

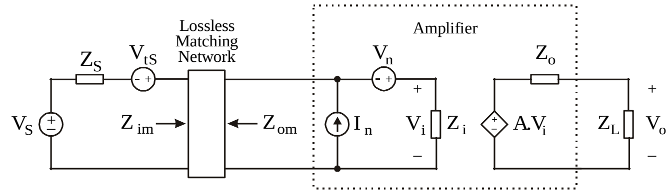

Fig. 1. Amplifier noise model with an input matching network.

The noise source V_{ts} models the thermal noise generated by the signal source V_s having output impedance $Z_s = R_s + jX_s$. The amplifier input impedance is $Z_i = R_i + jX_i$ and its output impedance $Z_o = R_o + jX_o$, respectively. The matching network has an input impedance $Z_{im} = R_{im} + jX_{im}$ and output impedance $Z_{om} = R_{om} + jX_{om}$. The noise generated by the amplifier is modeled by the noise sources V_n and I_n.

The effect of a matching network between the source and the amplifier on noise can be studied analyzing the model in Fig. 1.

A. Equivalent amplifier noise input voltage

The total power delivered to the matching network by the source is

$$P_{im} = \left|\frac{V_s + V_{ts}}{Z_s + Z_{im}}\right|^2 Re(Z_{im}) = \frac{v_s^2 + 4kTR_s\Delta f}{\left|Z_s + Z_{im}\right|^2} R_{im} \tag{3}$$

where T is the absolute temperature in K, k is Boltzmann's constant, and Δf is the bandwidth over which the noise is measured.

Let V_{is} and V_{its}, respectively, be the voltages at the amplifier input due to V_s and V_{ts}. The output power from the matching network is

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$$P_{om} = \left| \frac{V_{is} + V_{its}}{Z_i} \right|^2 Re(Z_i) = \frac{v_{is}^2 + v_{its}^2}{|Z_i|^2} R_i \ . \qquad (4)$$

Because the matching network is lossless, it follows that $P_{im} = P_{om}$. This leads to the relation

$$v_{is}^2 + v_{its}^2 = \frac{R_{im}}{R_i} \left| \frac{Z_i}{Z_s + Z_{im}} \right|^2 \left(v_s^2 + 4kTR_s \Delta f \right) . \qquad (5)$$

Equation (5) can be rewritten as

$$v_{is}^2 + v_{its}^2 = \left| \frac{Z_i}{Z_i + Z_{om}} \right|^2 \left[\frac{R_{im}}{R_i} \left| \frac{Z_i + Z_{om}}{Z_i + Z_{im}} \right|^2 \left(v_s^2 + 4kTR_s \Delta f \right) \right]$$

$$= \left| \frac{Z_i}{Z_i + Z_{om}} \right|^2 \left[v_{is(oc)}^2 + v_{its(oc)}^2 \right] \qquad (6)$$

where $v_{is(oc)}^2$ and $v_{its(oc)}^2$ are the open-circuit values v_{is}^2 and v_{its}^2. The latter is given by

$$v_{its(oc)}^2 = \frac{R_{im}}{R_i} \left| \frac{Z_i + Z_{om}}{Z_i + Z_{im}} \right|^2 4kTR_s \Delta f \ . \qquad (7)$$

This equation must be of the form $v_{its(oc)}^2 = 4kTR_{om}\Delta f$, where $R_{om} = Re(Z_{om})$. In this case, the output resistance of the matching network can be expressed as

$$R_{om} = \frac{v_{its(oc)}^2}{4kT\Delta f} = \frac{R_{im}}{R_i} \left| \frac{Z_i + Z_{om}}{Z_s + Z_{im}} \right|^2 R_s \ . \qquad (8)$$

When this equation is solved for $\frac{R_{im}}{R_i}$ and the result used in Eq. (5), it follows that

$$v_{is}^2 + v_{its}^2 = \left| \frac{Z_i}{Z_i + Z_{om}} \right|^2 \frac{R_{om}}{R_s} \left(v_s^2 + 4kTR_s \Delta f \right) . \qquad (9)$$

It might seem a contradiction that R_{om} in Eq. (8) is a function of Z_i. However, the dependence cancels because Z_{im} is also a function of Z_i. It can be concluded from Eq. (9) that the mean-square open-circuit output voltage from the matching network is given by

$$v_{is(oc)}^2 + v_{its(oc)}^2 = \frac{R_{om}}{R_s} \left(v_s^2 + 4kTR_s \Delta f \right) . \qquad (10)$$

