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Performances of Receiver with MRC, SC and SSC Combiner in the Presence of Fading and Gaussian Noise

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Abstract **- In this paper work we consider statistical characteristics of signal at the output of combiner with two diversity branches in channel with Nakagami-m fading. We analyze MRC, SC i SSC combining. Fading is identical and independent**.

I. Introduction

In this work we consider diversity system with two branches and predetection combining. Fading appears due to conveying signals over multiple paths. Amplitudes of signals over the branches are independent. Probability densities of the amplitudes are Nakagami-m with same parametres. In this paper is analyzed diversity system in presence of Nakagami-m fading which is not correlated between branches. At first we consider signal at output of MRC combiner. Input signals are in phase. Their squires of signal/noise ratios are collected and this sum is signal at output of MRC combiner. Probability density function of signal at output is combination of probability density functions of signals at its inputs.

Characteristic of signal function at output is product of characteristics of signal functions at inputs of MRC combiner. Based on probability density of signals at output, average value and variance of signals at output of combiner can be calculated. Average value and variance of output signal can be obtained by characteristic functions of signals at output of combiner. The output signal of SC combiner is equal to the value of its input which is higher. There is probability that any of input signals is higher than other. If the signal from the first input is higher than signal from the second, then the probability density function of output signal is equal to the probability density function of signal from first input multiplicated by the probability that the signal at second input is lower than the signal at first input. At this way the probability density function at the output of SC combiner is determinated. Characteristic function of output signal and its momentums can be also determinated.

If we analyze SSC combinig we consider that combiner first examines signal from one input. If this signal is higher than some beforehand determinated threshold then it is forwarded to circuit for demodulation. If this signal is lower than threshold than the signal from other branch is forwarded to circuit for demodulation. For example, in this paper, is determinated probability density function of signal at output of combiner. For calculating these probabilities we used Gilbert model.

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II. MRC combiner

Model MRC is presented on the figure 1. Input signals are r_1 i r_2, and the output signal is r. The squares of signals r_1 i r_2 are $\gamma_1 = r_1^2$ i $\gamma_2 = r_2^2$.
Signal power at the output of combiner is γ:
$\gamma = \gamma_1 + \gamma_2$ ili $\gamma_1 = \gamma - \gamma_2$.

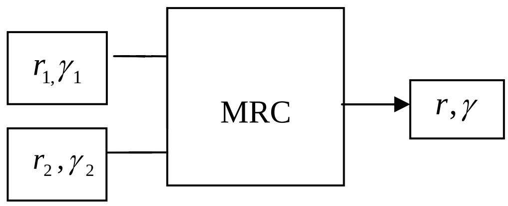

Fig.1. Model of MRC combiner

Conditional probability function of the signal γ at the output of the combiner is:

$$p(\gamma/\gamma_1) = \left|\frac{d\gamma_1}{d\gamma}\right| p_{\gamma_1}(\gamma - \gamma_2) \qquad (1)$$

The probability density function (PDF) of the signal can be acquired by averaging the previous function:

$$p_\gamma(\gamma) = \int_0^\gamma p_{\gamma_1}(\gamma - \gamma_2) p_{\gamma_2}(\gamma_2) d\gamma_2 \qquad (2)$$

Because $\frac{dr_1}{dr} = 1$, when the Nakagami-m fading is present, the probability density function of the amplitude of signals at the input of combiner are equal.

$$p_{r_1}(r_1) = \frac{2}{\Gamma(m)}\left(\frac{m}{\Omega}\right)^m r_1^{2m-1} e^{-\frac{m}{\Omega}r_1^2} \quad (3)$$

$$p_{r_2}(r_2) = \frac{2}{\Gamma(m)}\left(\frac{m}{\Omega}\right)^m r_2^{2m-1} e^{-\frac{m}{\Omega}r_2^2} \quad (4)$$

