Some Integral Properties of Nakagami-*m* distribution

Hana Popovic[1], Aleksandra Mitic[2], Ivan Stefanovic[3] and Dimitrije Stefanovic[4]

Abstract **– In this paper Nakagami fading channel model is described and some statistical characteristics of Nakagami-*m* distribution are analyzed. For analytical and numerical evaluation of system performance, the Nakagami probability density functions are analyzed like particular solutions of corresponding differential equation. The existence of singular solution is considered and analyzed under different conditions.**

Keywords **– The Nakagami fading channel model, probability density function, Nakagami distribution, singular solutions.**

I. INTRODUCTION

Mobile communication is burdened with particular propagation aspects that make reliable wireless communication more difficult than fixed communication. The propagation characteristics change from place to place and, if the terminal moves, from time to time. The mobile radio channel is usually evaluated from statistical propagation models: no specific terrain data is considered and channel parameters are modeled as stochastic variables. Two mutually independent, multiplicative propagation phenomena can usually be distinguished: multipath fading and large-scale path loss (shadowing). As the result of multipath reception, the mobile antenna receives a large number of reflected and scattered waves. Because of wave cancellation effects, the instantaneous received power seen by a moving antenna becomes a random variable. Multipath propagation causes rapid fluctuations of the phase and amplitude of the signal if the vehicle moves over a distance in the order of a wave length or more. Multipath fading thus has a small-scale effect. The rapid fluctuations of the instantaneous received power due to multipath effects are usually described with Rayleigh, Rician or Nakagami-m model [1].

The Rician model is useful for modeling mobile wireless communication systems when the transmitted signal can travel to the receiver along a dominant line-of-sight (LOS) or direct path. The reflected paths those the signal travels are modeled with multipath Rayleigh fading channel. Rayleigh model is used in cases when direct LOS doesn't exist. Besides Rayleigh and Rician models, the distribution of the signal amplitude and power can be well described with Nakagami-*m* probability density function (pdf). Rician and Nakagami fading models are two generalizations of the model for Rayleigh fading. In the literature, the Nakagami model is often used for analytical simplicity in cases where Rician fading would be a more appropriate model. The Nakagami distribution matches some empirical data better than other models, but it is not always an appropriate approximation for Rician fading model. It has an essentially different behavior for deep fades, such that results on outage probabilities or error rates can differ by orders of magnitude [6].

The main point in this paper is the numerical analysis of Nakagami-m pdfs, as particular solutions of corresponding differential equation, for one varying parameter, while the others are set to constant values. On the other side, the envelope of family of pdf curves shows that the initial differential equation has the singular solution. The existence of singular solution could be used during the analysis of system performance.

II. NAKAGAMI FADING CHANNEL

The Nakagami fading model was initially proposed because it matched empirical results for short wave ionospheric propagation. The Nakagami distribution is described by pdf:

$$p_z(z) = \frac{2}{\Gamma(m)}\left(\frac{m}{\Omega}\right)^m z^{2m-1} e^{-\frac{m}{\Omega}z^2}, z > 0, m \geq \frac{1}{2} \tag{1}$$

where z is the received signal level, $\Gamma()$ is the gamma function, and m is an integer shape factor called fading figure. It is defined by [4]:

$$m = \frac{E^2[z]}{Var[z^2]} \tag{2}$$

while Ω is defined as:

$$\Omega = E[z^2] \tag{3}$$

The expressions for Rician fading are less convenient, mainly due to the occurrence of a Bessel function in the Rician pdf of received signal amplitude. Approximations by Nakagami distribution, with the simpler mathematical expressions, have become very popular, but it is not always an appropriate model.

The Rician and Nakagami models behave approximately equivalently near their main value. This observation has been used in many recent papers to advocate the Nakagami model as an approximation for situations where a Rician model would be more appropriate [10].

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In the special case that m=1, Rayleigh fading is recovered, while for larger m the spread of the signal strength is less, and the pdf converges to a delta function for increasing m. When using the Nakagami model to approximate the pdf of the power of a Rician fading channel, matching the first and second moments of the Rician and Nakagami pdfs gives:

$$K = \sqrt{m^2 - m} \Big/ \left(m - \sqrt{m^2 - m} \right) \tag{4}$$

where is K Rician factor, defined as the ratio of LOS signal power to the random path signal power. The results are strikingly different for m>1. As the relation between K and m was based merely on the first and second moments, it is likely to be most accurate for values close to mean. The probability of deep fades differs for these two models, so an approximation the pdf of a Rician fading by a Nakagami pdf can be highly inaccurate.

III. NUMERICAL RESUTLTS

The graphic representation of numerical analysis of Nakagami pdf is given on following figures.