To obtain the total mean-square open-circuit voltage at the input to the amplifier, the contributions of the noise sources V_n and I_n must be added to Eq. (10). The addition of noise signals is accomplished mathematically by simply adding the functions, which describe these signals. Taking the effect of the noise sources into consideration [5], the following result is obtained

$$v_{i(oc)}^2 = v_{is(oc)}^2 + v_{its(oc)}^2 + v_n^2 + 2v_n i_n Re\left(cZ_{om}^*\right) + i_n^2 |Z_{om}|^2$$

$$= \frac{R_{om}}{R_s} \left(v_s^2 + 4kTR_s \Delta f \right) + v_n^2 + 2v_n i_n Re\left(cZ_{om}^*\right) + i_n^2 |Z_{om}|^2 \qquad (11)$$

where $c = c_r + jc_i$ is the correlation coefficient between the noise sources V_n and I_n.

This result shows that the mean-square equivalent amplifier noise input voltage in series with the signal source V_s can be expressed as

$$v_{ni}^2 = 4kTR_s \Delta f + \frac{R_s}{R_{om}} \left[v_n^2 + 2v_n i_n Re\left(cZ_{om}^*\right) + i_n^2 |Z_{om}|^2 \right] . \qquad (12)$$

B. Signal - to - Noise Ratio

When the source is modeled by a Thévenin equivalent circuit as in Fig.1, the signal – to – noise ratio (SNR) is $SNR = v_s^2 / v_{ni}^2$. Substituting Eq. (12) for v_{ni}^2 leads to the equation

$$SNR = \frac{v_s^2}{4kTR_s \Delta f + \frac{R_s}{R_{om}} \left[v_n^2 + 2v_n i_n Re\left(cZ_{om}^*\right) + i_n^2 |Z_{om}|^2 \right]} . \qquad (13)$$

C. Noise Factor

The noise factor for the amplifier model is

$$F = \frac{v_{ni}^2}{v_{ts}^2} = \frac{v_{ni}^2}{4kTR_s \Delta f} \ . \qquad (14)$$

Substituting Eq. (12) into Eq. (14) gives

$$F = 1 + \frac{v_n^2 + 2v_n i_n Re\left(cZ_{om}^*\right) + i_n^2 |Z_{om}|^2}{4kTR_{om} \Delta f} . \qquad (15)$$

This is the same as the noise factor calculated at the output of the matching network. The basic reason that the noise factors at the source and at the output of the matching network are equal is because a lossless matching network cannot add noise. Thus it follows that the signal-to-noise ratio is also the same at the input to the matching network as it is at the input to the amplifier. However, these conclusions do not hold for a lossy matching network. Equation (15) can be used to predict the noise factor for any matching network.

D. Signal Power Delivered to Z_i

It follows from Eq. (10), that the open-circuit value v_{is}^2 can be expressed by

$$v_{is(oc)}^2 = \frac{R_{om}}{R_s} v_s^2 . \qquad (16)$$

The signal power delivered to the amplifier input is given by

$$P_i = i_i^2 \, Re(Z_i) = \frac{v_{is(oc)}^2}{\left|Z_i + Z_{om}\right|^2} R_i = \frac{v_s^2}{\left|Z_i + Z_{om}\right|^2} \frac{R_i R_{om}}{R_s} . \qquad (17)$$

E. Noise characteristics for amplifier with different matching networks

Based on the above expressions and on the model in Fig. 1, a noise analysis for arbitrary matching network between the signal source and the amplifier input can be performed. In order to estimate the effect of a matching network on noise, the amplifier without and with a match is analyzed bellow.

Case 1 *No matching network is used at the amplifier input*

If a matching network is not used, $Z_{om} = Z_s$, and [5]

$$v_{ni}^2 = 4kTR_s\Delta f + v_n^2 + 2v_n i_n \, Re\left(cZ_s^*\right) + i_n^2 \left|Z_s\right|^2 . \qquad (18)$$

Under this condition, the SNR, and the noise factor can be expressed as follows

$$SNR = \frac{v_s^2}{4kTR_s\Delta f + v_n^2 + 2v_n i_n \, Re\left(cZ_s^*\right) + i_n^2 \left|Z_s\right|^2} , \qquad (19)$$

$$F = 1 + \frac{v_n^2 + 2v_n i_n \, Re\left(cZ_s^*\right) + i_n^2 \left|Z_s\right|^2}{4kTR_s\Delta f} . \qquad (20)$$

The power delivered to Z_i can be determined by

$$P_i = \frac{v_s^2}{\left|Z_i + Z_s\right|^2} R_i . \qquad (21)$$

Case 2 *Matching network with output impedance chosen to be optimum source impedance*

