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A Hybrid Population Based Method Solving Convex Integer Optimization Problems

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Abstract – **This paper presents a hybrid population based evolutionary programming method, designed to solve single objective convex integer optimization problems. The proposed method combines some ideas from scatter search and genetic algorithms, as well from the so-called ant systems and particle swarm optimization. The aim of the method is to explore in an efficient manner the whole feasible convex domain and to find out the global optimum of a multimodal objective function.**

Keywords – **evolutionary programming, scatter search, genetic algorithms, convex integer optimization problems.**

I. INTRODUCTION

The integer programming problem considered in this study can be stated in the following form:

$$\mathbf{Min}\ \boldsymbol{F(x)} \tag{1}$$

$$\textbf{subject to: } \boldsymbol{g_i(x) \le 0; \quad i = 1,\dots,m;} \tag{2}$$

$$\boldsymbol{l_j \le x_j \le u_j; \quad j = 1,\dots,n;} \tag{3}$$

$$\boldsymbol{x \in Z^n,} \tag{4}$$

where x is an n-dimensional vector of variables x_j, $j = 1,\dots,n$; which accept discrete values only. By l_j and u_j are denoted the bounds (lower and upper) of x_j, and $F(x)$ is the multimodal objective function. There is no necessary $F(x)$ to posses accessible to calculation derivatives in an explicit analytical form. The functions $g_i(x)$, $i = 1,\dots,m$; are convex nonlinear functions and m is the number of nonlinear constraints **(2)**.

The convex integer problems (see [6, 16]) belong to the class of NP-hard optimization problems. There does not exist an exact algorithm, which can solve these problems in time, depending polynomially on the problem input data length or on the problem size. For this reason many efficient approximate evolutionary algorithms and metaheuristic methods have been created to find out the global optimum of such complex optimization problems. The most successful and efficient methods usually hybridize two or more metaheuristics.

In this study is proposed a new hybrid method, combining different ideas of such evolutionary techniques. It is organized to manage the system of constraints preserving the feasibility of the new obtained solutions. This is very important, as shown in [18]. The paper is organized as follows: Some common features of the population based methods are considered in Section II. The new hybrid population based method is described in Section III. An illustrative example of this new method is given in Section IV. Some conclusions are drawn in Section V.

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II. COMMON FEATURES OF POPULATION BASED METHODS

The name „Adaptive Memory Programming" was proposed by Fred Glover in connection with the metaheuristic Tabu Search (TS) (see [10]). Many metaheuristics (i. e. methods designed to obtain a global optimum) can be classified as "adaptive memory methods" (see [19]) or "population based methods" (see [12]). The most familiar and powerful among them are Genetic Algorithms (GA) (see [11, 13]), Scatter Search (SS) (see [7, 9]), Tabu Search (TS) (see [8, 10]), Ant Systems (AS) (see [1, 2, 3]) and Particle Swarm Optimization (PSO) (see [5, 14, 15]).

The above mentioned metaheuristics possess the following common features (see [19]):

- They memorize solutions (or characteristics of solutions) in a population of individuals. Each individual is associated with a feasible solution of the problem at hand.
- They include a generating solutions search procedure, which uses the information stored in the memory.
- They apply some kind of local search method (a greedy improvement method, an elementary tabu search or simulated annealing) to improve the obtained solutions.

There are some shortcomings in GA. For example the mutation has unexpected results on the objective function value of an individual, i. e. it does not necessarily improve it (see [12]). Another crucial moment is the sufficient diversity of the population, i. e. the availability of diverse enough genetic material, which permits the exploration of the whole feasible domain and which would not restrict the search in a near part of the feasible domain around a local minimum (see [17]). Also GA are not designed for precisely locating the optimal solution(s) and the combination of genetic approach and of some form of local search is advisable in general, because it could improve the efficacy of GA (see [17, 19]).

The creation of successful global search methods is connected very often with the combination of two or more metaheuristics in hybrid methods. For example GA are combined with Tabu Search methods, or with a faster local search procedure, AS – with local search techniques (see [19]), GA – with clustering procedure (see[4]), SS – with TS or SS – with GA (see [9]).

All population based methods alternate during the search periods of self adaptation (the search process is intensified in some region of the search space) with periods of co-operation (information collective gathered during the search process is used to direct further the search). The periods of self adaptation correspond to execution of mutation, improvement or local search procedure, and the periods of co-operation are connected with the selection, trace updating or generation of trial points. The major difficulty with the creation of global search methods is to overcome the premature convergence towards local optima. To obtain the global optimum of the problem at hand a diversification phases of the search process are necessary, so that new areas of feasible domain, that are remained still unexplored, can be investigated.

