Fiber Nonlinearity Limitations in WDM CATV Systems

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Abstract – **The paper deals with some problems referring to fiber linearity effects on the parameters of the WDM CATV systems. Mathematical models of the scattering and Kerr effects in optical fibers are suggested taking into consideration both the parameters of the fiber chosen and the main characteristics of the system. Dependencies are given to determine the maximum launch power in a way to provide for acceptable worsening of the received information quality due to reduced carrier-to-noise ratio, intra-symbol distortion and cross-modulation.**

Keywords – **WDM system, SBS, SRS, SPM, XPM, FWM**

I. Introduction

Wavelength division multiplexing technology (WDM) allows increasing the transport capacity of CATV systems with the number of wavelengths used. A wavelength in this technology has the meaning of an optical carrier modulated with either an analogue or a digital signal. The most challenging in WDM systems is to achieve simultaneously smaller channel separation as well as higher modulation rates (bit rates) of the optical carrier in order to increase the transmission capacity. These goals begin to be contradictory at a certain point, due to dispersion and nonlinear effects in the single-mode optical fiber.

In WDM systems the laser beams of strictly defined wavelengths λ_1, λ_2, … λ_n are modulated and then combined by an optical multiplexer in the same single mode optical fiber at the transmitting side. At the receiving side an optical demultiplexer splits the optical channels into the same wavelengths and directs them to the optical receivers. Modern optical fibers have low attenuation coefficients (about 0.2 dB/km to 0.3 dB/km) but if the optical signals are carried at a long distance, they must be amplified every 80 to 120 km. A standard erbium-doped fiber amplifier (EDFA) has a total output power of 17 dBm (50 mW) at the maximum. When the power of the transmitted signal turns out to be higher than a predefined threshold level, some nonlinear effects can appear in the optical fiber thus causing noise and distortion.

There are two types of nonlinear effects in the optical fibers: scattering effects and Kerr effects. Stimulated Raman scattering (SRS) and stimulated Brillouin scattering (SBS) are associated with the first type. They are due to non-elastic interaction between a pump wave of wavelength λ_p and the fiber core that transfers most of the pump energy into a Stokes light wave of wavelength $\lambda_s > \lambda_p$. Kerr effects occur because of the dependence of the refractive index n on the power density inside the fiber core. Three types of Kerr effects are considered to be important for modern optical communications: four-wave mixing (FWM), self-phase modulation (SPM) and cross-phase modulation (XPM). The effects here mentioned are due to elastic interactions with which no energy transfer from wave to fiber takes place i.e. the total energy of the incident waves is transferred to the emerging waves only.

II. Launch Power Determination with Regards to Scattering Effects

Though very similar in origin SRS and SBS differ due to the fact that optical phonons participate in SRS while acoustic phonons participate in SBS. A fundamental difference is that in optic fibers SBS occurs only in the backward direction (with respect to the pump) whereas SRS dominates in the forward direction. Growth of the Stokes power due to SRS is characterized by the relation [1]

$$P_s(L) = P_s^*(0)\exp\left[g_R(\omega_s)P_p(0)L_{\text{eff}}/A_{\text{eff}} - \alpha L\right], \quad (1)$$

where $P_s(L)$ is the Stokes power at the fiber output, $P_p(0)$ is the input pump power, $g_R(\omega_s)$ is the value of the Raman-gain coefficient at a Stokes frequency ω_s that is downshifted from the pump frequency by about 13.2 THz, L is the actual length of the fiber, L_{eff} is its effective length, α is the fiber attenuation coefficient and A_{eff} is the effective core area.

Magnitude $g_R(\omega_s)$ corresponds to the Raman gain peak whose typical value is 1.10^{-13} m/W at a pump wavelength of 1550 nm. It is well known that the Raman gain spectrum is broad enough to extend up to 40 THz with a broad dominant peak at ω_s. The following formulae are used to calculate L_{eff} and A_{eff} :

$$L_{\text{eff}} = \left(1 - e^{-\alpha L}\right)\Big/\alpha, \; A_{\text{eff}} = \pi\left(MFD/2\right)^2 \quad (2)$$

where MFD is the mode field diameter of the fiber to be found in the manufacturer's data sheet.

