

# Substrate's Parameters Influence over the Directivity and Bandwidth at Microstrip Planar Arrays

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**Abstract** – In this paper are studied dependencies on dielectric substrate's parameters over the directivity and bandwidth at microstrip planar antenna arrays. It has been provided numerical calculations based on theoretical models for microstrip rectangular patch antennas.

**Keywords** – Microstrip antennas, Dielectric substrate, Bandwidth, Directivity

## I. INTRODUCTION

Microstrip antennas are widely used in modern radio communication systems. They are preferred for antenna arrays in microwave range (3-30 GHz) because of their small size, inexpensive and easy manufacture with printed-circuit technologies. Their main disadvantage is relatively narrow bandwidth typically about 5-6% [1,3,4,5], which is predetermined by resonant characteristic of radiating element. All basic characteristics of microstrip antennas depend on dielectric substrate's parameters [1-5]. This study describes mathematical models for analysis of microstrip antennas with rectangular patch. Simulative calculations based on these models are provided for determination of the bandwidth and directivity of microstrip antennas and their variation provoked by changes of dielectric substrate's parameter. It was explored the influence of dielectric substrate's height and permittivity over basic antenna characteristics.

## II. METHODS OF THEORETICAL ANALYSIS FOR MICROSTRIP ANTENNAS

There are many theoretical methods of analysis for microstrip antennas, which are used depending on the specific purpose of examination. In general the antennas could be most properly described by using full-wave model. It includes finding the solutions of primarily electromagnetic field equations for near and far zone at particular defined conditions. In almost all cases calculations are based on Moment Method. This method gives very accurate results, can be applied for all types of electrodynamics calculations but it is very complicated, difficult to implement and usually provides less physical insight for the processes. That is the reason why in most cases are used other models, which also

provides good and accurate results and are easier to implement. Most popular practical methods of analysis for microstrip antennas are transmission-line and cavity models [1,3,4,5]. They are relatively simple and give good physical interpretation of the electromagnetic processes in the antennas but less accurate results than full-wave model. In this section are presented main consideration of transmission-line and cavity model for microstrip antenna with rectangular patch (Fig.1).

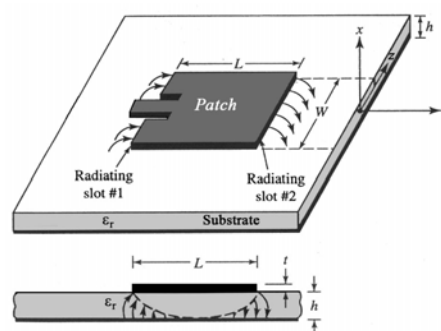


Fig.1. Microstrip antenna with rectangular patch

According the cavity model microstrip antenna is dielectric loaded cavity, which excite higher order resonances (Fig.2). In this model, the interior region of the patch is modeled as a cavity bounded by electric wall on the top and the bottom, and magnetic walls along the periphery. This is an approximate model which in principle leads to reactive input impedance and it does not radiate any power. This is the reason why there are introduced losses in the cavity. Results computed by this model are in good agreement with measurements [1],[5].

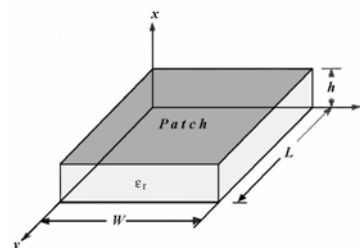


Fig.2 Cavity model – rectangular patch geometry

Analyses of the microstrip patch show that the type of electromagnetic field modes in the cavity is  $TM_{mnp}^x$ . The dominant mode is  $TM_{010}^x$  where  $x$  is direction normal to the patch according the geometry shown on Fig.2. The antenna dimensions are respectively  $W$  - width,  $L$  - Length,  $h$  - height.

