ICEST 2007
24 - 27 June 2007, Ohrid, Macedonia

Probability Density Function of M-ary FSK Signal in the Presence of Impulse Noise and Nakagami Fading

Mihajlo Stefanović[1], Dragana Krstić[2], Nataša Kapacinović[3], Edis Mekić[4] and Fatih Dizdarević[5]

***Abstract* - In this paper the receiver for the demodulation of M-FSK signals in the presence of Gaussian noise, impulse noise and Nakagami fading is considered. Probability Density Function of M-ary FSK Signal in the Presence of Impulse Noise and Nakagami Fading is derived. These interferences can seriously degrade the communication systems performance.**

***Keywords* - *M*-ary Frequency Shift Keying, Impulse noise, Nakagami Fading, Probability Density function.**

I. INTRODUCTION

In this paper we consider a system for coherent demodulation of *M*-ary FSK signals in the presence of Gaussian noise, impulse noise and variable signal amplitude. These annoyances can seriously degrade the performance of communication systems [1]-[3]. In the paper [4], the performance evaluation of several types of FSK and CPFSK receivers was investigated in detail using the modified moment's method. Also, the error probability of the cross-correlator receiver for binary digital frequency modulation detection is studied using theoretical analysis and computer simulations [5]. In [6] average bit-error probability performance for optimum diversity combining of noncoherent FSK over Rayleigh channels is determined. Performance analysis of wide-band *M*-ary FSK systems in Rayleigh fading channels is given in [7]. The influence of impulse noise in noncoherent *M*-ary digital systems is observed in the paper [8].

In order to view an influence of the Gaussian noise, impulse noise and variable signal amplitude on the performances of an *M*-ary FSK system, we are derived the probability density function of *M*-ary FSK receiver output signal, the joint probability density function of output signal and its derivative, and the joint probability density function of output signal at two time instants. The bit error probability, the signal error probability and outage probability can be determined by the probability density function of output signal. Also, the moment generating function, the cumulative distribution of output signals and the moment and variance of output signals can be derived by probability density function of output signals. An average level crossing rate and an average fade derivation of an output signal process can be calculated by the joint probability density of an output signals and its derivative. The expression for calculation autocorrelation function can be derived by joint probability density function of an output signals at two time instants and by using Winner-Hinchine theorem the spectral power density function of an output *M*-ary FSK signal can be obtain. Based on this, the results obtained in this paper have a great significance.

This paper is organized as follows: first section is introduction. In second section the model of the *M*-ary FSK system is defined. The expressions for the probability density function of the output signal is obtained in the third section. In the next, fourth section, the numerical results in the case M=2 are given. The last section is the conclusion.

II. MODEL OF THE M-ARY FSK SYSTEM

The model of an M-ary FSK system, which we consider in this paper, is shown at Fig. 1. This system has *M* branches. Each branch consists of the bandpass filter and correlator. The correlator is consisting of multiplier and lowpass filter.

The signal at the input of the receiver is digital frequently modulated signal corrupted by additive Gaussian noise, impulse noise and Nakagami fading.

Transmitted signal for the hypothesis H_i is:

$$s(t) = A\cos\omega_i t \tag{1}$$

where *A* denotes the amplitude of the modulated signal and has Nakagami distribution.

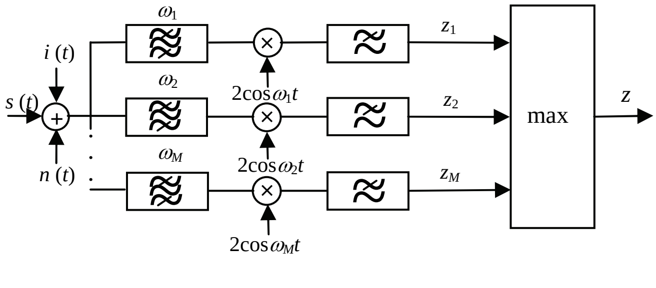

Fig. 1. Block diagram of the system for coherent demodulation of *M*-ary FSK signal

Gaussian noise at the input of the receiver is given with:

[1]Mihajlo Č. Stefanović is with Faculty of Electronic Engineering, A. Medvedeva 14, 18000 Nis, Serbia, e-mail: misa@elfak.ni.ac.yu
[2]Dragana Krstić is with the Faculty of Electronic Engineering, A. Medvedeva 14, 18000 Nis, Serbia, e-mail:dragana@elfak.ni.ac.yu
[3]Nataša D. Kapacinović is with the Faculty of Electronic Engineering, A. Medvedeva 14, 18000 Nis, Serbia, e-mail: nale@elfak.ni.ac.yu
[4]Edis Mekić is with the Faculty of Electronic Engineering, A. Medvedeva 14, 18000 Nis, Serbia.
[5]Fatih Dizdarević is with the Faculty of Electronic Engineering, A. Medvedeva 14, 18000 Nis, Serbia

$$n(t)=\sum_{i=1}^{M} x_i\cos\omega_i t+y_i\sin\omega_i t\ ,\quad i=1,2,...M \tag{2}$$

where x_i and y_i are components of the Gaussian noise, with zero means and variances σ^2.

