Coupled Linear Grounding Systems: Quasistationary Antenna Model

Nenad N. Cvetkovic[1] and Predrag D. Rancic[2]

Abstract – Characteristics of the linear grounding systems (LGSs) made of two coupled independently supplied linear ground electrodes, are analyzed in this paper. The unknown current distributions (UCDs) along the conductors are determined using quasistationary antenna model. Ground is assumed as linear, isotropic and homogeneous media having known parameters. UCDs are determined numerically solving system of integral equations (SIE) using point matching method and polynomial approximation for currents. Afterward, the self and mutual impedances (as the measure of mutual LGSs influence) as well as total impedance of connected grounding systems are calculated. For these calculations, so-called symmetric and anti-symmetric regimes of feeding are applied.

Keywords – Quasistationary EM field, grounding systems, linear electrode, antenna model.

I. Introduction

During last 15 years on the Faculty of Electronic Engineering in Nis, few number of models for analyze of grounding systems treating EM field in the ground as stationary ([1], [2]) and quasistationary ([3]) has been developed. By that, real inhomogeneous ground is modeled as homogeneous, stratified or sectoral structure ([4], [5]). For analyze of grounding systems, especially those ones having massive electrodes, one simple and efficient method named by author as equivalent electrodes method (EEM, [6], [7]) has been also developed. Before 90-ies, two PhD works are done at the Faculty under mentorship of prof. Velickovic in this area (Dr Jelenko Vlajic, Banja Luka and dr Panco Vrangalov, Skopje).

Quasistationary EM field structure in the LGS surrounding is analyzed in the paper, introducing model of longitudinal current along conductor axis, having unknown distribution (UCD). The LGS is fed with very low frequency currents, and domain where LGS is placed provides that surface of connected LGS conductors can be assumed as equipotential. This condition results with integral equation system of Rabenn's type (SIE-R) having UCD as unknown function. SIE-R is numerically solved using Point Matching Method (PMM, [9]) and polynomial approximation for UCD ([10]). By that, inhomogeneous ground is analyzed using various known models, and for each of them, Green function for Hertz vector and electrical scalar potential (ESP) is derived ([3], [8]). In this way is avoided rigorous and very complicated so-called Sommerfeld formulation. The described model corresponds to so-called quasistationary antenna model.

More rigorous quasistationary antenna model demands including of Hertz vector distribution in the way that result tangential electrical field component at the LGS surface is zero. This makes the mathematical model more complicated, and it result with so-called system of integral equation of two potentials (SIE-TP, [11]). If problem is analyzed for very low frequencies, SIE-TP approximately gets the form of SIE-R.

After introductory part, the paper is organized in three sections, followed by the conclusion and reference list.

In the second part of the paper theoretical basics are given, i.e. problem definition as well as expressions for electric scalar-potential (ESP) in the electrode system surrounding and general formulations of applied numerical procedures. The results obtained applying described numerical procedures are presented in the third part of the paper.

II. Theoretical basis of the model

A. Description of the system geometry

System made of total $N = N_1 + N_2$ linear conductors from which N_1 conductors are fed by common isolated earthing conductor with current I_{g1}, and the rest (N_2) of the conductors are fed with common supplying current I_{g2} is observed and illustrated in Fig. 1.

The linear ground conductors of length l_{ik} and circular cross-section radius a_{ik}, $a_{ik} << l_{ik}$, $k = 1, 2, ... N_i$, $i = 1, 2$ are placed in homogeneous ground.

The ground is treated as isotropic semi-conducting media having known electrical parameters σ_1, ε_1 and $\mu_1 = \mu_0$ (σ_1 - conductivity, ε_1-permittivity, μ_0- permeability). The following labels are also used in the paper: $\underline{\sigma}_i = \sigma_1 + j\omega\varepsilon_i$, $i = 0,1$- complex conductivities; $\omega = 2\pi f$ - low angular frequency; and R_{z10}, T_{z10}, $R_{z10} = T_{z10} - 1 = (\underline{\sigma}_1 - \underline{\sigma}_0)/(\underline{\sigma}_1 + \underline{\sigma}_0)$ are quasistationary reflection and transmission coefficients, respectively.