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Using The Field Equivalence Principle the four side slots of microstrip patch could be represented as radiating equivalent magnetic dipoles (Fig.3). The components of the electromagnetic fields, radiated from two of the slot separated by the length of the patch  $L \approx \lambda/2$ , in a direction perpendicular to the ground plane add in phase and give a maximum radiation. So these slots are referred as radiating slot. The fields radiated by other two of the slots, which are separated by the width of the patch  $W$  are canceled along the normal plane because the equivalent magnetic current densities on these sides are with same amplitude but in opposite phase. So these two slots are called non-radiating. On the Fig.3 are shown the equivalent current densities and field distribution for radiating slot for dominant mode  $TM_{010}^x$ . One drawback of the cavity model is that it does not take into account fringing from the end of the patch.

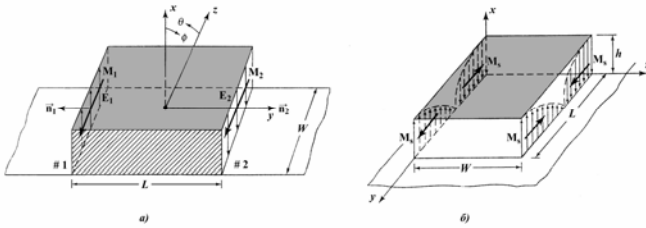


Fig. 3. Rectangular microstrip patch radiation and non-radiating slots at  $TM_{010}^x$  mode

Transmission Line model is the easiest and most popular of all other models. It gives relatively less accurate results, but very clear physical interpretation of the processes and is widely used for calculation of patch dimensions. According to transmission line model microstrip antenna is represented by two radiating slots separated by a low-impedance transmission line with length  $L$ . The radiating slots are equivalent magnetic dipoles, which form two elements antenna array. This model allows to work out good practical results for dependency between substrate's parameters ( $\epsilon_r$ ,  $h$ ) and geometrical dimension of the patch ( $W$ ,  $L$ ). It also takes into account fringing effects at the edges of the patch by introducing effective length  $L_{eff}$  and effective dielectric constant  $\epsilon_{reff}$ . Transmission-line model is simple and with some consideration gives nice practical results.

### III. SHORT DESCRIPTION OF THE EXAMINATION

This study is done for microstrip rectangular patch antenna at central working frequency 10 GHz and different parameters of dielectric substrate. The intervals of variation for dielectric constant and height of substrate are as follows:

- $0,5 \text{ mm} < h < 2 \text{ mm}$
- $2 < \epsilon_r < 10$

In these intervals are chosen 400 discrete equidistant points for substrate's height and permittivity. Therefore the steps of parameter's changes are:

$$\Delta \epsilon_r = 0,02$$

$$\Delta h = 0,004 \text{ mm}$$

These steps are small enough and allow very clearly studying of the variation of antenna's directivity and bandwidth with the substrate's parameters changes. The delta steps are practically so small that reach technological tolerance for dielectric material parameters. For each two discrete values of  $\epsilon_r$  and  $h$  are calculated the antenna parameters. For these calculations are used theoretical equations based on models described in previous section.

For the purpose of examination are accepted copper patch metallization and very low loss dielectric materials with tangent of dielectric losses  $\tan \delta \leq 9 \cdot 10^{-4}$

Antenna's Directivity is defined as the ratio of the radiation intensity in a given direction from the antenna to the radiation intensity averaged over all directions. If the direction is not specified it is implied the direction of maximal radiation intensity. The average radiation intensity is equal to the total power radiated by the antenna divided by  $4\pi$ . Therefore the directivity of antenna is the ratio of its radiation intensity in direction of the maximum radiation over that of an isotropic source. It could be mathematically expressed [1] as:

$$D = \frac{4\pi U_{\max}}{P_{\text{rad}}} \quad (1)$$

where

$U_{\max}$ , W/sr – maximum radiation intensity

$P_{\text{rad}}$ , W – total radiated power from the antenna

For a single slot total radiated power  $P_{\text{rad}}$  and maximum radiated intensity  $U_{\max}$  is computed by using equivalence principle (according cavity model) and representing the slot as equivalent magnetic dipoles [1],[4],[5]:

