Polar Diagram of Multiple Driver Loudspeaker Systems

Ekaterinoslav S. Sirakov

***Abstract* –In this work are present theoretical analysis polar characteristics and horizontal or vertical off–axis frequency responses of loudspeaker system. Discussed are the mathematical dependencies for generating of sound pressure from loudspeakers operating in a given frequency range, specified by crossover network.**

***Keywords* – Loudspeaker system, multiple driver, crossover network, polar diagram, Horizontal and Vertical off–axis frequency responses.**

I. THEORETICAL FUNDAMENTALS FOR DEFINITION OF SOUND FIELD

Sound field may be defined using the Huygen – Fresnel equation, known in acoustics as the Relay integral.

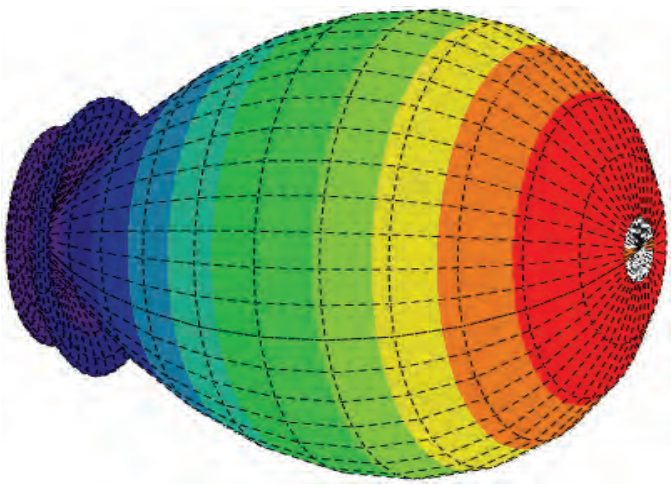

Fig.1. 3D polar diagram of a circular piston with diameter d=30cm for 2.5 kHz frequency, Eq. (1)

For establishment of sound pressure in a far-field region by a loudspeaker installed in the system we may use with success the mathematical expressions for circular piston surface situated on infinite absolutely solid screen.

For circular piston located in infinite screen, for the far-field region using the Frounhofer's approximation we have the following relative simple formula:

$$D(\theta)=\frac{1}{\pi.a^2}\int_0^a \rho \int_0^{2.\pi} \exp(-j.k.\rho.\sin(\theta).\cos(\gamma))d\gamma d\rho$$

where: a is the radius of circular piston, (1)

$\kappa=\frac{\varpi}{c}$ is the wave-number,

c is the speed of sound.

Illustrated on fig. 1 is a 3D polar diagram for Relay's integral Eq. (1), in the far-field region for a circular piston with diameter *d=2.a=30cm* for frequency *2.55 kHz.*

Illustrated on fig. 2 is the frequency response of the sound pressure created by a circular piston with diameter *d=30cm* for three values of the angle θ. The first zero is at $\theta=\pi/2$ $(\sin(\theta)=1)$ and is for frequency 1.4 kHz. The second zero is for frequency 2.55 kHz, represented by 3D polar diagram on fig. 1. By reducing the value towards emission axis, the frequency increases, as illustrated on fig. 2 for $\theta=45^0$, the first zero is at 2 kHz frequency.

$$D_{SPL}(f,\theta)=90+20.\log|D(f,\theta)| \quad (2)$$

$$D(f,\theta)=\frac{1}{\pi.a^2}\int_0^a \rho \int_0^{2.\pi} \exp(-j.k.\rho.\sin(\theta).\cos(\gamma))d\gamma d\rho$$

$$k=\frac{2.\pi.f}{c}$$

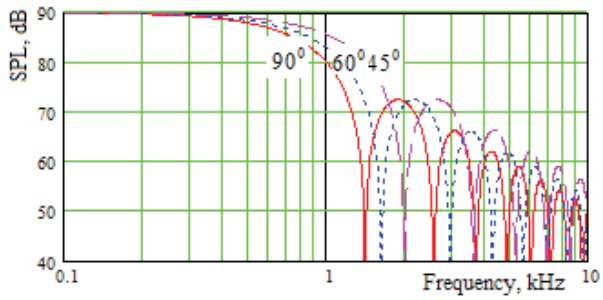

Fig. 2. Frequency response for sound pressure created by circular piston at parameter angle (90^0, 60^0 and 45^0), Eq. (2)

