Pricing the Internet Services Using Extensive Game Solution

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Abstract – **In this paper we described the application of extensive game model. We considered the case of the economic interactions between local Internet Service Providers in a region which we modeled as an extensive game. We founded three Nash equilibrium points as the solutions of the game.**

Keywords – **Pricing, Extensive game, Nash equilibrium.**

I. Introduction

The development of good pricing models for the Internet is a topic of current interest. Most Internet Service Providers (ISPs) currently employ flat rate pricing, i.e. user is charged a fixed amount per time unit, irrespective of usage [1]. Others, following the telephony model, price the time spent connected to the Internet (some dial-up services are priced this way). ISPs may also use percentile-based charging. Still others charge based on actual bytes transferred. In general, pricing schemes have to be defined and evaluated with respect to the heterogeneous technical, economic and social aspects.

Game theory, as a mathematical basis for the analysis of interactive decision-making processes, can be applied for solving various pricing problems in the current Internet. It is a collection of modeling tools that aid in the understanding of interactive decision problems.

In this paper we study how members of a group, who are in an identical position in the hierarchy, interact with each other. As an example, we consider the case of the economic interactions of local ISPs of a region with each other, which we model as an extensive game. The model of an extensive game defines the possible orders of the events. The players can make decisions during the game and they can react to other players' decisions. We focus on extensive game model as a possible solution for defining charges for exchanging traffic between local ISPs belonging to the same region.

The paper is organized in the following way. In Section 2 we briefly discuss the Internet as it looks like today and pricing issues in the current Internet. In Section 3 basic components of game theory and classification of games are presented. In Section 4 extensive game model is described and we give example of an extensive game in which we examine the case of the economic interactions of local ISPs of a region with each other. Conclusion is given in the Section 5.

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II. Pricing the Internet Services

A. Internet model

The Internet is a heterogeneous body of privately owned infrastructure. Roughly speaking, it consists of two types of networks: 1) densely meshed networks in geographically localized regions which specialize in providing consumers with connection points to the network and 2) networks traversing large geographical distances which provide connectivity between the local networks [2]. All the networks are connected by means of an inter-operable protocol stack agreed upon by the Internet Engineering Task Force (IETF). Fig. 1 illustrates the Internet as it looks like today.

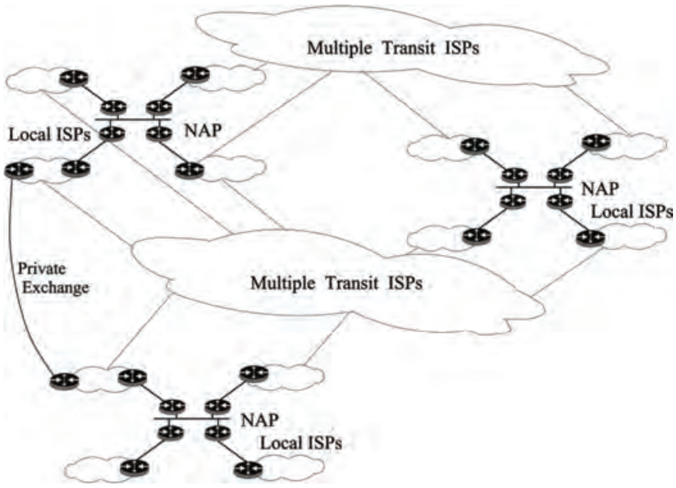

Fig. 1. Structure of the Internet consisting of local and transit ISPs[2]

There are local Internet Service Providers (ISPs) providing services in small regions, which compete for the same group of customers and transit ISPs which transfer data between such local groups. Local providers as well as transit providers establish a point of presence (shown as small "clouds" in Fig.1) at a Network Access Point (NAP) where traffic may be exchanged. Local ISPs in different regions have two options for traffic exchange—they can use the services provided by a transit ISP (shown as large transit „clouds") at a NAP, or can build their own means of private exchange (either at a NAP or independently). Recent maps of the Internet [3] indicate that private traffic exchanges are becoming popular.

B. Pricing issue

Pricing is one of the biggest challenges in in the current Internet. In general, pricing schemes have to be defined and evaluated with respect to the heterogeneous technical, economic and social aspects. The main evaluation criteria encompass efficiency in the sence of maximizing utilities of customers and the provider, fairness and feasibility [4].