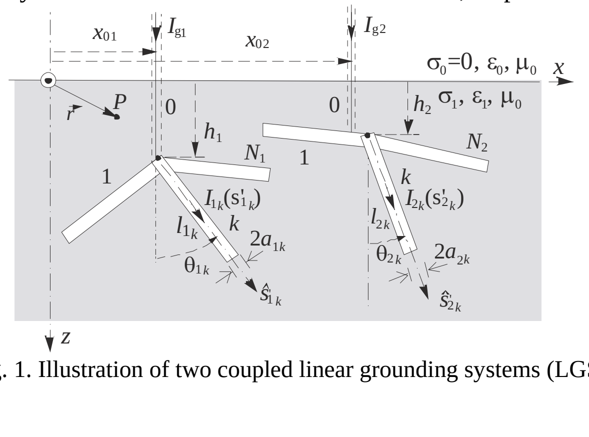

Fig. 1. Illustration of two coupled linear grounding systems (LGS).

[1]Nenad N. Cvetkovic is with the Faculty of Electronic Engineering, P.O.Box 73, 18000 Nis, Serbia, E-mail: csone@elfak.ni.ac.yu.
[2]Predrag D. Rancic is with the Faculty of Electronic Engineering, P.O.Box 73, 18000 Nis, Serbia, E-mail: prancic@elfak.ni.ac.yu.

ESP of the system in the ground point P, having position defined with vector $\vec{r}$, is

$$\varphi_1(\vec{r}) = \varphi_{10}(\vec{r}) + \varphi_{11}(\vec{r}) = \frac{1}{4\pi\underline{\sigma}_1}\left[\sum_{i=1}^{2}\frac{T_{z10}I_{gi}}{|\vec{r}-x_{0i}\hat{x}|} + \right.$$

$$\left. + \sum_{i=1}^{2}\sum_{k=0}^{N_i}\int_{s'_{ki}=0}^{l_{ki}} I_{ik}(s'_{ik})\frac{\partial}{\partial s'_{ik}}K(\vec{r},s'_{ik})\,\mathrm{d}\,s'_{ik}\right], \quad (1)$$

where x_{0i}, $i=1,2$ determine the positions of the supplying points at the ground surface, $I_{ik}(s'_{ik})$ -longitudinal current distribution and $K(\vec{r},s'_{ik})$ is kernel of the form,

$$K(\vec{r},s'_{ik}) = \frac{1}{r_{1k}} + \frac{R_{z10}}{r_{2k}}. \quad (2)$$

In Eq, (2), r_{1k} and r_{2k} are distances between the field point P and conductor current element, and its image in modified flat mirror, respectively.

B. Numerical procedure evaluation

Matching the ESP value (1) in the conductors surface points P_n, $\varphi_1 \cong U_n$, the SIE-Rs are obtained

$$U_n = \frac{1}{4\pi\underline{\sigma}_1}\left\{\sum_{i=1}^{2}\sum_{k=1}^{N_i}\left[I_{ik}(l_{ik})K(s_n,l_{ik}) - \right.\right.$$

$$\left.\left. - \int_{s'_k=0}^{l_k} I'_{ik}(s'_{ik})K(s_n,s'_{ik})\,\mathrm{d}\,s'_{ik}\right]\right\}, \; n=1,2 \quad (3)$$

where: $I_{ik}(l_{ik})$ are currents values at free ends of ground electrodes, $I'_{ik}(s'_{ik}) = -\partial I_{ik}/\partial s'_{ik}$ are leakage currents density per unit length and s_n denotes distance between the matching point P_n at the n-th conductor surface and its origin.