The solution of Riley's integral for circular piston, installed on infinite solid screen, [1], [3] and [4] is:

$$D(\theta)=\frac{2.J_1(k.a.\sin(\theta))}{k.a.\sin(\theta)}, \quad (3)$$

$J_1(k.a.\sin\gamma)$ is a first-order Bessel function.

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II. Definition of Sound Pressure with Two Sources

For two point sources in phase of equal amplitude, sound pressure may be defined with the expression [1], [2]:

$$D_2(f,\theta)=\frac{1}{2}.\left(e^{-\frac{j.\pi.\frac{d}{2}.f}{c}.\sin(\theta)}+e^{\frac{j.\pi.\frac{d}{2}.f}{c}.\sin(\theta)}\right) \quad (4)$$

To establish a model for the off-axis frequency response of two loudspeakers of the same type operating in a phase of the same amplitude in vertical plane, used with success may be multiplication theorem (on the diagram for circular piston, Eqs. (1) or (3) and the diagram for two point source, Eq. (4)).

$$D_{2\_LS}(f,\theta)=90+20.\log\left|D_2(f,\theta).D(f,\theta)\right| \quad (5)$$

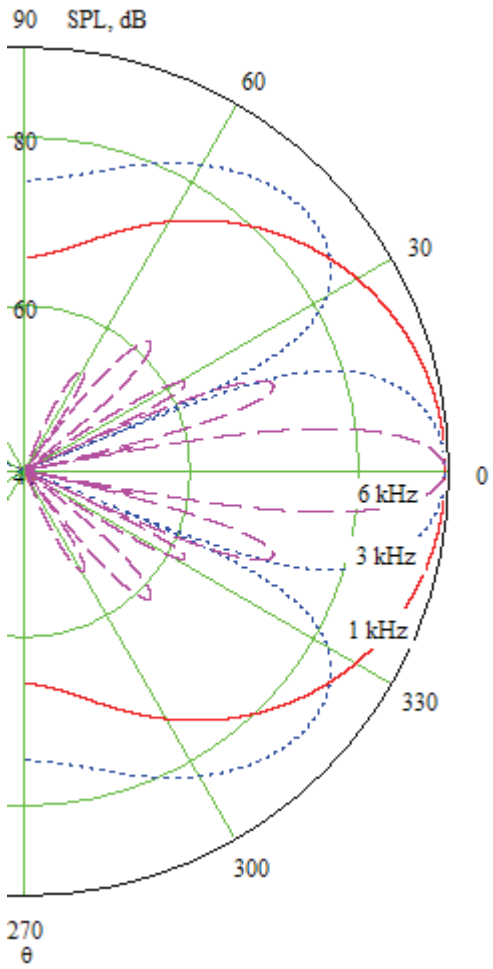

Fig.3. Vertical polar response of two source for frequencies 1, 3 and 6 kHz and distance between a acoustic centers 30 cm and equal diameter of circular piston 30 cm for both drivers, Eq. (5).

III. Model Frequency Response of Sound Pressure Created Loudspeaker

The frequency characteristic of every single loudspeaker may be presented with the band-pass function [2], [4] and [9]:

$$K(s)=K_1(s).K_2(s) \quad (6)$$

$$K_1(s)=\frac{s^2}{s^2+\frac{2.\pi.f_0}{Q}.s+(2.\pi.f_0)^2} \quad [2] \quad (7)$$

$$K_{21}(s)=\frac{1}{1+s} \quad [9]$$

$$K_{22}(s)=\frac{1}{s^2+\frac{2.\pi.f_h}{Q_h}.s+(2.\pi.f_h)^2} \quad [4] \quad (8)$$