In today's Internet there is a need for shifting from simple charging schemes such as flat rate based or duration based charging towards the usage based charging [5] with different tariffs assigned to different service classes (DiffServ). However, providers usually require simple charging schemes which enable them to recover costs fairly and effectively allocate network resources.

Independently of pricing scheme the ISP uses, he needs to price and re-price its services from time to time. The need for re-pricing arises typically with changes in prices of competitors in the market or in its efforts at continual service differentiation through the periodic introduction of new services. The periodicities of price changes can be range from once or twice a day to several months.

Game theory is a good basis for the analysis of pricing problems in the current Internet.

III. Game Theory

A. Assumptions and Definitions

Game theory is a field of applied mathematics that describes and analyzes interactive decision making situations. It consists of a set of analytical tools that predict the outcome of complex interactions among rational players [6].

Basic components of a game are players, the possible actions of the players and consequences of the actions. The players are decision makers and their actions result in a consequence or outcome. The players try to ensure the best possible consequence according to their preferences. The preferences of a player can be expressed either with a utility function, which maps every consequence to a real number, or with preference relations, which define the ranking of the consequences.

The most fundamental assumption in game theory is rationality. Rational players are assumed to maximize their payoff. If the game is not deterministic, the players maximize their expected payoff [6], [7]. It is also assumed that the players know the rules of the game well.

In game theory, a solution of a game is a set of the possible outcomes. A game describes what actions the players can take and what the consequences of the actions are. The solution of a game is a description of outcomes that may emerge in the game if the players act rationally and intelligently. Generally, a solution is an outcome from which no player wants to deviate unilaterally.

An outcome of a game is Pareto efficient, if there is no other outcome that would make all players better off. In implementation theory, the aim is typically to design a game that will end in a Pareto efficient outcome.

When a player makes a decision, he can use either a pure or a mixed strategy. If the actions of the player are deterministic, he is said to use a pure strategy. If probability distributions are defined to describe the actions of the player, a mixed strategy is used.

B. Classification of Games

Games can be classified into different categories according to their properties.

According to their focus, games can be divided into noncooperative and cooperative games. In noncooperative games, the actions of the single players are considered. In cooperative games the joint actions of groups are analyzed, i.e. what is the outcome if a group of players cooperate. In telecommunications, most game theoretic research has been conducted using noncooperative games, but there are also approaches using cooperative games.

According to their dynamics, games can be divided into strategic and extensive games. In strategic (or static) games, the players make their decisions simultaneously at the beginning of the game. While the game may last long and there can be probabilistic events, the players can not react to the events during the game. On the other hand, the model of an extensive game defines the possible orders of the events. The players can make decisions during the game and they can react to other players' decisions. Extensive games can be finite or infinite.

Games can be divided according to their payoff structures. A game is called zerosum game, if the sum of the utilities is constant in every outcome. Whatever is gained by one player, is lost by the other players. In telecommunications, the games are usually not zero-sum games.

Games can be divided on games with perfect and imperfect information. If the players are fully informed about each other's moves, the game has perfect information. Games with simultaneous moves have always imperfect information, thus only extensive games can have perfect information.

Games can also be divided on games with complete and incomplete information. In games with complete information the preferences of the players are common knowledge, i.e. all the players know all the utility functions. In a game of incomplete information, in contrast, at least one player is uncertain about another player's preferences.

In this paper we focus on extensive game model as a possible solution for defining charges for exchanging traffic between local ISPs belonging to the same region.

IV. Extensive Game

A. Basic model

The strategic game model is suitable for representing simple real life events such as auctions. A broader model is needed, when more complex interactions are occurring between the decision makers. Especially the possibility to react to the actions of the other players is essential in many applications. Extensive games eliminate the limitation of the simultaneous decisions, thus they make possible to model a wider range of real life situations.

For simplicity, the following formulation of extensive game does not allow simultaneous actions of the players, i.e. the game has perfect information.

Definition 1: An extensive game with perfect information has the following components.

- A set of players N ,
- A set H of sequences (finite or infinite) of actions that satisfies the following three properties:
  - The empty sequence Ø is a member of H .
  - If $(a^k)_{k=1,...K} \in H$ (where K may be infinite) and $L < K$ then $(a^k)_{k=1,...L} \in H$.
  - If an infinite sequence $(a^k)_{k=1}^{\infty}$ satisfies $(a^k)_{k=1,...L} \in H$ for every positive integer L then $(a^k)_{k=1}^{\infty} \in H$.