The pulse interference *i(t)* can be written as:

$$i(t)=\sum_{i=1}^{M}(A_i+cn)\cos(\omega_i t+\theta_i) \tag{3}$$

c is constant and *n* has Poisson's distribution:

$$p(n)=\frac{\lambda^n}{n!}e^{-\lambda} \tag{4}$$

λ is intensity of impulse process. Phases θ_i, i=1,2,...,M have uniform probability density function.

These signals pass first through bandpass filter whose central frequencies ω_1, ω_2,..., ω_M correspond to hypotheses $H_1,H_2,...,H_M$. Then, after multiplying with signal from the local oscillator, they pass through lowpass filter which cut all spectral components which frequencies are greater than the border frequency of the filter.

If z_1, z_2, …z_M are the output signals of the branch of receiver, then *M*-FSK receiver output signal is:

$$z=\max\{z_1,z_2...z_M\} \tag{5}$$

The probability density of output signal is

$$p_z(z)=\sum_{i=1}^{M}p_{z_i}(z)\cdot\prod_{\substack{j=1\\ j\neq i}}^{M}F_{z_j}(z) \tag{6}$$

III. Probability density function

In the case of the hypothesis H_1, transmitted signal is:

$$s(t)=A\cos\omega_1 t \tag{7}$$

while the output branch signals of the receiver are:

$$z_1=A+x_1+A_1\cos\theta_1 \tag{8}$$

$$z_k=x_k+A_k\cos\theta_k\ ,\qquad k=2,3,\dots,M \tag{9}$$

It is necessary to define the probability density functions on the output of branches and the cumulative density of these signals to obtain output probability density function of *M*-ary FSK receiver.

The conditional probability density functions for the signals z_1, z_2,..., z_M are:

$$p_{z_1/A,\theta_1}(z)=\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(z-A-(A_1+cn)\cos\theta_1)^2}{2\sigma^2}} \tag{10}$$

σ denotes standard deviation.

$$p_{z_k/A,\theta_k}(z)=\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(z-(A_k+cn)\cos\theta_k)^2}{2\sigma^2}}\ ,\quad k=2,3,\dots,M \tag{11}$$

By averaging (11) and (12) we obtain the probability density functions of the signals on the output of the branches:

$$p_{z_1}(z)=\int_0^{\infty}\int_{-\pi}^{\pi}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(z-A-(A_1+cn)\cos\theta_1)^2}{2\sigma^2}}\cdot$$

$$\cdot\sum_{n=0}^{\infty}\frac{\lambda^n}{n!}e^{-\lambda}\frac{2}{\Gamma(m_1)}\left(\frac{m_1}{\Omega_1}\right)A^{2m_1-1}e^{-\frac{m_1}{\Omega_1}A^2}dA\frac{1}{2\pi}d\theta_1 \tag{12}$$

$$p_{z_k}(z)=\int_0^{\infty}\int_{-\pi}^{\pi}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(z-(A_k+cn)\cos\theta_k)^2}{2\sigma^2}}\cdot$$

$$\cdot\sum_{n=0}^{\infty}\frac{\lambda^n}{n!}e^{-\lambda}\frac{2}{\Gamma(m_k)}\left(\frac{m_k}{\Omega_k}\right)A^{2m_k-1}e^{-\frac{m_k}{\Omega_k}A^2}dA\frac{1}{2\pi}d\theta_k \tag{13}$$