The solution of the SIE-Rs given with Eq. (3) is current distribution along the conductors, $I_{ik}(s'_{ik})$ $k=1,2,...N_i$, $i=1,2$, where feeding currents satisfy conditions

$$\sum_{k=1}^{N_i} I_{ik}(0) = I_{\mathrm{g}i}, \; i=1,2. \quad (4)$$

Assuming the current distribution in polynomial form,

$$I_{ik}(s'_{ik}) = \sum_{m=0}^{M_k} I_{ikm}\left(\frac{s'_{ik}}{l_{ik}}\right)^m, \; k=1,2,...N_i, \; i=1,2, \quad (5)$$

where I_{km}, $k=1,2,...N_i$, $m=0,1,...M_k$ are unknown current coefficients, the SIE-Rs (3) got the form

$$U_n = \frac{1}{4\pi\underline{\sigma}_1}\left\{\sum_{i=1}^{2}\sum_{k=1}^{N_i}\sum_{m=0}^{M_k} I_{ikm}\left[K(s_n,l_{ik}) - \right.\right.$$

$$\left.\left. - \frac{m}{l_{ik}}\int_{s'_{ik}=0}^{l_{ik}}\left(\frac{s'_{ik}}{l_{ik}}\right)^{m-1} K(s_n,s'_{ik})\,\mathrm{d}\,s'_{ik}\right]\right\}, \; n=1,2. \quad (6)$$

The total number of unknown coefficients in (6) is $N_u = \sum_{i=1}^{2}\sum_{k=1}^{N_i}(M_k+1)$ and there are N_1+N_2 conditions for currents having form [3],

$$I_{ik}(s'_{ik}=l_{ik}) = -a_{ik}I'_{ik}(s'_{ik}=l_{ik}), \; k=1,2,...N_i, \; i=1,2. \quad (7)$$

The rest of the linear equations is formed matching (6) at the conductors surfaces points defined with

$$s_{nj} = \frac{j}{M_k+1}l_{ik}, \; j=1,2,...M_k, \; k=1,2,...N_i, \; i=1,2. \quad (8)$$

After solving the system given with Eqs. (6) and (7), the current coefficients in (5), i.e. current distribution along the conductors are obtained.

The SIE-Rs (6) with conditions (7) are solved for two regimes of feeding ([12]) so-called: symmetric ($U_1 = U_2 = = U_s = 1V$) and anti-symmetric ($U_1 = -U_2 = U_a = 1V$). After determining coefficients in (5) the feeding currents using (4) are obtained, i.e. $I_{gk}^{s/a}$, $k=1,2$. Labels "*s/a*" denote solution for the currents corresponding to the supplying potential U_s - symmetric, i.e. U_a - anti-symmetric. Using the same procedure as in [12], after determining "*Y*" parameters, corresponding "*Z*" parameters of the LGSs are obtained.

In the general case the known following relations between electrodes voltage and their feeding currents are,

$$U_1 = Z_{11}I_{g1} + Z_{12}I_{g2},$$

$$U_2 = Z_{21}I_{g1} + Z_{22}I_{g2}. \quad (9)$$

In (9), Z_{11} and Z_{22} are self-impedances of the LGS-1, i.e. LGS-2, while $Z_{12} = Z_{21}$ are mutual-impedances between two systems. "*Z*" parameters represent integral grounding system characteristics and measure of mutual influence level between coupled LGSs.

If two LGSs are connected, i.e. formed the unique grounding system, substituting $U_1 = U_2 = U_s (=1V)$ in (9), grounding impedance can be determined as

$$Z_g = R_g + \mathrm{j}X_g = 1/(I_{g1}^s + I_{g2}^s). \quad (10)$$

III. Numerical results

In order to illustrate describe general model for coupled LGSs system analyze, two linear grounding electrode having length l_1 and l_2 and circular cross section of radius a_1 and a_2 ($N_1 = 1$, $N_2 = 1$), arbitrary placed in homogeneous ground in the plane $y=0$, are observed. Electrodes are fed over isolated earthing conductors of vertical length in the ground h_1 and h_2 with currents $I_{\mathrm{g}1}$ and $I_{\mathrm{g}2}$, respectively. The distance between feeding point is $d = x_{02} - x_{01}$.