III. THE NEW SYSTEMATICALLY DIVERSIFIED SEARCH METHOD SDS

Considering the search process of a global optimum there is no sense to direct the search in the region of the best found so far (local optimal or near optimal) solution, because in the most cases it will not coincide with the global optimal solution. For this reason information sharing as in AS or in PSO will be necessary only after the whole feasible domain has been roughly explored. The exploration of the whole feasible domain means that there is a guaranteed systematically diversification of the search process. The basic idea of the proposed hybrid SDS (systematically diversified search) is to divide the feasible domain in sub-regions (cones having a common vertex) and to explore each of them applying systematically diversification of the search. Around the best found solution for each sub-region, a simple local search procedure is performed. Then the obtained best solutions from each sub-region form a final population, i. e. the information gathered during the search so far is shared like in AS or PSO. By means of this final population a few scatter search iterations are performed to obtain the global optimal solution. At the end a precise local search procedure is used in the neighborhood of the best obtained solution in order to locate precisely the global optimum.

Let the feasible domain be denoted by X. The method SDS starts from the Tchebicheff center of the feasible domain. The Tchebicheff center $x_{tch} \in X$ is the point located at the maximal Euclidean distance from the constraint surfaces. We assume that the Tchebicheff center is obtained by means of a method for solving convex problems with continuous variables. Then x_{tch} is rounded to the nearest integer point i_{tch}. The "pseudo-code" form of the SDS method is presented below:

<u>Hybrid method SDS:</u>

Round off the components of the Tchebicheff center x_{tch} rounded to their nearest integer values and use them to obtain the starting integer point i_{tch}.

Generate a regular simplex with n+1 vertices, using i_{tch} as one vertex. Generate the other vertices on the base of the elementary geometry in the following manner:

$$v^{(i)}_j = \begin{cases} i_{tch} + \varphi_1 & \text{if } j \neq i \\ i_{tch} + \varphi_2 & \text{if } j = i \end{cases} \quad (5)$$

$$\varphi_1 = \alpha.[\frac{\sqrt{(n+1)}+n-1}{n\sqrt{2}}] \quad (6)$$

$$\varphi_2 = \alpha.[\frac{\sqrt{(n+1)}-1}{n\sqrt{2}}] \quad (7)$$

Let i_{tch} be denoted as $v^{(0)}$. Round off each $v^{(j)}$, $j = 1,\ldots,n$; to its nearest integer point. There are (n+1)combinations of n vertices, correspondingly for each facet of the simplex. The rays starting at the weight center of all simplex vertices cs and passing through the vertices belonging to each facet determine $K^{(i)}$ cones, $i = 1,\ldots, n+1$; in the feasible domain.

Calculate the weight center of the simplex:

$$cs = \frac{\sum_{j=0}^{n} v^{(j)}}{n+1} \quad (8)$$

Round off each cs to its nearest integer point.

FOR $i = 1,n+1$; **DO**

Explore the cone $K^{(i)}$ as follows:

Create a population $P^{(i)}$ of (n+1) points, including the simplex vertices of the current simplex facet and their weight center cv, where

$$cv = \frac{\sum_{v^{(i)} \in K^{(i)}} v^{(i)}}{n} \quad (9)$$

Round off each cv to its nearest integer point.

Calculate the mean fitness value FV of the population $P^{(i)}$.

Let: $xold^{(j)} = v^{(j)}$, $j = 1,\ldots, n$; and $xold^{(0)} = cv$.

Calculate $p^{(j)} = \gamma(xold^{(j)} - cs)$, $j = 1,\ldots, n$; and $p^{(0)} = \gamma(cv-cs)$, where γ is a scale multiplier, tuned according to the concrete problem.

ITERATION

Calculate the points $xnew^{(j)} = xold^{(j)} + p^{(j)}$, $j= 0,\ldots, n$; Round off each $xnew^{(j)}$ to its nearest integer point. In case there is a violated constraint from the system **(2)-(3)** reduce the corresponding $p^{(j)}$ as follows:

- If a constraint of type $\boldsymbol{x_k + a = 0}$ is violated, where $\boldsymbol{a}$ can have positive or negative value, then the corresponding component p_k of $p^{(j)}$, is used to reduce $p^{(j)}$:

$$p^{(j)} = |\frac{a-p_k}{p_k}|\, p^{(j)} \quad (10)$$

- If a constraint of type $\boldsymbol{g_i(x) \leq 0}$ is violated then $p^{(j)} = 0.8\, p^{(j)}$. If it is necessary repeat this reduction until the rounded off integer $xnew^{(j)}$ becomes feasible.