We discuss the case when the fading between the branches is independent and identical. Parameter m can take the following values (0.5, ... ∞). For $m=1$ Nakagami-m fading becomes Rayleigh fading, and for $m=\frac{1}{2}$ Nakagami-m feding becomes Gaussian fading. For $m\rightarrow\infty$ there is no fading. According to: $\gamma_1 = {r_1}^2$, $r_1=\sqrt{\gamma_1}$, $\frac{dr_1}{d\gamma_1}=\frac{1}{2\sqrt{\gamma_1}}$, $r_1 \geq 0$, $\gamma_1 \geq 0$, we acquire:

$$p_{\gamma_1}(\gamma_1)=\frac{1}{\Gamma(m)}\left(\frac{m}{\Omega}\right)^m {\gamma_1}^{m-1} e^{-\frac{m}{\Omega}\gamma_1} \quad (5)$$

At the same way we acquire:

$$p_{\gamma_2}(\gamma_2)=\frac{1}{\Gamma(m)}\left(\frac{m}{\Omega}\right)^m {\gamma_2}^{m-1} e^{-\frac{m}{\Omega}\gamma_2} \quad (6)$$

Probability density function of the signal γ is:

$$p_\gamma(\gamma)=\int_0^\gamma \frac{1}{\Gamma(m)}\left(\frac{m}{\Omega}\right)^m (\gamma-\gamma_2)^{m-1} e^{-\frac{m}{\Omega}(\gamma-\gamma_2)}$$
$$\frac{1}{\Gamma(m)}\left(\frac{m}{\Omega}\right)^m {\gamma_2}^{m-1} e^{-\frac{m}{\Omega}\gamma_2} d\gamma_2 = \quad (7)$$
$$\frac{1}{\Gamma^2(m)}\left(\frac{m}{\Omega}\right)^{2m} e^{-\frac{m}{\Omega}\gamma}\gamma^{2m-1}\sum_{i=0}^{m-1}\binom{m-1}{i}\frac{1}{i+m}$$

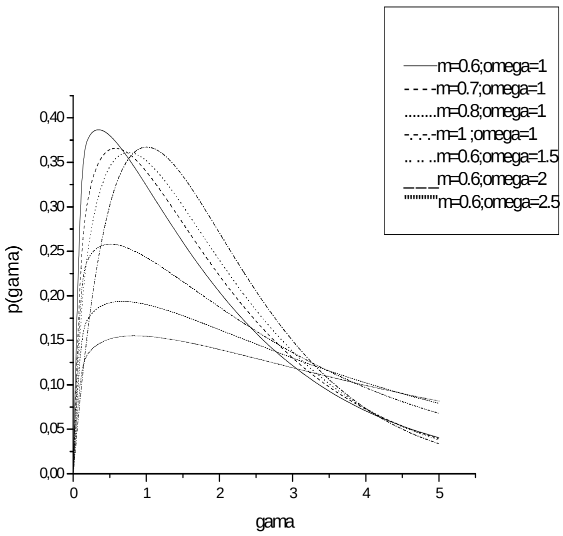

Fig.2.Probability density function of the signal γ

Characteristic function of the signal at the output of the combiner is :

$$M_\gamma(s)=\int_0^\infty e^{\gamma s} p_\gamma(\gamma)d\gamma =$$
$$=\int_0^\gamma e^{\gamma s}\frac{1}{\Gamma^2(m)}\left(\frac{m}{\Omega}\right)^{2m} e^{-\frac{m}{\Omega}\gamma}\gamma^{2m-1}$$
$$\sum_{i=0}^{m-1}\binom{m-1}{i}\frac{1}{i+m}=\frac{1}{\Gamma^2(m)}\left(\frac{m}{\Omega}\right)^{3m} \quad (8)$$
$$\sum_{i=0}^{m-1}\binom{m-1}{i}\frac{1}{i+m}\frac{\Omega^{2m}}{(m-\Omega s)^{2m}}\Gamma(2m)$$