Similarly to SRS, the Stokes power grows exponentially in the backward direction because of the Brillouin amplification due to SBS. That growth can be described by the same relation as (1) if $g_R(\omega_s)$ is replaced with the peak value of the Brillouin-gain coefficient $g_B(\omega_s)$. It should be pointed out that

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Brillouin-gain spectrum is extremely narrow with a bandwidth of about 10 MHz. The peak value of Brillouin gain $g_B(\omega_s)$ is downshifted from the pump frequency by about 10 GHz, i.e., the Stokes shift in SBS is smaller (by order of three) if compared with SRS. For a narrow-bandwidth pump $g_B(\omega_s) \approx 5.10^{-11}$ m/W at a pump wavelength of 1550 nm, i.e. g_B is higher by more than two orders of magnitude if compared with g_R .

Both types of scattering effects reveal a threshold-like behavior, i.e. significant conversion of pump energy into Stokes energy occurs only when pump power exceeds a certain threshold level. The Raman threshold is defined as the input pump power at which Stokes power at the fiber output becomes equal to that of the pump, i.e.

$$P_s(L) = P_p(L) = P_p(0)\exp\left(-\alpha L\right). \tag{3}$$

The same formula applies to determine the Brillouin threshold if $P_s(L)$ is replaced with Stokes power at the fiber input $P_s(0)$.

The critical pump power required to reach the Raman threshold P_{RT} in a single mode fiber with $\alpha L >> 1$ can be calculated by the following formula:

$$P_{RT} = 16 A_{\text{eff}} / g_R\left(\omega_s\right) L_{\text{eff}}. \tag{4}$$

The SBS power threshold P_{BT} is given by the equation

$$P_{BT} = 21 A_{\text{eff}} / g_B\left(\omega_s\right) L_{\text{eff}}. \tag{5}$$

The typical value of the SBS threshold is less than 10 mW, while the SRS threshold is higher by about two orders and can reach 1 W.

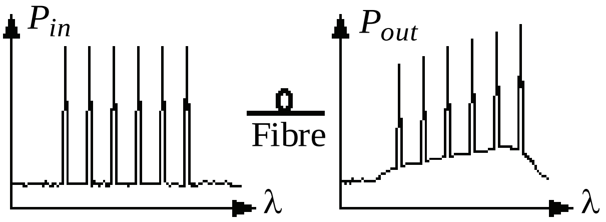

Fig. 1

The Raman-gain spectrum being very broad, SRS can cause problems in WDM systems and does not affect the parameters of the single-channel systems. Due to SRS an energy transfer from lower channels (shorter wavelengths) to higher channels (longer wavelengths) is observed, as shown in Fig. 1. This results in worsening the CNR in lower channels and limiting the transport capacity of CATV systems. The power penalty due to SRS is characterized by

$$PP_{SRS} = -10\lg\left(1 - P_k\right), \tag{6}$$

where P_k is the fraction of the power coupled from channel k to all other channels.

The following relation [2] can be used to calculate the total power P_{tot} of all WDM channels that is transferred to the fiber in order to provide 0.5 dB power penalty ($P_k < 0.1$):

$$P_{\text{tot}}\left(N-1\right)\Delta\lambda_s L_{\text{eff}} < 40\,000\,[\text{mW.nm.km}], \tag{7}$$

where N is the number of channels and $\Delta\lambda_s$ is the channel spacing.

III. Limitation of the intra-symbol distortion by SPM and XPM

SPM refers to the self-induced phase shift experienced by an optical field during its propagation along optical fibers. Its magnitude can be defined on the basis of the optical field's phase Φ according to the formula

$$\Phi = \Phi_o + \Phi_1 = \left(n_o + n_1 P/A_{\text{eff}}\right) k_o L_{\text{eff}}, \tag{8}$$

where n_o is the refractive index, n_1 is the nonlinear index coefficient, P/A_{eff} is the optical intensity inside the fiber, $k_o = 2\pi/\lambda$ and λ is the optical wavelength. SPM is responsible for broadening the pulses' spectrum and for producing the optical solitons in the fibers' anomalous-dispersion regime.