$$P_{\text{rad}} = \frac{|V_0|^2}{2\pi Z_0} \int_0^\pi \left[ \sin\left(\frac{\pi W}{\lambda_0} \cos \theta\right) / \cos \theta \right]^2 \sin^3 \theta d\theta \quad (2)$$

$$U_{\max} = \frac{|V_0|^2}{2\pi Z_0} \left( \frac{\pi W}{\lambda_0} \right) \quad (3)$$

where

$V_0$ , Volts - equivalent voltage of the radiating slot

$Z_0$ ,  $\Omega$  - characteristic (free space) impedance

Now using the transmission-line model the microstrip patch can be represented as two elements antenna array with distance between elements  $L \approx \lambda/2$ . Consequently according antenna array theory the total directivity is:

$$D = \left( \frac{2\pi W}{\lambda_0} \right)^2 \frac{\pi}{I} \quad (4)$$

$$I = \int_0^\pi \int_0^\pi \left[ \frac{\sin(0,5kW \cos \theta)}{\cos \theta} \right]^2 \sin^3 \theta \times \cos^2(0,5kL_e \sin \theta \cos \phi) d\theta d\phi \quad (5)$$

The bandwidth of an antenna is defined as range of frequencies within which the performance of the antenna

conforms to specified requirements. For narrowband antennas bandwidth is expressed as a percentage of the frequency difference over the central working frequency. The antenna bandwidth is inversely proportional of its total quality factor  $Q$ . Therefore to define the bandwidth at first should be computed total quality factor.

Total quality factor is characteristic which represent the power transformation in antenna system. It is connected with losses mechanism. Typically there are radiations, conduction (ohmic), dielectric and surface wave losses. So the total quality factor  $Q$  is depended on all these losses and in general is written as [1],[3],[4],[5]:

$$\frac{1}{Q} = \frac{1}{Q_{rad}} + \frac{1}{Q_c} + \frac{1}{Q_d} + \frac{1}{Q_{sw}} \quad (6)$$

where different quality factors could be computed by using the following formulas [1,3]:

-  $Q_{rad}$  - quality factor due to radiation losses:

$$Q_{rad} = \frac{2\omega_0 \varepsilon_r W}{h G_{rad}} K \quad (7)$$

-  $Q_c$  - quality factor due to conduction losses:

$$Q_c = h \sqrt{\pi f \mu \sigma} \quad (8)$$

-  $Q_d$  - quality factor due to dielectric losses:

$$Q_d = 1/tg\delta \quad (9)$$

-  $Q_{sw}$  - quality factor due to surface wave losses:

$$Q_{sw} = \frac{\omega_0 W_t}{P_{sur}} \quad (10)$$

In the formulas above there are used the following parameters:

$\sigma$ , [S/m] – specific conductance of the patch;

$tg\delta$  – tangent of dielectric losses in substrate;

$P_{sur}$ , [W] – power of surface waves;

$\omega_0 = 2\pi f_0$ , [rad/s] – resonant frequency;

$W_t$ , [W] – total energy stored in the cavity for one period, which for rectangular patch is:

$$W_t = \frac{1}{4} \varepsilon_0 \varepsilon_r V = \frac{1}{4} \varepsilon_0 \varepsilon_r h L W \quad (11)$$

$G_{rad}$ , [S] - Radiation conductance of the antenna, which is computed by using expression for radiated power  $P_{rad}$  (2) and antenna array factor, according transmission line model [1],[3]:

$$G_{rad} = \frac{1}{30\pi^3} \cdot \int_0^\pi \int_0^\pi \left[ \frac{\sin(0,5kW \cos \theta)}{\cos \theta} \right]^2 \times \sin^3 \theta \cos^2(0,5kL_e \sin \theta \cos \phi) d\theta d\phi \quad (12)$$

The parameter  $K$  for a rectangular patch at the dominant mode is:

$$K = \frac{\iint_{Area} |E|^2 dA}{\oint_{Perimeter} |E|^2 dl} = 0,25L \quad (13)$$

For thin substrates ( $h \ll \lambda$ ) the losses due to surface wave are negligible and in most of cases are not taken into account. However for thicker substrates they need to be considered and the power of surface waves  $P_{sur}$  could be computed [3].