The cumulative distributions of the signals z_1, z_2,..., z_M are:

$$F_{z_1}(z)=\int_{-\infty}^{z}\int_0^{\infty}\int_{-\pi}^{\pi}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(r-A-(A_1+cn)\cos\theta_1)^2}{2\sigma^2}}\cdot$$

$$\cdot\sum_{n=0}^{\infty}\frac{\lambda^n}{n!}e^{-\lambda}\frac{2}{\Gamma(m_1)}\left(\frac{m_1}{\Omega_1}\right)A^{2m_1-1}e^{-\frac{m_1}{\Omega_1}A^2}dA\frac{1}{2\pi}d\theta_1 dr \tag{14}$$

$$F_{z_k}(z)=\int_{-\infty}^{z}\int_0^{\infty}\int_{-\pi}^{\pi}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(r-(A_k+cn)\cos\theta_k)^2}{2\sigma^2}}\cdot$$

$$\cdot\sum_{n=0}^{\infty}\frac{\lambda^n}{n!}e^{-\lambda}\frac{2}{\Gamma(m_k)}\left(\frac{m_k}{\Omega_k}\right)A^{2m_k-1}e^{-\frac{m_k}{\Omega_k}A^2}dA\frac{1}{2\pi}d\theta_k dr \tag{15}$$

The probability density function of the *M*-ary FSK receiver output signal in the case of the hypothesis H_1 can be obtained from:

$$p_z(z)=\sum_{i=1}^{M}p_i(z)\cdot\prod_{\substack{j=1\\ j\neq i}}^{M}F_j(z) \tag{16}$$

IV. Numerical resultats

We now consider the dual branch FSK receiver because of its easy implementation and very good performances. It is employed in many practical telecommunication systems.

The probability density function, in case of dual branch, has form:

$$p_z(z)=p_{z_1}(z)\cdot F_{z_2}(z)+p_{z_2}(z)\cdot F_{z_1}(z) \tag{17}$$

The probability density functions $p(z)$ versus output signal z, with and without interference and impulse noise, for various values of fading severity parameter m and average power Ω are shown at Figs. 2 and 3, respectively.

At Figs. 4. to 6. probability density functions $p(z)$ versus output signal z, are given depends on parameter λ.

The probability density functions $p(z)$ versus output signal z, when dependence is on standard deviation σ.

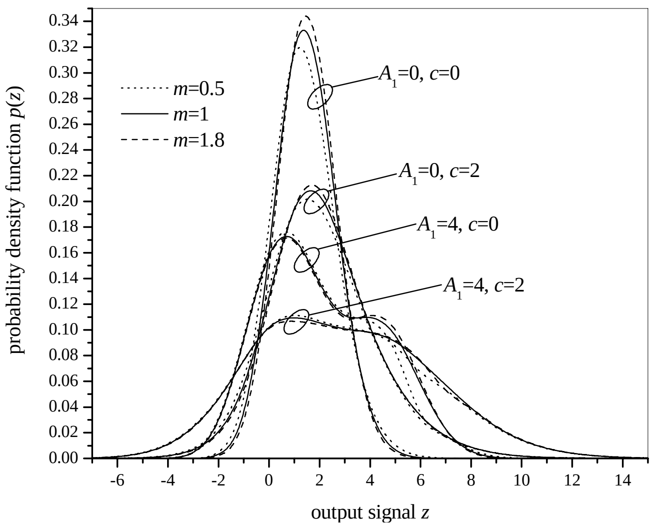

Fig. 2. The probability density functions $p(z)$ for the parameters σ=1, λ=1 and Ω=2

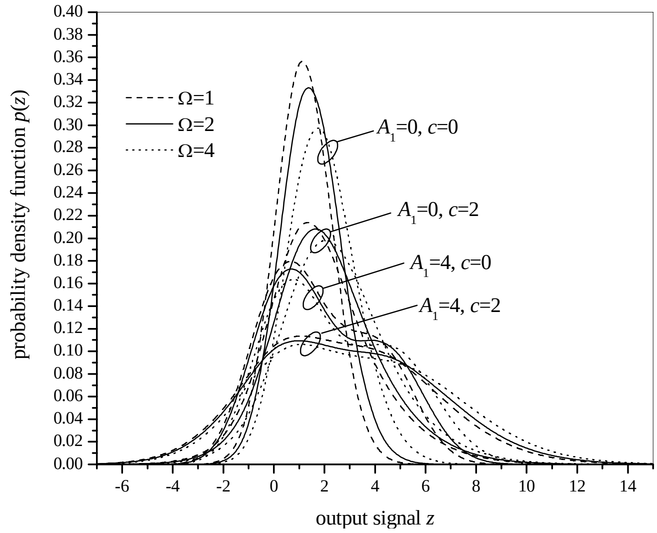

Fig. 3. The probability density functions $p(z)$ for the parameters σ=1, λ=1 and m=1