If the effect of group-velocity dispersion (GVD) on SPM is negligible then the intensity-dependent nonlinear phase shift at a point *z* arbitrarily chosen along the fiber at a moment *t* can be described by

$$\Phi_1\left(z,t\right) = \left|U\left(0,t\right)\right|^2 P_o \gamma L_{\text{eff}}, \tag{9}$$

where $U(0,t)$ is the normalized optical field amplitude at $z = 0$, P_o is the peak power and $\gamma = 2\pi n_1/\lambda A_{eff}$ is the nonlinear propagation coefficient. Parameter γ can be calculated by the formula

$$\gamma = 2\pi n_1 / \lambda A_{\text{eff}}, \tag{10}$$

where $n_1 \approx 3.2 \times 10^{-20}$ m^2/W.

Since Φ_1 is proportional to $|U(0,t)|^2$ its temporal variation is identical to that of the pulse intensity. The maximum phase shift occurs at the pulse center located at $t = 0$ and is given by

$$\Phi_{1\max} = \gamma P_o L_{\text{eff}}. \tag{11}$$

In order to avoid inadmissible intra-symbol distortion in NRZ digital systems the requirement $\Phi_{1max} \le \pi/2$ must be fulfilled.

The SPM-induced spectral broadening is a consequence of the time dependence of Φ_1. A temporally varying phase implies that the instantaneous optical frequency differ across the pulse from its central value ω_o. The difference $\delta\omega(t)$ is given by

$$\delta\omega(t) = -\frac{\partial\Phi_1}{\partial t} = -\gamma P_o L_{\text{eff}} \frac{\partial}{\partial t}\left|U\left(0,t\right)\right|^2. \tag{12}$$

The time dependence of $\delta\omega$ can be viewed as a frequency chirp increasing in magnitude with the distance propagated. In other words, new frequency components are continuously generated as the pulse propagates down the fiber. These SPM-generated frequency components broaden the spectrum over its initial width at $z = 0$.

As shown through analysis, the temporal variation of the induced chirp $\delta\omega$ is negative near the leading edge of the pulse (red shift) and becomes positive near the trailing edge (blue shift). In other words, the result is a shift towards longer wavelengths at the leading edge of the pulse along with a shift to shorter wavelengths at the trailing edge. The SPM effect is illustrated in Fig. 2.

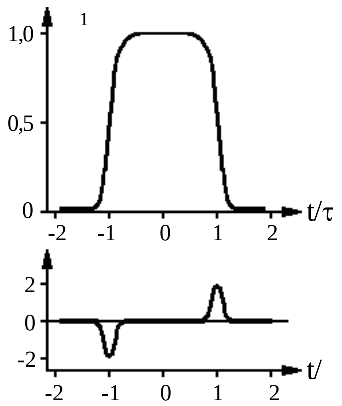

Fig. 2

In the CATV systems design the mutual influence of GVD and SPM effects on the pulse width must be taken into consideration. It is known that in the normal-dispersion regime (when GVD parameter is positive) the wavelength shift caused by GVD is the same as with SPM. Hence, in that case GVD leads to an enhanced rate of pulse broadening compared with that expected for SPM alone. The situation differs for pulses propagating in the anomalous-dispersion regime of the fiber (when GVD parameter is negative). In that case the spectrum is narrowing rather than broadening as expected by SPM in the absence of GVD. This behavior can be explained with the fact that the SPM-induced chirp is positive while the dispersion-induced chirp is negative, i.e. the two chirp contributions nearly cancel each other.