  (Each member of H is a history; each component of a history is an action taken by a player.) A history $(a^k)_{k=1,...K} \in H$ is terminal if it is infinite or if there is no a^{K+1} such that $(a^k)_{k=1,...K+1} \in H$. The set of terminal histories is denoted Z.
- A function P that assigns to each nonterminal history (each member of $H \setminus Z$) a member of N. (P is the player function, $P(h)$ being the player who takes an action after the history h.)
- For each player $i \in N$ a utility function U_i on Z.

In strategic games, the behavior of the player is defined by the action the player takes. In order to define the player's behavior in an extensive game, more information is needed. A strategy describes the action of the player in every possible situation of the game.

Definition 2: A strategy of player $i \in N$ in an extensive game with perfect information. $\{H, N, P, (U_i)\}$ is a function that assigns an action in $A(h)$ to each nonterminal history $h \in H \setminus Z$ for which $P(h) = i$.

The solution of an extensive game is a Nash equilibrium from which no player has an incentive to deviate unilaterally.

Definition 3: A Nash equilibrium of an extensive game $\{H, N, P, (U_i)\}$ is a profile $a^* = (a_1^*, \dots a_N^*)$ of actions with the property that for every player $i \in N$ we have:

$$U_i(a^*) \geq U_i(a_1^*, \dots, a_{i-1}^*, a_i, a_{i+1}^*, \dots, a_N^*) \text{ for all } a_i \in A(h).$$

When a game is played, the rationality assumption will force the game into a Nash equilibrium outcome. If the outcome is not a Nash equilibrium, at least one player would gain a higher payoff by choosing another action. If there are multiple equilibriums, more information on the behavior of the players is needed to determine the outcome of the game. It is important to notice that while an equilibrium is a result of the optimization of the individual players, it does by no means imply that the result is "good" or globally optimum.

Extensive games with two players can be illustrated with matrices, as it is given in table 1. In this simple example, there are two cases:

1) Player 1 takes his first move choosing between 1 or 2. After observing player 1's decision, player 2 decides to choose 1 if the player 1's first move was 1 and 1.5 if the player 1's first move was 2.
2) Player 2 takes his first move choosing between 1 or 2. After that, player 1 decides to choose 0.5 if the player 2 played 1 and 1.5 if the player 2 played 2.

Table 1. Example extensive game with two players in matrix form

Strategies	Player 2	
Player 1	(1,1)	(2,1.5)
	(0.5,1)	(1.5,2)

The solution of the example game can be deducted easily. In the first case, player 1 prefers option that leads to higher utility, i.e. 2, which means that optimal strategy is (2,1.5). Respectively, in the second case player 2, as a rational player, chooses 2 (which is higher utility for him), hence the optimal strategy is (1.5,2). Clearly, the game has two Nash equilibrium points.

B. Example: Interaction among local ISPs

Next we consider the case of the economic interactions of local ISPs of a region with each other. Since they belong to a single group of the hierarchy, we observe this case as an interaction among equals. We study only intra-regional traffic and consider bilateral interactions of ISPs 1, 2 and 3 (Fig. 2.).

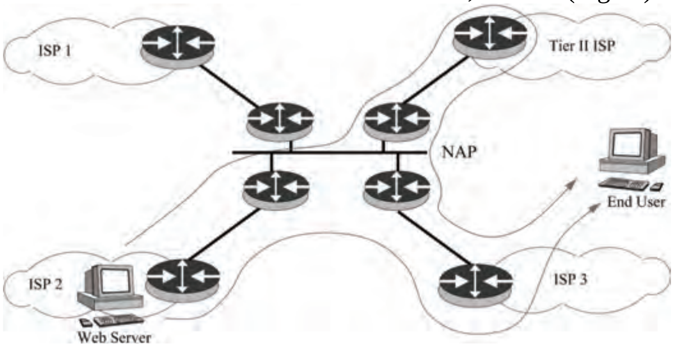

Figure 2. Local ISP structure

We suppose a small number of data carriers in a local region. This corresponds to the situation in the current Internet, where the number of data carriers is usually limited to a couple of ISPs providing a few QoS levels (DSL, cable/T1 and T3 seem to be most popular [2]). Local ISPs compete for consumers in a geographical region. They have to set prices for intra-regional and inter-regional[1] traffic. For intra-regional traffic, ISP i charges prices $\tilde{p}_i^r$ and p_i^r per unit traffic to websites and end-users, respectively. Each ISP must provide a guarantee of connectivity to all other users. This means that ISPs have to exchange traffic. They do this by means of a bilateral settlement, i.e., ISP i charges ISP j an access charge t_{ji}^r for termination of traffic, where ISPs i and j both belong to region r. The costs for all ISPs in the region assumed to be identical. All ISPs have identical fixed costs, which we take to be equal to zero (since it makes no difference to any of the results).