$$s=j.\varpi=j.2.\pi.f$$

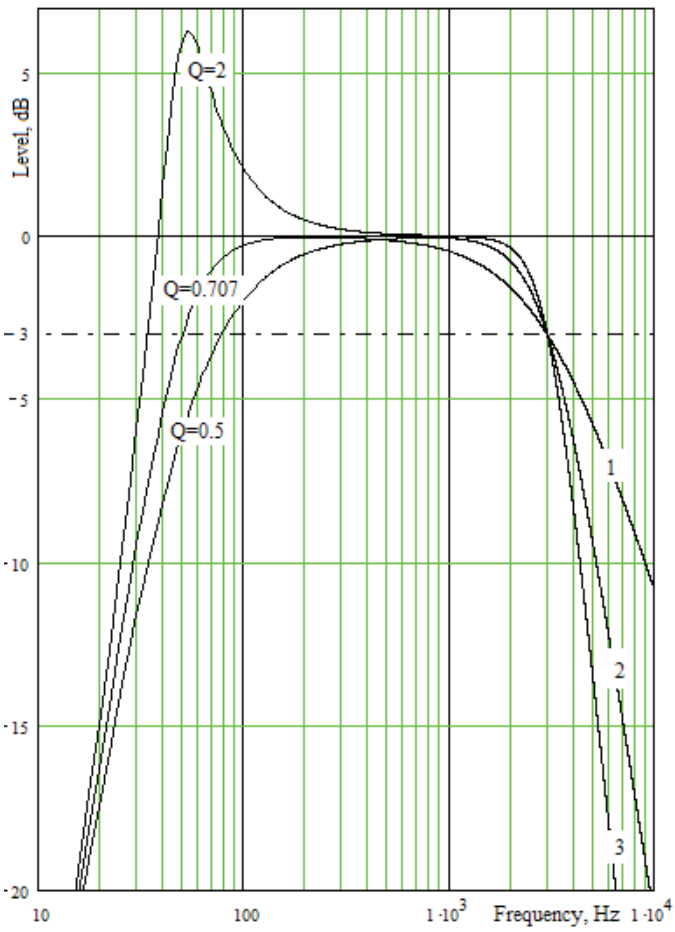

Fig.4. Model for frequency response of sound pressure created by loudspeaker, Eqs. (6), (7) and (8)

IV. Target functions approach of crossover (filters) for two frequency bands

Frequency separation for loudspeaker systems is achieved by passive (for high signal level), active (for low signal level) and digital (for digital loudspeaker systems) separation circuits (crossover) or in a mixed mode.