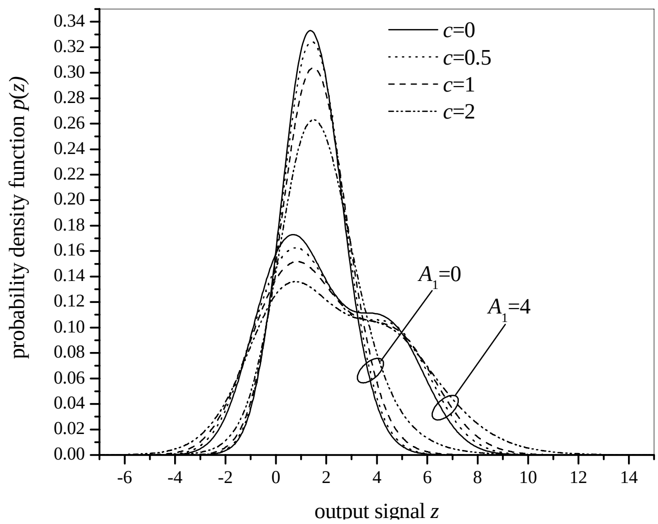

Fig. 4. The probability density functions p(z) for the parameters σ=1, λ=0.5, Ω=2 and m=1

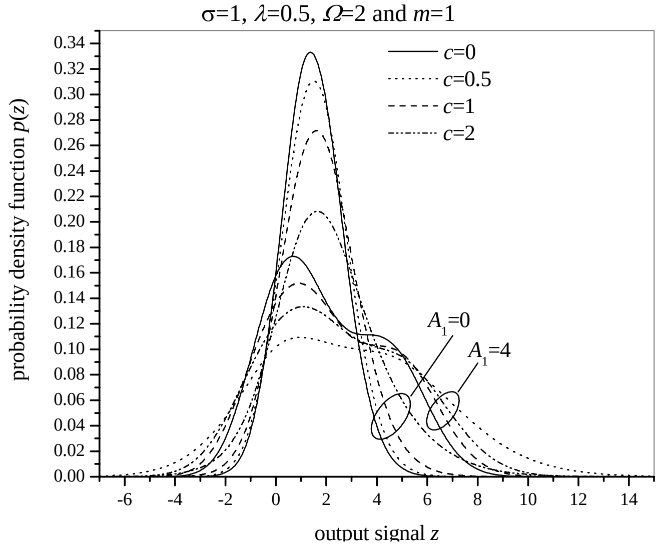

Fig. 5. The probability density functions p(z) for the parameters σ=1, λ=1, Ω=2 and m=1

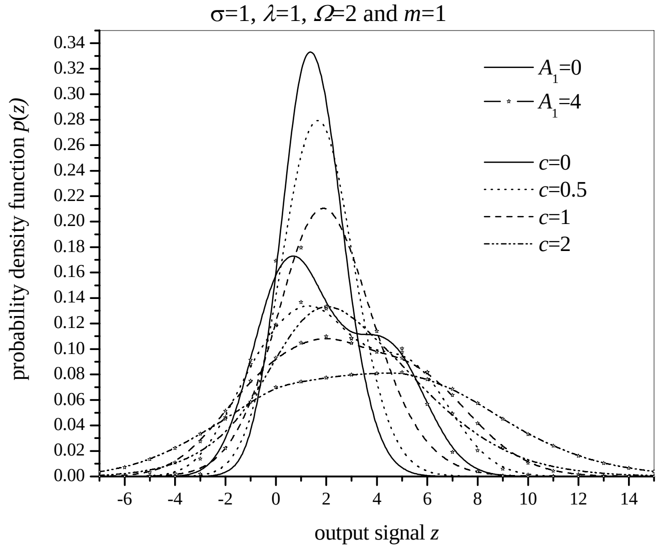

Fig. 6. The probability density functions p(z) for the parameters σ=1, λ=2, Ω=2 and m=1

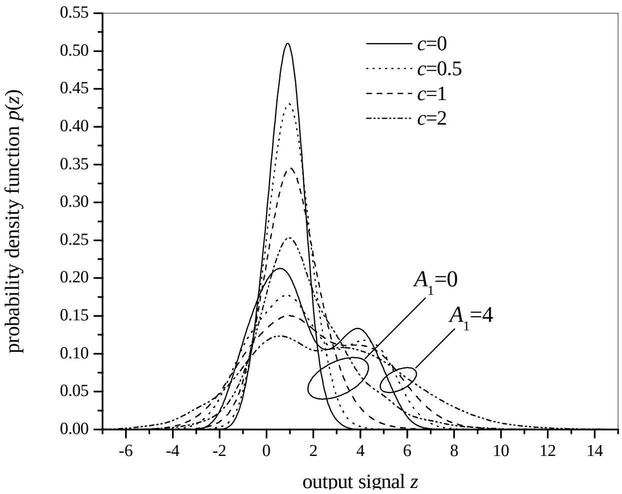

Fig. 7. The probability density functions p(z) for the parameters σ=0.5, λ=1,Ω=2 and m=1