The bandwidth of the antenna  $\Delta f$  is inversely proportional to its total quality factor and can be defined as [1],[3],[4],[5]:

$$\Delta f = f_0 / Q_t \quad (14)$$

This expression is approximate and is not useful for real purposes because does not take into account impedance matching between the antenna and feeder line. For practical calculations is used definition of the bandwidth where the VSWR (Voltage Standing Wave Ratio) at the feeder line is equal or less than desired maximum value. Therefore the antenna bandwidth is calculated by the formula:

$$\Delta f = f_0 \frac{VSWR - 1}{Q_t \sqrt{VSWR}} \quad (15)$$

For practical usage the antenna bandwidth is measured at  $VSWR=2$ , which means that 90% of the transmitter power reached the antenna and the rest 10% reflected back.

Therefore the bandwidth is measured on level  $1/\sqrt{2}$  from the maximum:

$$\Delta f = \frac{f_0}{Q_t \sqrt{2}} \quad (16)$$

#### IV. RESULTS

The main purpose of this study is to find out numerical relations between substrate's parameters (height and permittivity) and one of the most important antenna characteristics - the directivity and the bandwidth. Major results of examination are shown on the figures below (Figs. 4-7). They illustrate the changes of bandwidth and directivity caused by dielectric substrate's parameters changes.

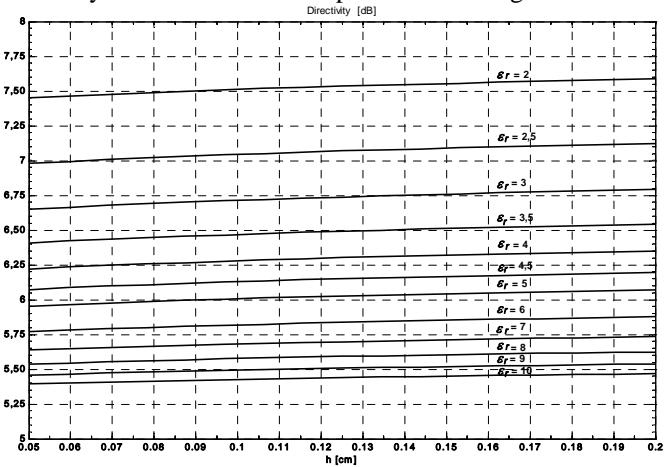


Fig. 4. Influence of substrate's height  $h$  over the Antenna Directivity

## V. CONCLUSION

As a result of examination could be taken out many conclusions about dielectric substrate's parameters impact on microstrip antenna characteristics. It is possible to study out not only changes of bandwidth and directivity, but also changes of geometrical dimension, variation of fringing effects and quality factor. Analyses of the results lead to the following main conclusions:

- The antenna directivity is greater when materials with low value of dielectric constant  $\epsilon_r$  (about  $2 \div 3$ ) are used. An increase of  $\epsilon_r$  will concentrate more of the power in the patch and decrease the radiation, but at the same time leads to smaller size of the antenna. Variation of directivity in the examined interval  $2 < \epsilon_r < 10$  is about 25 %.

- Usage of thicker substrates leads to small increase of Directivity, which about 2% for whole explored interval.

- Directivity is to a large extent more dependent on dielectric material respectively  $\epsilon_r$  than on substrate's height.

- Substrate's parameters influence over the bandwidth is more or less like over directivity. When the height is increased the bandwidth also increases with close to linear function. The change of bandwidth depending on substrate's height is relatively large - from 0,9% to 9% at different values of  $\epsilon_r$ .

- The bandwidth is more sensitive to  $\epsilon_r$ . When the dielectric constant is increasing in diapason  $2 < \epsilon_r < 10$  the bandwidth is narrowed approximately with 50%.

- General conclusion is that maximum directivity and bandwidth are achieved with thin substrates and materials with low dielectric constant at the expense of bigger antenna size.

- Thicker substrates and low dielectric constant lead to extension of patch length (about 15%) and width (about 45%).