In the following text, angles θ_1 and θ_2 are defined as

$$\theta_k = \pm\arccos(\hat{s}'_k \hat{z}),\ k = 1,2, \tag{11}$$

where $\hat{s}'_k$ is the ort defining direction of k-th conductor, $k = 1,2$. The sign "-" is used when angle between $\hat{x}$ and $\hat{s}'_k$-ort is less than 90^0.

The convergence of "Z" parameters (real parts values) with increasing polynomial degrees of the approximation (5) are presented in Table I. System geometry parameters are: $h_1 = h_2 = 0.5\,\mathrm{m}$, $l_1 = 1.5\,\mathrm{m}$, $l_2 = 2\,\mathrm{m}$, $a_1 = a_2 = 25\,\mathrm{mm}$, $d = 4\,\mathrm{m}$, $\theta_1 = 30^0$, $\theta_2 = 45^0$, $\sigma_1 = 0.01\,\mathrm{S/m}$ and $\varepsilon_{r1} = 10$.

TABLE I

REAL PARTS OF "Z" PARAMETERS, $Z_{..}$ IN $[\Omega]$, VERSUS POLYNOMIAL DEGREES

$M_1 = M_2$	$\mathrm{Re}\{Z_{11}\}$	$\mathrm{Re}\{Z_{12}\}$	$\mathrm{Re}\{Z_{21}\}$	$\mathrm{Re}\{Z_{22}\}$
1	46.4306	3.4715	3.4445	37.8645
2	45.8434	3.4651	3.4402	37.4187
3	44.5614	3.4592	3.4579	36.4543
4	44.3778	3.4594	3.4569	36.3150
5	43.9354	3.4588	3.4586	35.9826

Real parts of self impedance Z_{11} and mutual impedance Z_{12} versus angle θ_2, with angle θ_1 as parameter, are shown in Fig. 2 and Fig. 3, respectively. The parameters values are: $h_1 = h_2 = 0.5\,\mathrm{m}$, $l_1 = 1.5\,\mathrm{m}$, $l_2 = 2\,\mathrm{m}$, $a_1 = a_2 = 25\,\mathrm{mm}$, $d = 4\,\mathrm{m}$, $\sigma_1 = 0.01\,\mathrm{S/m}$ and $\varepsilon_{r1} = 10$. The polynomial degrees are $M_1 = M_2 = 3$.

Real parts of self Z_{22} and mutual impedance Z_{21}, versus angle θ_2 are shown in Fig. 4. The parameter values are: $h_1 = h_2 = 0.5\,\mathrm{m}$, $l_1 = 1.5\,\mathrm{m}$, $l_2 = 2\,\mathrm{m}$, $a_1 = a_2 = 25\,\mathrm{mm}$, $d = 2\,\mathrm{m}$, $\theta_1 = 0$, $\sigma_1 = 0.01\,\mathrm{S/m}$ and $\varepsilon_{r1} = 10$. The polynomial degrees are $M_1 = M_2 = 3$.

Real parts of self Z_{22} and mutual impedance Z_{21} versus distance d are presented in Fig. 5. The parameter values are: $h_1 = h_2 = 0.5\,\mathrm{m}$, $l_1 = 1.5\,\mathrm{m}$, $l_2 = 2\,\mathrm{m}$, $a_1 = a_2 = 25\,\mathrm{mm}$, $\theta_1 = 30^0$, $\theta_2 = 45^0$, $\sigma_1 = 0.01\,\mathrm{S/m}$ and $\varepsilon_{r1} = 10$. The polynomial degrees are $M_1 = M_2 = 3$.

The real part of the impedance of the unique grounding system formed by two conductors versus angle θ_2 for $M_1 = M_2 = 3$ is shown in Fig. 6. The parameter values are: $h_1 = h_2 = 0.5\,\mathrm{m}$, $l_1 = 1.5\,\mathrm{m}$, $l_2 = 2\,\mathrm{m}$, $a_1 = a_2 = 25\,\mathrm{mm}$, $d = 2\,\mathrm{m}$, $\theta_1 = 0$, $\sigma_1 = 0.01\,\mathrm{S/m}$ and $\varepsilon_{r1} = 10$.