- If there are more than one constraints, violated by $p^{(j)}$, then chose the most reduced vector $p^{(j)}$, so that the rounded off integer $xnew^{(j)}$ becomes feasible.

Evaluate the objective function values $\boldsymbol{F(xnew^{(j)})}$, $j = 0,\ldots, n$; In case $F(xnew^{(j)}) < FV$ then replace the point of P having worst (greatest) fitness value by the point $xnew^{(j)}$.

Calculate the mean fitness value FV of the updated population $P^{(i)}$.

Update $xold^{(j)} = xnew^{(j)}$, $j = 1,\ldots, n$; In case some $p^{(j)}$ has been reduced at the current iteration then

$$xold^{(0)} = \frac{\sum_{j=1}^{n} xold^{(j)}}{n}$$; Round off $xold^{(0)}$ to its nearest

integer point;

else $xold^{(0)} = xnew^{(0)}$.

ENDofITERATION

Repeat the *ITERATION* until it is not possible to generate any new feasible points by means of $p^{(j)}$, $j = 0,\dots, n$;

At the obtained end points along the cone rays try to make a step with $p^{(0)} = xold^{(0)} - cs$.

Around the best obtained point of *P* perform a simple local search by means of vectors *d*, having only one nonzero component: $d_j = \pm 1$.

ENDFOR

Create a final population *FP* of (*n*+1) points, including the best found points from the populations $P^{(i)}$, $i = 1,n+1$;. Calculate the mean fitness value *FFV* of the population *FP*.

For each two pints $x^{(1)}$ and $x^{(2)}$ of FP perform scatter search as follows:

$xnew = x^{(1)} + w(x^{(2)} - x^{(1)})$, where $w = \pm 1/\|x^{(2)} - x^{(1)}\|$.

In case the obtained points *xnew* have better objective function values than *FFV*, replace the worst points in *FP* by them.

Around the best obtained point of *FP* perform more precise local search by means of vectors *df*, having two nonzero components : $df_j = \pm 1$.

END

Performing consecutive steps from the weight center *cs* towards the boundaries of the feasible domain, no matter are they improving the objective function value or not, the search process in the SDS method is further diversified in different way, besides the separately exploration of each cone, generated in the feasible region.

IV. Illustrative Example

Let us consider the following example. The objective function $\boldsymbol{F}(\boldsymbol{x})$ is given in a tabular form (see Table I), *x* is two-dimensional and there are simple constraints on each variable:

$$0 \le x_1 \le 10;$$
$$0 \le x_2 \le 8;$$

TABLE I. VALUES OF THE OBJECTIVE FUNCTION $F(x)$

x_2 \ x_1	0	1	2	3	4	5	6	7	8	9	10
0	11	11	10	11	11	10	10	9	7	6	8
1	7	8	10	11	12	13	12	10	7	3	7
2	6	4	7	10	15	14	13	11	9	7	9
3	7	8	9	10	13	12	14	13	10	9	10
4	8	9	12	11	13	12	17	16	11	12	13
5	16	14	12	11	12	11	10	14	12	13	14
6	15	14	12	12	12	10	8	9	8	11	15
7	13	12	11	10	11	9	8	4	6	10	13
8	10	12	11	11	10	9	7	0	7	9	11

Starting from the Tchebicheff center $i_{tch} = (5,5)$ the simplex with vertices (5,4), (6,7) and (8,5) is generated. The weight center of the simplex is $cs = (6,5)$.

The population $P^{(1)}$ includes the points (5,4), (8,5) and the middle point of the segment, determined by them – the point $cv = (7,5)$. The obtained mean fitness value $FV = 12.67$. For this problem $\gamma = 3$ is chosen, so that $p^{(0)} = (3,0)$, $p^{(1)} = (-3,-3)$, $p^{(2)} = (6,0)$; The last vector is reduced to $p^{(2)} = (3,0)$. Only two new points are obtained: (2,1) and (10,5). The point (2,1) enters in *P*, replacing the point (7,5). The new mean fitness value $FV = 11.33$. A reduction of $p^{(2)}$ is made, so that the point $xold^{(0)} = (6,3)$ with objective function value $F = 14$. The obtained point $xnew^{(0)}$.= (9,3) with $F = 9$. It enters in *P* replacing the point (5,4). The new mean fitness value $FV = 10.33$. Reducing $p^{(1)}$ to $p^{(1)} = (-1,-1)$ the point (1,0) is obtained. It has objective function value 11 and does not enter in *P*. The point $xold^{(0)} = (6,3)$. At the point (10,5) the step with $p^{(0)} = xold^{(0)} - cs = (6,3)-(6,5) = (0,-2)$ leads in point (10,3) with $F=10$. This point replaces the point (8,5) in *P* and $FV = 9.67$. A simple local search is performed around the point (9,3). The local optimal solution (9,1) is obtained with $F=3$.