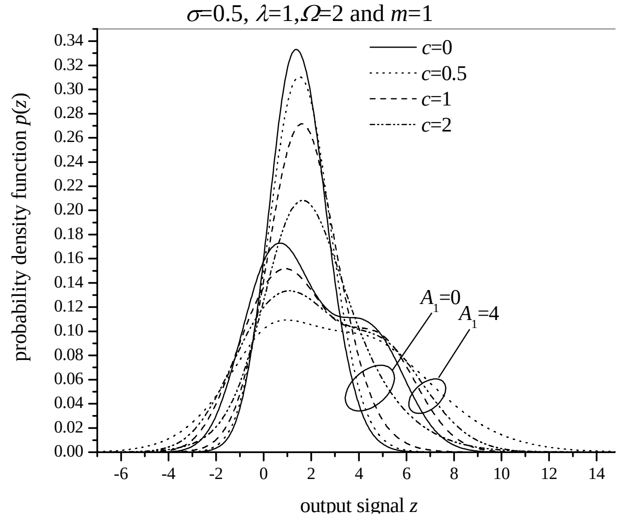

Fig. 8. The probability density functions p(z) for the parameters σ=1, λ=1, Ω=2 and m=1

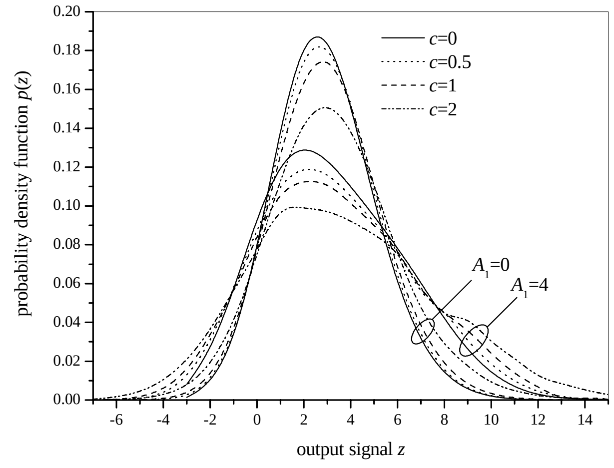

Fig. 9. The probability density functions p(z) for the parameters σ=2, λ=1, Ω=2 and m=1

V. CONCLUSION

The statistical characteristics of signal at the output of receiver for coherent FSK demodulation are derived in this paper. The input signal of the receiver is digital frequently modulated signal corrupted by additive Gaussian noise, impulse noise and Nakagami fading. The interferences appear in each receiver branch. In this paper the probability density function of *M*-ary FSK receiver output signal is derived. The bit error probability and the outage probability can be determined by the probability density function of output signal.

REFERENCES

[1] T. Okoshi and K. Kikuchi, *Coherent Optical Fiber Communications*, Tokyo, Japan: KTW Scientific, 1988.

[2] J. M. Senior, *Optical Fiber Communications*, New York: Prentice Hall, 1992.

[3] J. G. Proakis, *Digital Communications 2nd ed*. New York, McGraw-Hill, 1989.

[4] H. Miin-Jong and B. Stephen Wicker, “Performance evaluation of FSK and CPFSK optical communication systems: a stable and accurate method”, J. Lightwave Technol., vol. 13, no. 8, Aug. 1995.

[5] K. A. Farrell and P. J. McLane, “Performance of the cross-correlator receiver for binary digital frequency modulation”, IEEE Trans. Commun., vol. 45, no 5, May 1997.

[6] M..K Simon, and M. Alouini, “Average Bitt-Error Probability Performance for Optimum Diversity Combining of Noncoherent FSK Over Rayleigh channels”, ”, IEEE Trans. on Commun., vol. 51, no 4, pp.566-569, April 2003.

[7] Y. K. Jeong, K. B. Lee, “Performance analysis of wide-band M-ary FSK systems in Rayleigh fading channels”, IEEE Trans. on Commun., vol.48, Issue 12, pp. 1983–1986, Dec. 2000.

[8] H. Huynh, M. Lecours, “Impulsive Noise in Noncoherent M-ary Digital Systems”, IEEE Trans. on Commun., vol. 23, Issue 2, pp. 246 – 25, Feb 1975.