Finally, ESP distribution at the ground surface in the conductors surrounding for grounding geometry parameters from Fig. 6 is presented in Fig. 7, for $\theta_2 = 45^0$.

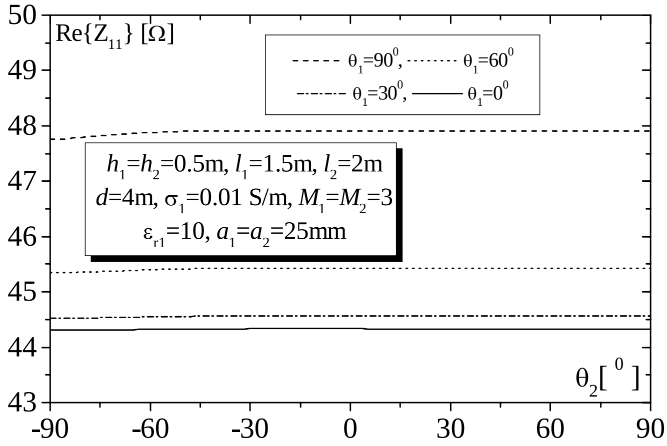

Fig. 2. The real part of self impedance Z_{11} of two coupled ground wire conductors, versus angle θ_2 and angle θ_1 as parameter.

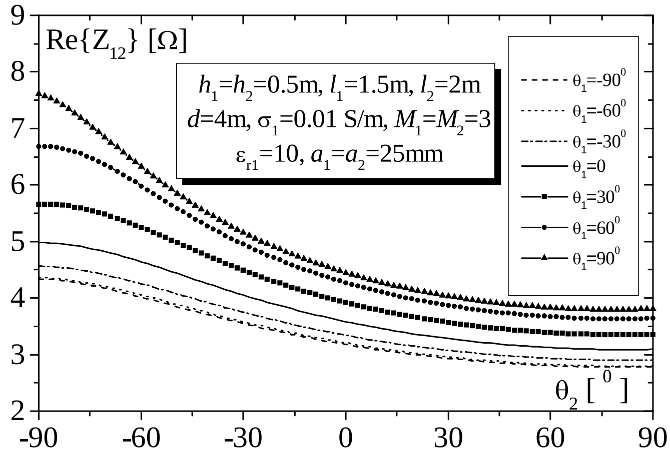

Fig. 3. The real part of mutual impedance Z_{12} of two coupled ground wire conductors, versus angle θ_2 and angle θ_1 as parameter.

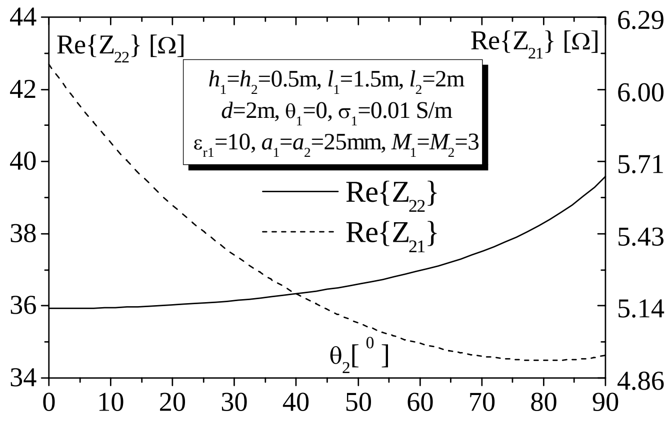

Fig. 4. The real part of self Z_{22} and mutual impedance Z_{21} of two coupled ground wire conductorss versus angle θ_2.