Cross-phase modulation appears when two or more waves propagate inside the fiber and interact between them in result of the nonlinearity of the refractive index produced by the total power inside the fiber. This effect is similar to SPM but the phase shift of one channel depends on the power of other channels. The XPM phase shift Φ_i associated with each channel ($i = 1,2, \dots, \mathrm{N}$) can be estimated by adapting the formula used for SPM phase shift as follows

$$\Phi_i = \gamma L_{\text{eff}} \left[P_i + 2\sum_{n \neq i}^{N} P_n \right]. \tag{13}$$

As seen from (13), XPM is always accompanied by SPM. If the optical fields are of equal intensity the XPM contribution to the nonlinear phase shift is twice as big as that of SPM. Like SPM, the XPM phase shift in a NRZ digital system becomes significant when $\Phi_i > \pi/2$.

In WDM systems both SPM and XPM can cause significant phase changes. When information is transmitted through amplitude modulation and is then incoherently demodulated, nonlinear phase changes are of little consequence. However, if coherent demodulation techniques are employed, such phase changes can limit the system performance.

Let's consider the case of amplitude-modulated coherent CATV system. If a phase sensitive (homodyne) detection technique is used, the phase Φ_i would vary from bit to bit depending on the bit pattern of the neighboring channels. In the worse case the XPM-induced phase shift of a given channel will be

$$\Phi_{i\,\max} = 2\gamma L_{\text{eff}} (N-1)P\,, \tag{14}$$

where P is the power assumed to be the same in each channel. If we take $\Delta\Phi_{i\,\max} \leq 0.1$ as an acceptable value, the power in each channel is restricted to

$$P \leq 0.05\left[\gamma L_{\text{eff}} (N-1)\right]^{-1}. \tag{15}$$

IV. FWM EFFECT ON THE SYSTEM PARAMETERS

Four-wave mixing is the interaction between three transmitted channels of different frequencies f_i, f_j and f_k, producing a fourth product frequency

$$f_{ijk} = f_i + f_j - f_k\,. \tag{16}$$

There are number of ways in which channels can combine to form a new channel according to the formula above. Figure 3 illustrates the possible frequency combinations that can be generated. With N-channel system the number M of unwanted signals known as ghost channels can be calculated by

$$M = 0.5\left(N^3 - N^2\right). \tag{17}$$

For example, a four-channel system would produce 24 ghost channels and an eight-channel system would produce 224 unwanted channels.

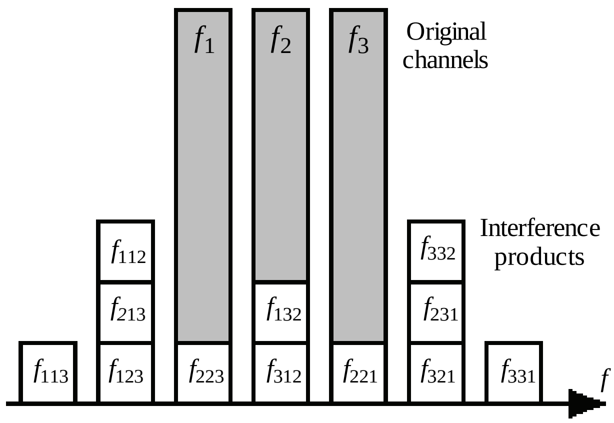

Fig. 3

FWM products reduce the energy in the transmitted channels, thus causing the carrier-to-noise ratio (CNR) to go down at the receiver input. In addition, if the resulting frequency product is within the bandwidth of the transmitted channel it will cause crosstalk at the receiver. As shown in Fig. 3, four of the products generated through FWM drop within the frequency band of the channels transmitted while the remaining six products are out of it and can be suppressed through optical filtering.

Although the equation (16) indicates the position of the potential FWM products it provides no information as to whether the product will be viable, i.e. if the process will be efficient enough for the product to have significant power. The effect of FWM depends on the phase relationship between the interacting signals. That's why the efficiency of the FWM process is determined by the phase matching condition. Phase matching depends on the frequencies of the incident and resultant signals and the chromatic dispersion of the fiber.