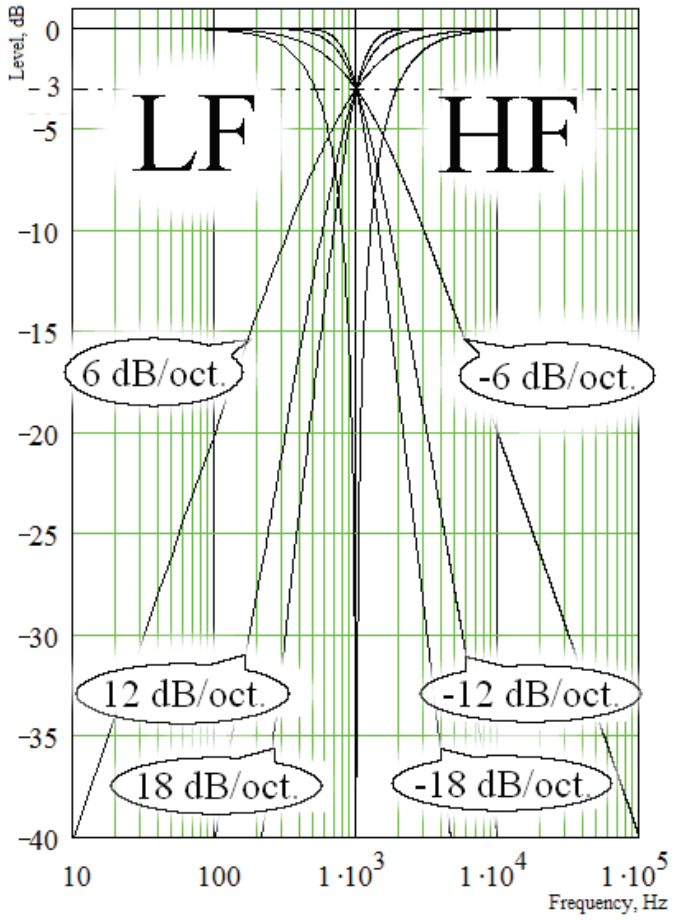

Fig.5. Two-way crossover response (6, 12 and 18 dB/octave rolloffs), Eqs. (9), (10) and (11)

All crossover networks (filters) may be described by their target functions approach. Selected for filters is Linkwitz-Riley, Bessel, Butterworth or different polynomial approximation. The rolloff of filters defined by polynomial approximates, is: 6 dB/oct. for first order, 12 dB/oct. for second order, 18 dB/oct. for third order etc.

The selection of filter sequence and type as well their polynomial approximation depend on a number of factors and is subject to compromises in actual realization.

For symmetrical filters of 1st order Butterworth type, the channels are in phase, and crossover networks (filters) in passive realization has constant input resistance [2]:

$$T_{L_1}(s)=\frac{1}{1+s},\ T_{H_1}(s)=\frac{s}{1+s},\ T_{L_1}(s)+T_{H_1}(s)=1 \qquad (9)$$

For Butterworth of the second order target functions approach:

$$T_{L_2}(s)=\frac{s/\sqrt{2}+1}{1+\sqrt{2}.s+s^2} \qquad T_{H_2}(s)=\frac{s^2+s/\sqrt{2}}{1+\sqrt{2}.s+s^2} \qquad (10)$$

$$T_{L_2}(s)+T_{H_2}(s)=1$$

For third order:

$$T_{L_3}(s)=\frac{2.s+1}{1+2.s+2.s^2+s^3}$$

$$T_{H_3}(s)=\frac{s^3+2.s}{1+2.s+2.s^2+s^3} \qquad (11)$$

$$T_{L_3}(s)+T_{H_3}(s)=1$$

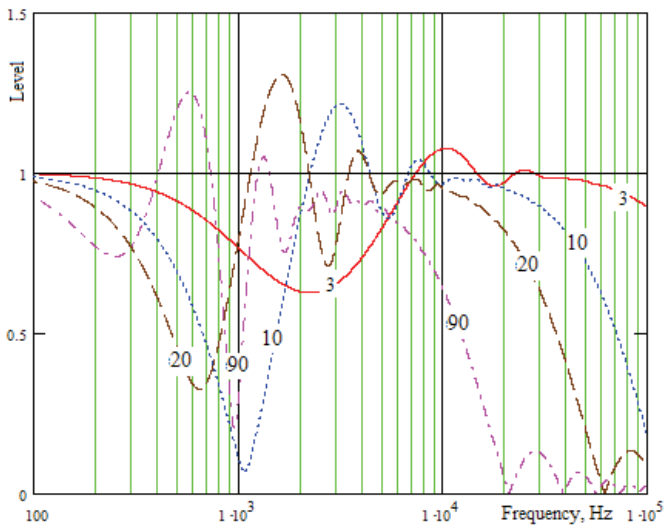

Fig.6. Frequency characteristic of sound pressure from loudspeaker system with two frequency bands at angle θ= 3°, 10°, 20° and 90°, Eq. (12).

Illustrated on fig. 6 is frequency response of sound pressure from loudspeaker system with two frequency bands at angle $\theta=3^0$, 10^0, 20^0 and 90^0 and crossover with Butterworth polynomial approximation of first order. Low frequency loudspeaker effective circular piston diameter is $D_{LF}=20cm$, and for high frequency loudspeaker $D_{HF}=2cm$ respectively, with minimum distance between both loudspeaker centers is $d=15cm$.