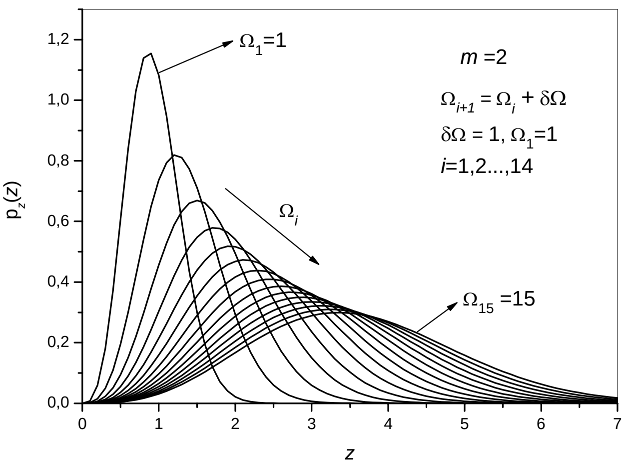

Fig. 1. The Nakagami pdf versus signal level z, for case m=2 with Ω taking values from 1 to 15

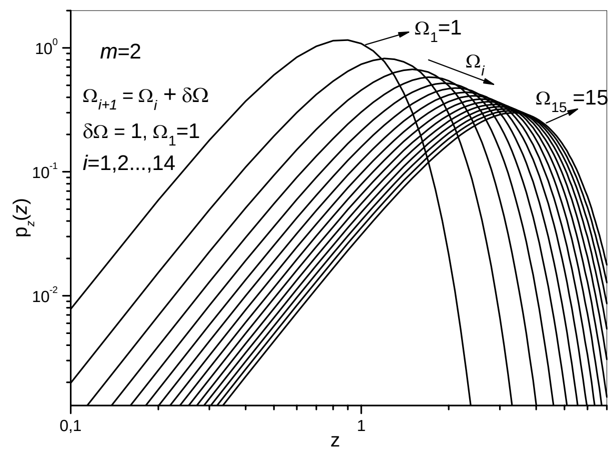

Fig. 2. The Nakagami pdf versus signal level z, in logarithmic scale, for case m=2 with Ω taking values from 1 to 15

It can be seen on Fig. 1. that with the increase of Ω the maximums of pdfs achieve smaller values for larger values of z. Also, the width of the curves is getting larger for larger values of Ω.

The same dependency is shown on Fig. 2. in logarithmic scale, where it can be seen that envelope of pdf curves family is a straight line.

The existence of the envelope of the family of pdf curves enables the pdfs to be regarded as particular solutions of some differential equation in further analysis.

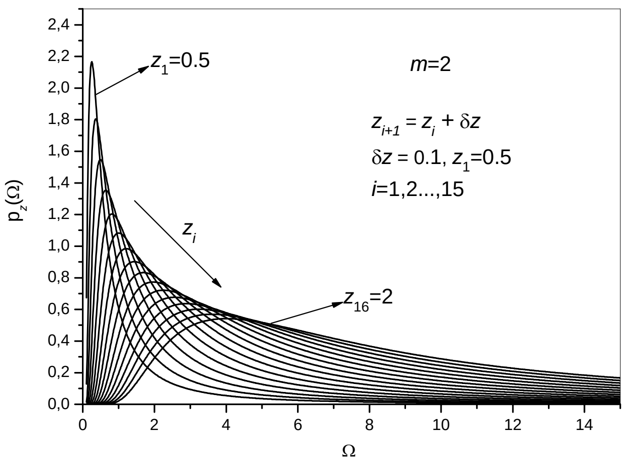

Fig. 3. The Nakagami pdf versus Ω, for case m=2 with z taking values from 0.5 to 2

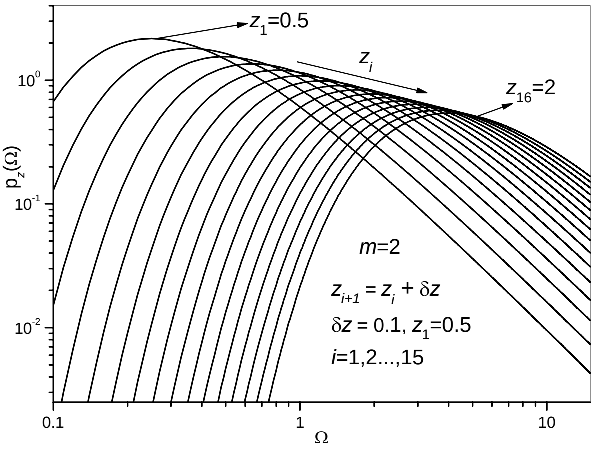

Fig. 4. The Nakagami pdf versusΩ, in logarithmic scale, for case m=2 with z taking values from 0.5 to 2

The Figs 3. and 4. show the families of pdf curves where z is taken as parameter. It can be seen that similar conclusions exist as in previous case. With the increase of z the maximums of pdfs achieve smaller values for larger values of Ω, and also, the width of the curves is getting larger for larger values of z. In logarithmic scale, it can be seen that envelope of pdf curves family is a straight line.

Analytical and numerical analysis shows that as well as particular solutions to the aforementioned differential equation, the singular solutions also exist, and they represent the envelopes of the analyzed families of pdf curves. Since these solutions can not be found through analysis of the initial differential equation, they were found by grapho-analythical means. As the result, the solutions were found to be exponential functions, or in logarithmic scale, straight lines.

The dependency of Nakagami pdf versus z and Ω is presented by three-dimensional graph, as shown on Fig. 5.