We also assume identical access charges for termination of traffic for all ISPs in the region (i.e. $t_{ji}^r = t_{ij}^r \quad \forall i, j$ in region r) at the beginning of the game. We will find the optimum

[1] In this example, we will not examine charging for inter-regional traffic.

change of access charges, which would maximize the profit of individually rational ISPs.

Possible actions for any ISPi , $i = \overline{1,3}$ are:

- a_i^1 - not to change access charge to ISPj, $\forall j = \overline{1,3}$, $j \neq i$
- a_i^2 - higher access charge to ISPj, $\forall j = \overline{1,3}$, $j \neq i$ and
- a_i^3 - lower access charge to ISPj, $\forall j = \overline{1,3}$, $j \neq i$.

No change of access charge gives ISP utility 0, higher access charge gives him utility 1 and lower access charge gives him utility -1. We assume that ISP1 takes his first move choosing between a_1^1, a_1^2 and a_1^3. After observing ISP1's decision, ISP2 and ISP3 decide to choose their actions, respectively. This example is ilustrated in table 2.

Table 2. Extensive game with three ISPs

		a_3^1	a_3^2	a_3^3
a_1^1	a_2^1	(0,0,0)	(0,0,1)	(0,0,-1)
	a_2^2	(0,1,0)	(0,1,1)	-
	a_2^3	(0,-1,0)	-	(0,-1,-1)

		a_3^1	a_3^2	a_3^3
a_1^2	a_2^1	(1,0,0)	(1,0,1)	-
	a_2^2	-	(1,1,1)	-
	a_2^3	-	-	-

		a_3^1	a_3^2	a_3^3
a_1^3	a_2^1	(-1,0,0)	-	(-1,0,-1)
	a_2^2	-	-	-
	a_2^3	(-1,-1,0)	-	(-1,-1,-1)

There are three possible cases for ISP2: 1) If ISP1 decides for action a_1^1, ISP2 choose between all three possible actions; 2) If ISP1 decides for a_1^2, ISP2 will eliminate third possibility, a_2^3 and 3) If ISP1 decides for a_1^3, ISP2 will eliminate second possibility a_2^2.

ISP3 plays by the following rules: 1) If ISP1 decides for a_1^1 and ISP2 decides for a_2^1, ISP3 will think over all three possible actions; 2) If ISP1 decides for a_1^1 and ISP2 decides for a_2^2, ISP3 will eliminate third possibility, a_3^3; 3) If ISP1 decides for a_1^1 and ISP2 choose a_2^3, ISP3 will eliminate second possibility a_3^2. 4) If ISP1 decides for a_1^2 and ISP2 choose a_2^1, ISP3 will eliminate third possibility, a_3^3; 5) If ISP1 decides for a_1^2 and ISP2 choose a_2^2, ISP3 will choose a_3^2 and 6) If ISP1 decides for action a_1^3, ISP3 will eliminate a_3^2 independently of what ISP2 choose.

There are three Nash equilibrium points in this game: (0,1,1), (1,1,1) and (-1,0,0).

V. Conclusion

This paper described the application of extensive game model as a mathematical basis for the analysis of various pricing problems in the current Internet.

Extensive game model is suitable for representing complex interactions, that occures between the decision makers. The model of an extensive game defines the possible orders of the events. The players can make decisions during the game and they can react to other players' decisions, which is essential in many applications. Extensive games eliminate the limitation of the simultaneous decisions, thus they make possible to model a wider range of real life situations. This model is a good basis for the analysis of pricing the Internet services.

We consider the case of the economic interactions of local ISPs of a region with each other. The problem of finding the optimum change of ISPs access charges for termination of traffic is modeled as an extensive game. We found three Nash equilibrium points as the solutions of the game.

Acknowledgments

This paper resulted from the researching project *Planning, Forecasting and Pricing in Telecommunication Networks* that is supported by Serbian Ministry of Science and Environmental Protection (project designation TP-6106A).

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