Momentum of n-order of the signal at the output of combiner is :

$$m_n=\overline{\gamma^n}=\int_0^\infty \gamma^n p_\gamma(\gamma)d\gamma=\int_0^\infty\gamma^n\frac{1}{\Gamma^2(m)}\left(\frac{m}{\Omega}\right)^{2m}$$
$$\gamma^{2m-1}e^{-\frac{m}{\Omega}\gamma}\sum_{i=0}^{m-1}\binom{m-1}{i}\frac{1}{i+m}d\gamma=$$
$$\frac{1}{\Gamma^2(m)}\left(\frac{m}{\Omega}\right)^{2m}\sum_{i=0}^{m-1}\binom{m-1}{i}\frac{1}{i+m} \quad (9)$$
$$\left(\frac{m}{\Omega}\right)^{n+2m}\Gamma(n+2m)$$

For n=1 the average value of the signal at the output of MRC combiner is determinated like:

$$m_1=\frac{1}{\Gamma^2(m)}\sum_{i=0}^{m-1}\frac{1}{i+m}\left(\frac{\Omega}{m}\right)^{2m}\Gamma(2m+1)\left(\frac{\Omega}{m}\right) \quad (10)$$

For n=2 mean squared value is acquired:

$$m_2=\frac{1}{\Gamma^2(m)}\sum_{i=0}^{m-1}\binom{m-1}{i}\frac{1}{i+m}\Gamma(2m+1)\left(\frac{\Omega}{m}\right)^2 \quad (11)$$

Cumulative probability function of the output signal is:

$$F_\gamma(\gamma)=\int_0^\gamma p_\gamma(x)dx=\int_0^\gamma\frac{1}{\Gamma^2(m)}\left(\frac{m}{\Omega}\right)^{2m}x^{2m-1}$$
$$e^{-\frac{m}{\Omega}x}\sum_{i-0}^{m-1}\binom{m-1}{i}\frac{1}{i+m}dx=$$
$$\left(\frac{m}{\Omega}\right)^{2m}\Gamma\left(2m,\frac{m}{\Omega}\gamma\right)\frac{1}{\Gamma^2(m)} \quad (12)$$
$$\sum_{i=0}^{m-1}\binom{m-1}{i}\left(\frac{1}{i+m}\right),$$

where the function $\Gamma\left(2m, \frac{m}{\Omega}\gamma\right)$ is incomplete gamma function. The probability density function of the signal at the output of the combiner can be used for evaluating the probability of the signal by the formula :

$$P_0 = \int_0^{\gamma_t} p = F_\gamma(\gamma_t) = \frac{1}{\Gamma^2(m)} \sum_{i=0}^{m-1} \frac{1}{i+m} \Gamma\left(2m, \frac{m}{\Omega}\gamma_t\right) \quad (13)$$

III. SC COMBINER

The output signal of SC combiner is determinated like maximum of its input signals:

$$\gamma = \max(\gamma_1, \gamma_2)$$

When the signal γ_1 is higher than the signal γ_2 than the probability density function of the signal γ is determinated like production of its probability density functions. When the signal γ_2 is higher than the signal γ_1 then the probability density function of the signal γ is counted like production of probability density function (PDF) of the signal γ_2 and the probability that the signal γ_1 is lower than the signal γ_2. According to this:

$$\begin{aligned} p_\gamma(\gamma) &= p_{\gamma_1}(\gamma) P(\gamma_2 < \gamma_1) + \\ & p_{\gamma_2}(\gamma) P(\gamma_1 < \gamma_2) = \\ &= p_{\gamma_2}(\gamma) F_{\gamma_2}(\gamma) + p_{\gamma_1}(\gamma) F_{\gamma_1}(\gamma) \end{aligned} \quad (14)$$