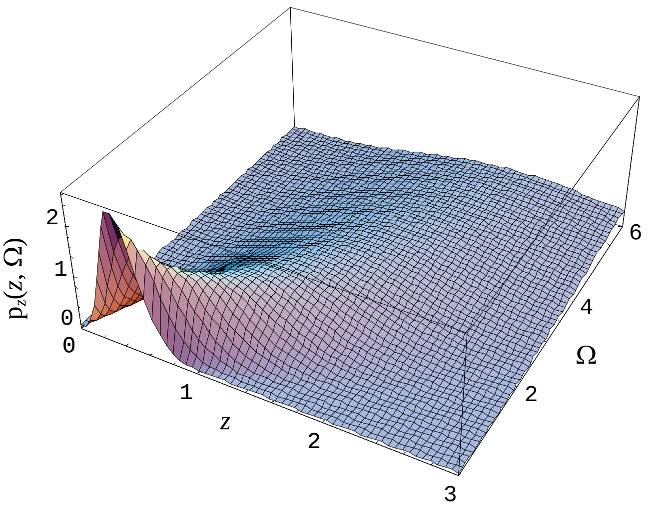

Fig. 5. A three-dimensional graph showing the dependency of Nakagami pdf versus z and Ω

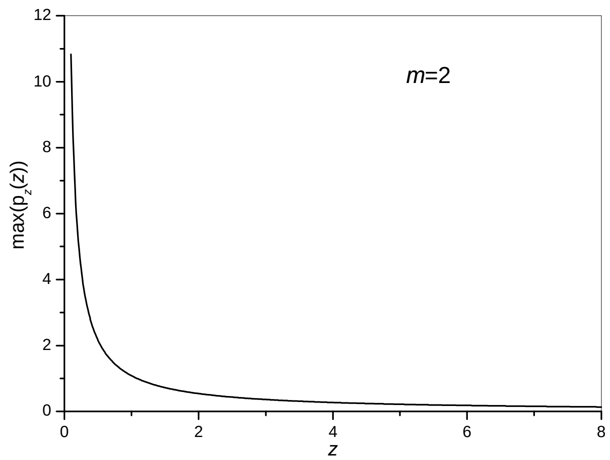

Fig. 6. The max Nakagami pdf versus z, for case m=2

The dependency of pdfs maximums versus z is shown on Fig. 6. That dependency can be described by equation:

$$\log(\max p_z(z)) = k \log z + n \qquad (5)$$

In logarithmic scale, it is presented by a straight line. All of pdfs maximums rest on a straight line, defined by Eq. (5), where k=-1 and n=0.0345.

The dependency of pdfs maximums versus Ω is shown on Fig. 7. That dependency is also described by Eq. (5), but with different parameters: k=-0.5 and n=0.064. In logarithmic scale it is also presented by a straight line, which means that all of pdfs maximums rest on a straight line.

It can be shown that pdfs maximums versus z rest on envelope of pdf curves family given on Fig. 3. and the pdfs maximums versus Ω rest on envelope of pdf curves family given on Fig. 1.

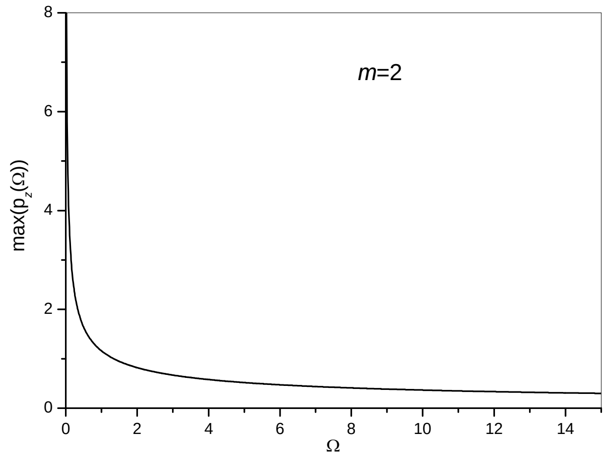

Fig. 7. The max Nakagami pdf versus Ω, for case m=2

Analytical and numerical analysis of integral properties of pdfs is very important for predicting the system performance. The theoretical analysis of performance (like Bit-Error-Rate and outage probability) for various modulation techniques, with the different pre-detection and post-detection diversity combining techniques, is based on analysis of properties of pdf.

This approach could also be used in the analysis of the other physical processes involved in the radio-diffusion, optoelectronic and physics generally.

IV. Conclusion

An advantage of the Nakagami distribution is that it can be reduced to the Rayleigh distribution and can model fading conditions more severe or less severe than those in Rayleigh case. In some situations, Nakagami model can be used as an approximation for Rician fading model. Analytical and numerical analysis of Nakagami pdf that describes the received signal envelope, is very important for predicting the performance of a modulation or coding scheme and is essential for the efficient evaluation and validation of system design [8].

Analytical and numerical analysis of Nakagami-m pdfs, as particular solutions of corresponding differential equation, for one varying parameter, while the others are set to constant values, shows the existence of singular solution, which could be used during the analysis of system performance.

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