- The thicker substrates improve the bandwidth and directivity, but allow excitation of surface waves. Influence of surface waves could be neglected. For a single patch it leads to probable error of 1-2 %, but have to be taken into account at the antenna arrays.

This research make possible to be illustrated physical processes in the microstrip patches. The results of the examination allows making correct choose of materials for dielectric substrates with a view to achieve specific parameters of printed antennas and antenna arrays.

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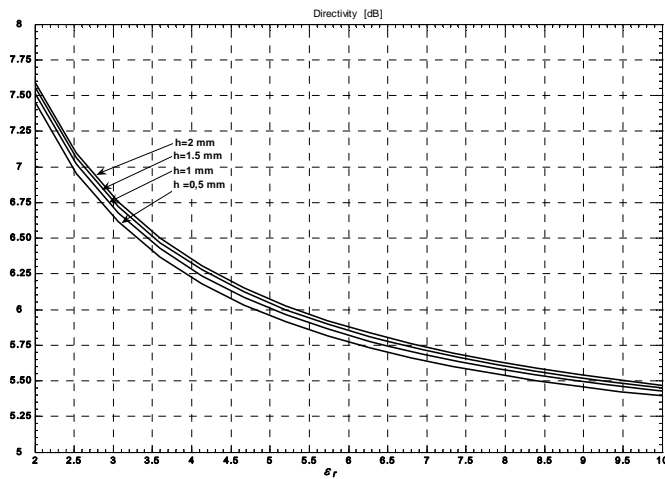


Fig. 5. Influence of substrate's permittivity  $\epsilon_r$  over the Antenna Directivity

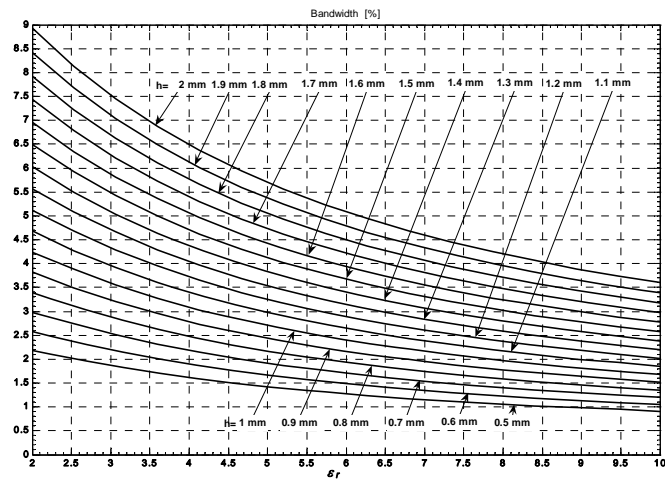


Fig. 6. Influence of substrate's permittivity  $\epsilon_r$  over the Antenna Bandwidth

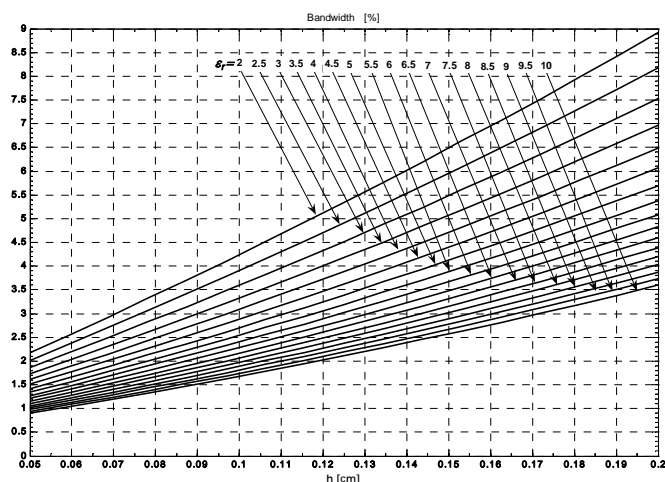


Fig. 7. Influence of substrate's height  $h$  over the Antenna Bandwidth