Cumulative probability function of the signal γ_1 is:

$$F_{\gamma_1}(\gamma) = \int_0^{\gamma_1} p_{\gamma_1}(x) dx = \int_0^{\gamma_1} \frac{1}{\Gamma(m)} \left(\frac{m}{\Omega}\right)^m x^{m-1} e^{-\frac{m}{\Omega}x} dx = \frac{1}{\Gamma(m)} \left(\frac{m}{\Omega}\right)^m \left(\frac{\Omega}{m}\right)^m \Gamma\left(m, \frac{m}{\Omega}\gamma_1\right) \quad (15)$$

At the same way, the cumulative probability function of the signal γ_2 is acquired:

$$F_{\gamma_2}(\gamma_2) = \int_0^{\gamma_2} p_{\gamma_2}(x) dx = \frac{1}{\Gamma(m)} \Gamma\left(m, \frac{m}{\Omega}\gamma_2\right) \quad (16)$$

According to all of this the probability density function of the output signal from SC combiner is:

$$p_\gamma(\gamma) = \frac{2}{\Gamma^2(m)} \left(\frac{m}{\Omega}\right)^m \gamma^{2m-1} e^{-\frac{m}{\Omega}\gamma} \Gamma\left(m, \frac{m}{\Omega}\gamma\right) \quad (17)$$

Joint cumulative probability of the input signals γ_1 and γ_2 is:

$$F_{\gamma_1\gamma_2}(\gamma_1\gamma_2) = F_{\gamma_1}(\gamma_1) F_{\gamma_2}(\gamma_2) \quad (18)$$

Cumulative probability function of the signal γ at the output of the SC combiner is :

$$F_\gamma(\gamma) = F_{\gamma_1\gamma_2}(\gamma_1\gamma_2) = F_{\gamma_1}(\gamma_1) F_{\gamma_2}(\gamma_2) = \frac{1}{\Gamma^2(m)} \Gamma^2\left(m, \frac{m}{\Omega}\gamma\right) \quad (19)$$

Cumulative probability function of the output signal γ can be also accomplished at the following way :

$$p_\gamma(\gamma) = \frac{dF_\gamma(\gamma)}{d\gamma} = \frac{d\left(F_{\gamma_1}(\gamma) F_{\gamma_2}(\gamma)\right)}{d\gamma} = p_{\gamma_1}(\gamma) F_{\gamma_2}(\gamma) + p_{\gamma_2}(\gamma) F_{\gamma_1}(\gamma) \quad (20)$$

IV. SSC COMBINER

At first, the signal at one branch is examined, analyzing SSC combiner. If this signal is higher than some beforehand determinated threshold then it is forwarded to circuit for demodulation of signal. If this signal is lower than threshold then the signal from other branch is forwarded to circuit for demodulation. Probability p_1 is the probability that the signal from first branch is firstly examined and p_2 is probability that the signal from second branch is firstly examined. Let assume $\gamma < \gamma_T$. If the signal from the first branch is firstly examined than the signal from the second branch is forwarded to circuit for demodulation because of the given condition $\gamma < \gamma_T$. Based on this:

$$p_\gamma(\gamma) = p_1 p_{\gamma_2}(\gamma) + p_2 p_{\gamma_1}(\gamma) \quad (21)$$

If the condition $\gamma > \gamma_T$ is filled at least in one of two branches then the signal is higher than γ_T. The signal from the first branch is firstly examined. If the signal is higher than γ_T then it is forwarded to the circuit for demodulation. If the signal is lower than γ_T then the signal from the second branch is forwarded to the circuit for demodulation because of the condition $\gamma > \gamma_T$. The same thing is done when the signal from the second branch is firstly examined. Based on this the probability density function at the output of SC combiner, according to $\gamma > \gamma_T$, is determinated:

$$\begin{aligned} p_\gamma(\gamma) &= p_1 p_{\gamma_1}(\gamma) + p_1 p_{\gamma_2}(\gamma) F_{\gamma_1}(\gamma_T) + \\ & p_2 p_{\gamma_2}(\gamma) + p_2 p_{\gamma_1}(\gamma) F_{\gamma_2}(\gamma_T) \end{aligned} \quad (22)$$