The population $P^{(2)}$ includes the points (8,5), (6,7) and the middle point of the segment, determined by them – the point $cv = (7,6)$. The obtained mean fitness value $FV = 9.67$. The search vectors are: $p^{(0)} = (3,3)$, $p^{(1)} = (6,0)$, $p^{(2)} = (0,6)$; All the three vectors $p^{(0)}$, $p^{(1)}$ and $p^{(2)}$ are reduced and the obtained points are (9,8) with $F=9$, (10,5) with $F=14$ and (6,8) with $F=7$. Point (9,8) replaces the point (8,5) in P, and point (6,8) replaces the point (7,6). The new mean fitness value $FV = 8$. The calculated point $xold^{(0)} = (8,7)$ with $F=6$. It replaces the point (9,8). The new mean fitness value $FV = 7$. The vector $p^{(0)}$ is reduced to $p^{(0)} = (1,1)$ and the new point (9,8) is obtained, which has been already explored. A simple local search is performed around the point (8,7). The local optimal solution (7,8) is obtained with $F=0$.

The population $P^{(3)}$ includes the points (5,4), (6,7) and the middle point of the segment, determined by them – the point $cv = (6,6)$. The obtained mean fitness value $FV= 9.33$. The search vectors are: $p^{(0)} = (0,3)$, $p^{(1)} = (-3,-3)$, $p^{(2)} = (0,6)$; Both vectors $p^{(0)}$ and $p^{(2)}$ are reduced and the points (6,8), (2,1) and (6,8) are obtained. The point (6,8) has $F=7$ and the point (2,1) has $F=10$. Point (6,8) enters in *P*, replacing the point (5,4) and the new mean fitness value $FV= 7.67$. The calculated point

$xold^{(0)}$ = (4,5) with F=12. It does not enter in P. The vector $p^{(1)}$ is reduced to $p^{(1)}$ = (-1,-1) and the vector $p^{(2)}$ is reduced to $p^{(2)}$ = (0,0). The new points (1,0) with F=11 and (4,8) with F=10 are obtained. They do not enter in P. At the point (1,0) the step with $p^{(0)} = xold^{(0)} - cs$ = (4,8)-(6,5) = (-2,3) leads after reducing $p^{(0)}$ in point (0,2) with F=6. The last found point replaces point (6,7) in P and the new FV= 7. A simple local search is performed around the point (0,2). The local optimal solution (1,2) is obtained with F=4.

The points (9,1), (7,8) and (1,2) form the final population FP. The mean fitness value FFV = 2.33. After the scatter and the precise local search no better feasible solution is obtained. The found global optimal solution is x = (7,8) with F=0.

V. CONCLUSIONS

The presented new hybrid method SDS has the following good features and advantages:

- During the exploration of each sub-region the SDS method systematically diversifies the search process, avoiding in this manner the trap of local minima.
- The SDS method performs search in all defined sub-regions of the search space, so that the whole feasible domain is roughly explored.
- The formed final population contains diverse enough individuals, so that it is expected that the final search phase would lead to the global optimal solution.
- The applying of simple local search technique at the end of sub-regions exploration and of more precise local search at the end of the search process guarantees the good quality of the obtained final best solution.
- The SDS method can be efficient in comparison to other global search methods, because it explores only a small percent of all integer points in the feasible domain. This percent decreases with the increase of n and with growth of the feasible area.
- The populations used in SDS method have relatively small size, so that no great memory will be necessary for its implementation.
- The SDS method guarantees the feasibility of the obtained solutions.
- A great part of the integer points located on the rays forming each cone, which have been explored during the search in the corresponding sub-region, can be used during the exploration of the next sub-region. This may be used for creation of efficient program realizations of SDS method.

The SDS method will be tested on a set of test examples and may be further refined.

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