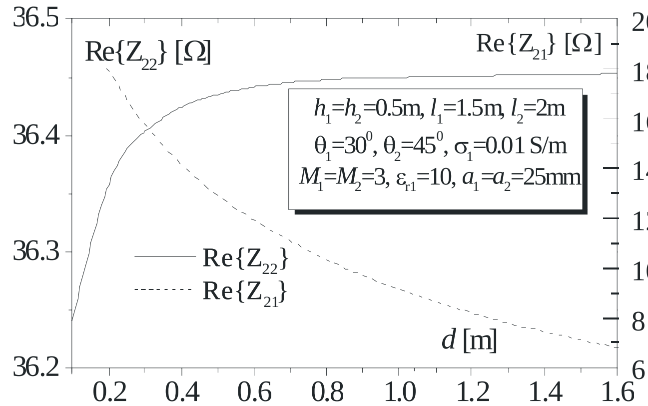

Fig. 5. The real part of self Z_{22} and mutual impedance Z_{21} of two coupled wire ground conductors, versus distance d.

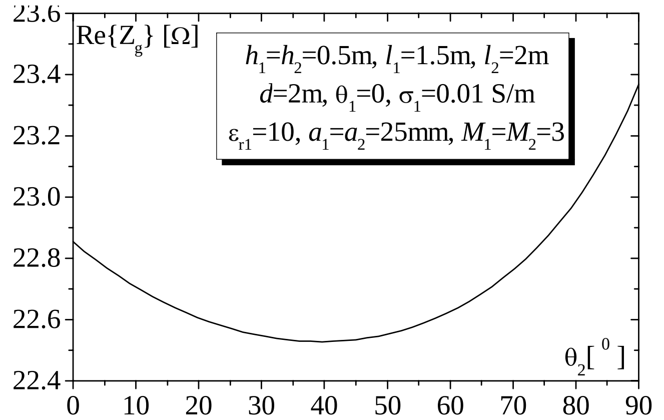

Fig. 6. The real part of grounding impedance of the unique grounding system formed by two conductors versus angle θ_2.

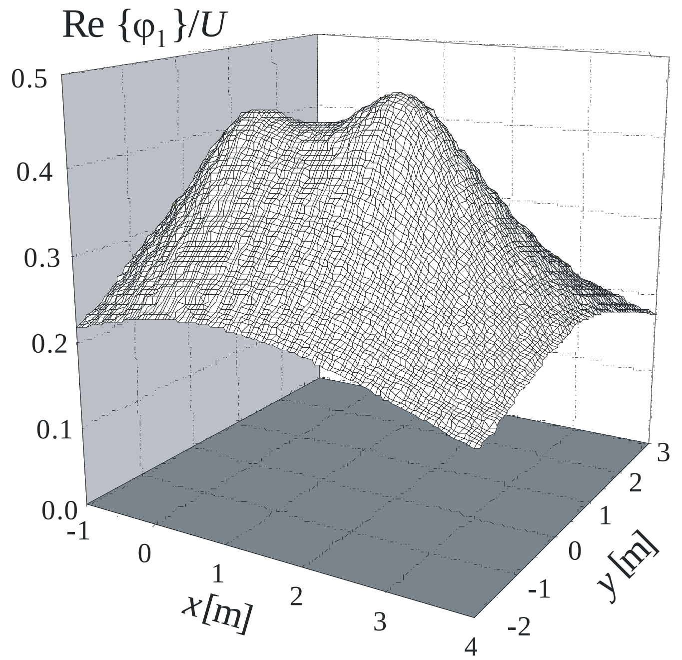

Fig. 7. Potential distribution at the ground surface.

IV. CONCLUSION

Unknown distribution of longitudinal current along the LGS conductors axis are numerically determined in the paper. It is carried out by numerical solving SIE-R using PMM and polynomial approximation for current distribution.

The described procedure is illustrated applying it on the problem of two coupled linear grounding electrode in symmetric and anti-symmetric regime. For this case, self and mutual impedance (determined from current distribution) are numerically calculated. ESP distribution at the ground surface is also determined.

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