Moreover :

$$p_\gamma(\gamma) = p_1 \frac{2}{\Gamma(m)}\left(\frac{m}{\Omega}\right)\gamma^{m-1} e^{-\frac{m}{\Omega}\gamma} + p_2 \frac{2}{\Gamma(m)} \gamma^{m-1} e^{-\frac{m}{\Omega}\gamma}$$
$$\frac{1}{\Gamma(m)}\Gamma\left(m, \frac{m}{\Omega}\gamma_T\right)$$
$$+ p_2 \frac{2}{\Gamma(m)}\left(\frac{m}{\Omega}\right)\gamma^{m-1} e^{-\frac{m}{\Omega}\gamma} + p_1 \frac{2}{\Gamma(m)}\left(\frac{m}{\Omega}\right)\gamma^{m-1} e^{-\frac{m}{\Omega}\gamma} \qquad (23)$$
$$\Gamma\left(m, \frac{m}{\Omega}\gamma_T\right) = \frac{2}{\Gamma(m)}\left(\frac{m}{\Omega}\right)\gamma^{m-1} e^{-\frac{m}{\Omega}\gamma}(p_1 + p_2)$$
$$\left[1 + \Gamma\left(m, \frac{m}{\Omega}\gamma_T\right)\right]$$

Probabilities p_1 i p_2 are determinated by the Gilbert's model which is presented on figure 3:

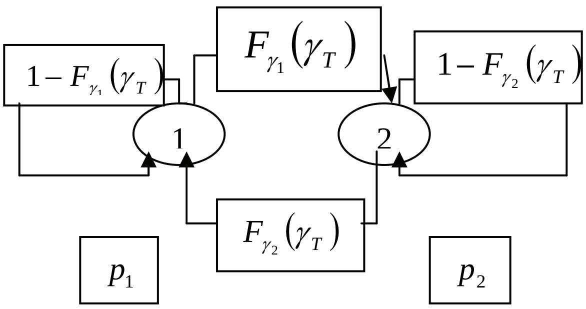

Fig.3. Gilbert's model

Equations of conditions for this model are :

$$\begin{aligned} p_1 &= p_1\left[1 - F_{\gamma_1}(\gamma_T)\right] + p_2 F_{\gamma_2}(\gamma_T) \\ p_2 &= p_1 F_{\gamma_1}(\gamma_T) + p_1\left[1 - F_{\gamma_2}(\gamma_T)\right] \\ p_1 + p_2 &= 1 \end{aligned} \qquad (24)$$

Evaluating these equations it can be acquired:

$$\begin{aligned} p_1 &= \frac{F_{\gamma_2}(\gamma_T)}{F_{\gamma_1}(\gamma_T) + F_{\gamma_2}(\gamma_T)} \\ p_2 &= \frac{F_{\gamma_1}(\gamma_T)}{F_{\gamma_1}(\gamma_T) + F_{\gamma_2}(\gamma_T)} \end{aligned} \qquad (25)$$

V. CONCLUSION

In this paper we consider diversity system with two branches and predetection combining. There is the Nakagami-m fading at input branches of the combiner. The spatial diversity system is used for reducing the influence of the fading on system performances. There are several ways of combining. The amplitude of signals can be collected, but also signal to noise ratios. In both cases it is needed to bring signals in phase at input. Because of this, these two ways of combining are expensive for realization. The process of combining where the signal to noise ratios are collected is the optimal. There is another process of combining, where the signal on input which is minimal is forwarded to the circuit for determination. The process isn't the optimal one but is simple for realization. The process of combining, where the signal from the input is examined and if it is higher than the threshold then it is forwarded to the circuit for demodulation and if it is lower then the other one is forwarded to the circuit, is the simplest for realization. In this paper are also determinated statistical characteristic of output signal of MRC, SC and SSC combiner. The most important is probability density function which is used to determinate bit error rate and state probability of the signal.

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