When $Z_{om} = Z_{opt}$, where Z_{opt} is given by [5]

$$Z_{opt} = \left[\sqrt{1 - c_i^2} - jc_i\right] \frac{v_n}{i_n} , \qquad (22)$$

the voltage v_{ni}^2 is

$$v_{ni}^2 = 4kTR_s\Delta f + \frac{R_s}{R_{opt}} \left[v_n^2 + 2v_n i_n \, Re\left(cZ_{opt}^*\right) + i_n^2 \left|Z_{opt}\right|^2\right], \qquad (23)$$

In this case, the SNR is defined by

$$SNR = \frac{v_s^2}{4kTR_s\Delta f + \frac{R_s}{R_{opt}} \left[v_n^2 + 2v_n i_n \, Re\left(cZ_{opt}^*\right) + i_n^2 \left|Z_{opt}\right|^2\right]} . \qquad (24)$$

This matching network minimizes the noise factor so that

$$F_{min} = 1 + \frac{v_n i_n \left(1 + c_r \sqrt{1 - c_i^2} - c_i^2\right)}{2kT\sqrt{1 - c_i^2}\,\Delta f} . \qquad (25)$$

If the matching network is designed so that the amplifier sees its optimum source impedance, Eq. (17) reduces to

$$P_i = \frac{v_s^2}{\left|Z_i + Z_{opt}\right|^2} \frac{R_i R_{opt}}{R_s} . \qquad (26)$$

Case 3 *Conjugate impedance matching network*

For a conjugate match, the condition $Z_{om} = Z_i^*$ must hold [4], and that's way $R_{om} = R_i$. Under this condition, the expression for v_{ni}^2 in Eq. (12) reduces to

$$v_{ni}^2 = 4kTR_s\Delta f + \frac{R_s}{R_i} \left[v_n^2 + 2v_n i_n \, Re\left(cZ_i\right) + i_n^2 \left|Z_i^*\right|^2\right] . \qquad (27)$$

The corresponding SNR and noise factor are, respectively

$$SNR = \frac{v_s^2}{4kTR_s\Delta f + \frac{R_s}{R_i} \left[v_n^2 + 2v_n i_n \, Re\left(cZ_i\right) + i_n^2 \left|Z_i^*\right|^2\right]} , \qquad (28)$$

$$F = 1 + \frac{v_n^2 + 2v_n i_n \, Re\left(cZ_i\right) + i_n^2 \left|Z_i^*\right|^2}{4kTR_i\Delta f} . \qquad (29)$$

For a conjugate match, the maximum value of P_i occurs and it is given by

$$P_{i\,max} = \frac{v_s^2}{4R_s} . \qquad (30)$$

III. Numerical Simulations

In order to illustrate a matching network effect on noise, the noise characteristics for the amplifier having a resistive input impedance, driven from a Thévenin source with a resistive output impedance, have been simulated. The simulation for the above analyzed cases has been implemented. The bellow presented characteristics illustrate the matching network effect on noise with $\Delta f = 1Hz$, $R_i = 25\,\Omega$, $i_n / \sqrt{\Delta f} = 31pA/\sqrt{Hz}$, $v_n / \sqrt{\Delta f} = 0{,}447nV/\sqrt{Hz}$, and $c = 0{,}12 - j0{,}44$.

Fig.2 shows the equivalent noise voltage change versus the source resistance. It is clear that if a matching network is used, the voltage is lower than in case1. For example, if $R_S = 50\,\Omega$, then the noise voltage in case1 is 1,271 times grater than that in case2, and 1,178 times grater than the voltage in case3. The results also show that the matching network, whose output impedance is equal to the optimum impedance of the source, provides the lowest noise voltage.

The effect of a match between the source and the amplifier on the noise figure is illustrated in Fig.3.

It follows from the simulated waveforms that as the source resistance increases, the noise figure increases in case1, only. In the other two cases the noise figures do not change. The comparison between the noise figures with $R_S = 50\,\Omega$, shows

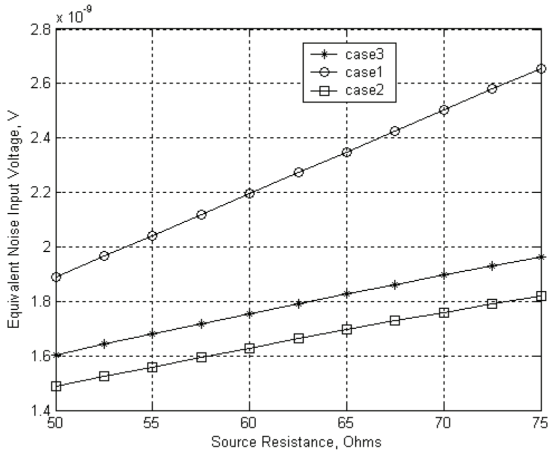

Fig.2. Equivalent noise input voltage versus source resistance.