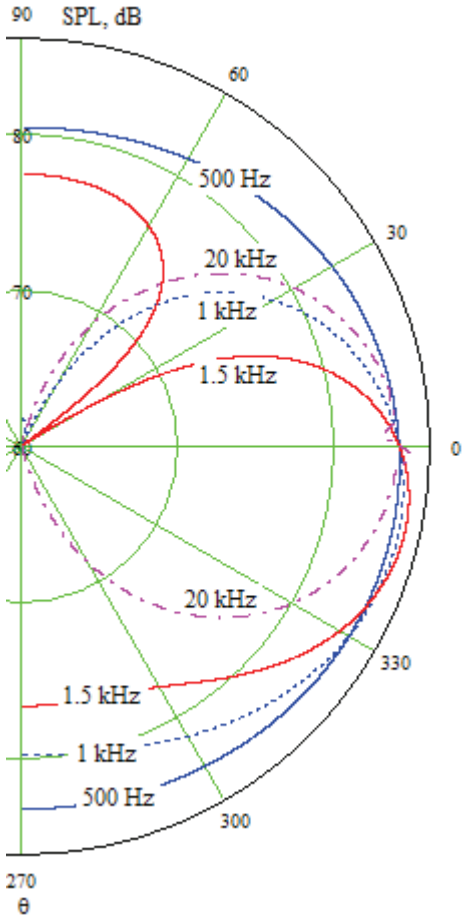

Fig.7. SPL polar response of loudspeaker system with two frequency bands for frequencies 0.5, 1, 1.5 and 20 kHz, Eq. (12).

Illustrated on fig. 7. is the vertical polar plot of two-way loudspeaker system for frequencies 0.5, 1, 1.5 and 20 kHz.

$$D_{V\_dB}(f,\theta) = 90 + 20.\log\left|D_V(f,\theta)\right| \tag{12}$$

$$D_V(f,\theta) = \frac{1}{2}.\left\{ \begin{array}{l} D_L(f,\theta).\left[T_{L_1}(f).e^{-\frac{j.\pi.\frac{d}{2}.f}{c}.\sin(\theta)}\right] + \\ D_H(f,\theta).\left[T_{H_1}(f).e^{\frac{j.\pi.\frac{d}{2}.f}{c}.\sin(\theta)}\right] \end{array} \right\} \tag{13}$$

Driver piston diameter is $d_L=2.a_L=20cm$ for low frequency loudspeaker (for $D_L(f,\theta)$, Eqs. (1), (2) or (3)) and $d_H=2.a_H=2cm$ for high frequency loudspeaker (for $D_H(f,\theta)$, Eqs. (1), (2) or (3)), with distance between acoustic centers d=20 cm

Crossover frequency is $f_0=3kHz$. Butterworth approximation of crossover network of the first order is, Eq. (9).

V. Conclusion

The components of a single loudspeaker system: separation circuits (crossover) for respective frequency bands, usually two or three, geometry of loudspeaker arrangement – one or two for respective band, frequency characteristic of every single loudspeaker, influence the general polar characteristics of the system.

The presentation of loudspeaker as a circular piston Eq. (1) facilitates the investigation of frequency characteristic irregularity depending on angle parameter diameter (fig. 2), and the selection of upper limit frequency.

With two loudspeakers for the same frequency range according to classical geometry (one over the other normally at specified distance) or in the so called D'Appolito geometry aimed to enhance the generated sound pressure will sharply narrow the vertical polar response (fig. 3).

Used successfully for measuring the frequency characteristics of specific loudspeaker may be the mathematical dependencies Eq. (6) taking into account the specification T/S parameters (fig. 4) or actually measured characteristic values [Martin J. King, 6].

Of considerable importance is the selection of a specific separation circuit (crossover) type and filter sequence, not only for the generated sound pressure along emission axis, but for different angles too (Eq. (13), fig. 6) as well as the common diagram, depending on frequency, especially for frequencies close to separation (Eq. (12), fig. 7).

The results obtained in practice may be used for theoretical analysis, and in loudspeaker system design and production.

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