The FWM phase mismatch $\Delta\beta = \beta_i + \beta_j - \beta_k - \beta_{ijk}$ can by expressed as [3]

$$\Delta\beta = \left(\frac{2\pi c}{\lambda^2}\right)\left(\frac{dD}{d\lambda}\right)\left(\frac{\lambda_i+\lambda_j}{2}-\lambda_o\right)\left(\lambda_i-\lambda_k\right)\left(\lambda_j-\lambda_k\right) \quad (18)$$

where λ_i, λ_j and λ_k are the three original wavelengths; λ_o is the zero-dispersion wavelength and $dD/d\lambda$ is the slope of the dispersion curve at λ = 1550 nm.

The following formula is used to determine FWM efficiency η:

$$\eta = \left(\frac{\alpha^2}{\alpha^2+\Delta\beta^2}\right)\left(1+\frac{4e^{-\alpha L}\sin^2\left(\Delta\beta L/2\right)}{\left(1-e^{-\alpha L}\right)^2}\right) \mathrm{x} \quad (19)$$

$$\mathrm{x}\left(\frac{\sin\left(N\Delta\beta L/2\right)}{\sin\left(\Delta\beta L/2\right)}\right)^2,$$

where α is the attenuation constant, L is the actual fiber length, N is the number of transmitted channels and $\Delta\beta$ is as calculated above. Analysis shows that the effect of FWM is reduced whenever a dispersion is available and $\eta \rightarrow 0$ for a channel spacing of 100 GHz.

V. OPTIMIZATION OF THE MAXIMUM POWER PER CHANNEL

Two types of fiber have been used to carry out the simulation studies – Fujikura Ltd. SMF and LEAF. The SMF chosen is designed to work within the wavelength range 1310-1600 nm and its parameters are as follows: α = 0.21 dB/km at 1550 nm; MFD = 10.4 µm (A_{eff} = 85 µm^2) and D_{ch} = 17 ps/nm.km at 1550 nm. The parameters of the chosen LEAF are as follows: α = 0.22 dB/km, MFD = 9.6 µm (A_{eff} = 72 µm^2), D_{ch} = 2.0 to 6.0 ps/nm.km at 1530 – 1565 nm.

LEAF refer to the non-zero dispersion shifted fibers (NZ-DSF), whose MFD is smaller than that of the standard SMF and makes them more susceptible to non-linear effects. LEAF offer system designers a higher power-handling capability, improved optical signal-to-noise ratio, longer amplifier spacing and maximum WDM channel plan flexibility as compared with NZ-DSF. It should be pointed out that fibers of a large A_{eff} provide a critical performance advantage – the ability to uniformly reduce all non-linear effects.

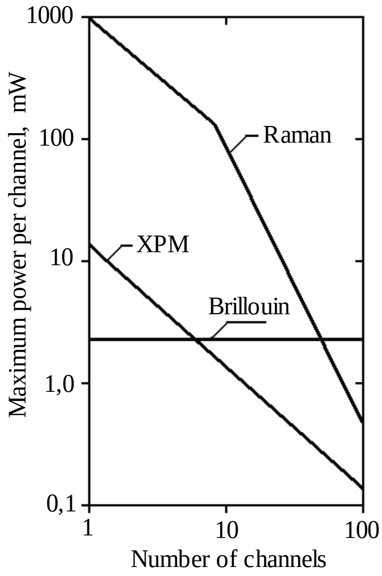

Fig. 4

The relation curves of the maximum power per channel of a WDM system investigated for various nonlinear effects are shown in Fig. 4. The experiment is based on the mathematical models here described and refers to a system applying an optical fiber of Fujikura Ltd LEAF type, the channel spacing being 200 GHz. The maximum power fed to the fiber in an 8-channel system is limited by a SBS power threshold of 3 dB, while limitations in a 16-channel system are due to the XPM effect.

VI. CONCLUSION

The dependencies here proposed enable the designers to determine the maximum launch power that provides the desired information quality if the fiber parameters and the number of transferred channels are given. The formulae have been developed into software products for the design of WDM CATV systems.

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