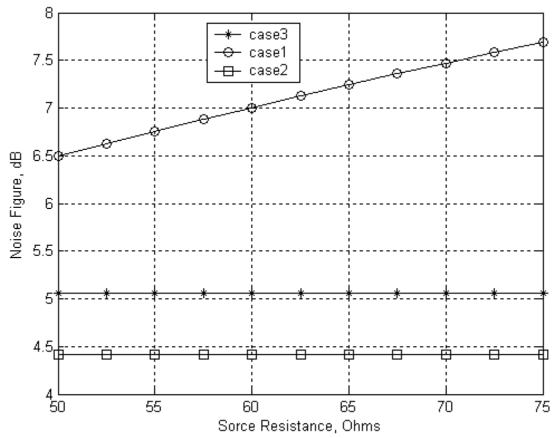

Fig.3. Noise figure dependence on source resistance.

that $NF_{case1} - NF_{min} = 2{,}08$ dB, $NF_{case3} - NF_{min} = 0{,}649$ dB where $NF_{min} = 4{,}413$ corresponds to case2.

Knowing the signal power delivered to the amplifier input, the power gain decrease in percents can be expressed as

$$P_{decrease}\% = \left(P_i / P_{i\,max}\right)100\,\% . \tag{31}$$

Because the maximum power gain occurs with a conjugate impedance match, in case1 the power gain drops compared to $P_{i\,max} = P_{icase3}$ by the factor P_{icase1} / P_{icase3}, and in case2 it drops by the factor P_{icase2} / P_{icase3}, respectively. The power gain decrease is presented in Fig. 4. It can be concluded that for the amplifier without a matching network the percent decrease in power gain is a linear function of the source resistance, while for a matching network with optimum source impedance, the decrease is a constant. If the source resistance is $R_S = 50\,\Omega$, the decrease in power gain in case1 is 11,11%, or 0,512 dB. In case2 it is 12,53%, or 0,581 dB, respectively.

The results, obtained with different values of the correlation coefficient, show that the correlation coefficient effects on the input power if and only if a matching network with optimum source resistance is used.

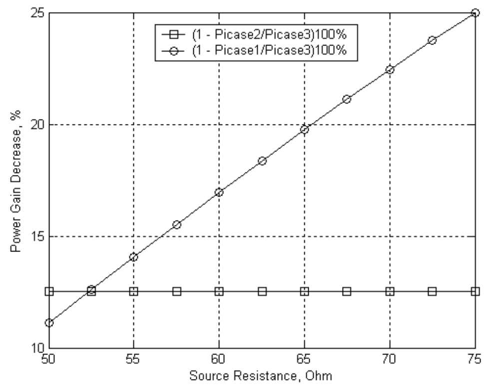

Fig.4. Power gain decrease as a function of source resistance.

IV. Conclusion

An effective approach for analysis of a matching network effect on noise is presented in this paper. The approach is based on the use of an amplifier noise model with a Thévenin input source and a lossless matching network. The analysis is applied to the amplifier without an input matching network, as well as, to a conjugate impedance matching network and to a matching network with optimum source impedance. It is seen from the simulation results that the addition of a conjugate matching network at the amplifier input maximizes the power gain, and the addition of a matching network with optimum source impedance minimizes the noise factor.

It follows from the analysis that because of the dependence of v_{ni}^2 and F of Z_i, it is difficult to predict from the Eqs. (27) and (29) how changes in Z_i affect the noise. This is because V_n, I_n, and c, in the model are, in general, related to Z_i. For example, V_n, I_n, c, and Z_i may all be functions of the bias current in the amplifier input stage. A change in the bias current to vary Z_i can cause a change in V_n and c. Thus the effects cannot be examined in detail unless the relations between the variables are known.

References

[1] S. Maas, "*Noise in Linear and Nonlinear Circuits*", Boston, Artech House, 2004.

[2] Jieh - Tsorng Wu, "*Noise Analysis and Modeling*", National Chiao - Tung University, 2001.

[3] A. Demir and A. Vincentelli, "*Analysis and Simulation of Noise in Nonlinear Electronic Circuits and Systems*", Kluwer Academic Publishers, Boston/Dordrecht/London, 1998.

[4] A. Mehrotra and A. Vincentelli, "*Noise Analysis of Radio Frequency Circuits*", Kluwer Academic Publishers, MA, USA, Hardbound, 2003.

[5] P. Petrova, "Amplifier Noise Model with Thévenin Input Source", ICEST 2005, Paper Proceedings, vol. 1, pp.105-108, Niŝ, Serbia and Montenegro, 2005.