Nonlinear Systems Modeling in Broadband Communications

Oleg Borisov Panagiev[1]

***Abstract* — In this report modeling of nonlinear systems is investigated. Nonlinear models can be divided to ad hoc type block models, and analytical models, most notably the Volterra series. In broadband communications systems related modeling it is convenient to split nonlinear models to memoryless ones, and to those with memory. In order to obtain optimum results from Volterra methods, it helps to understand a little about the method and the way nonlinear system are modeled.**

***Keywords* — CATV/HFC, Volterra series, single-frequency dependent (SFD) model.**

I. INTRODUCTION

Recently, CATV/HFC networks have received much attention in the context of interactive application: telephony, interactive television, home-shopping, video-on demand, high-speed Web browsing, etc. The interactive nature of these services requires however two-way communication on a network that initially was only intended for a one-way broadcasting of television signals. This huge information capacity requires communication system to be very broadband, witch set using of a big number of amplifiers. Each amplifier adds noise and intermodulation distortions to the signal, the number of cascaded trunk amplifiers in the primary lines is thus limited to 40 to 45, in order to ensure a good reliability of the network and a good quality of the signal delivered to the subscribers. In order to provide other services such as digital television, it is necessary to increase the capacity of existing networks [1]. In this case CATV/HFC networks contain many other cascaded nonlinear elements, coaxial and/or optical with coaxial amplifiers too. Each of these nonlinear elements would have deteriorated C/N, respectively BER and CSO/CTB in the transmitted channels. The CSO/CTB frequency spectrum and C/N are adequate characteristics of distortions and noises in channels of CATV network. They can be determinate for the cascaded net elements from the CSO/CTB frequency spectrum and C/N for each of cascaded net elements and coaxial amplifiers too [2]. This leads to make investigation and classification of model types. The next section is devoted to classification and choosing the model of broadband communication systems as CATV/HFC.

[1]Oleg B. Panagiev is whit the Technical University of Sofia, Bulgaria, E-mail: olcomol@yahoo.com

II. THE MODEL'S CLASIFICATION

In the process of CATV/HFC nonlinearities research is necessary to choose a good model. The nonlinear system modeling problem can be formulated as follows: Find a nonlinear function f that well describes the input-output mapping obtained by measurements. In general case the function is a mapping from an m -dimensional space to an n – dimensional one, a multiple-input multiple-output system. Usually some noisy measurement results of the input-output mapping are available.

A nth degree polynomial

$$f\,[x(t), a] = a_0 + a_1 x(t) + a_2 x^2(t) + a_3 x^3(t) + \ldots + a_n x^n(t) \qquad (1)$$

is an example of a nonlinear mapping $f : \Re \rightarrow \Re$ with (n+1) parameters. It is assumed that the model is time-invariant. In this report it is assumed that input and output are one-dimensional.

Depending on the amount of prior information known about the modeling task we may talk about white-box, grey-box, or black-box modeling [3]. In white-box modeling the model is perfectly known from prior knowledge. An example of a white-box model is the simple voltage-current law of a diode. In a grey-box modeling something is known a priori, but some model parameters must be determined from measurements. In black-box modeling no physical insight or a priori information is available. The nonlinear black-box modeling problem is among the most difficult in the field of system identification.

The central problem is how to find a proper model class, and model order within the chosen class, of all possible functions. After this it is only a matter of optimizing the parameters of the model. Due to nonlinear nature of the problem, several local optimums usually exist, complicating the optimization phase. Examples of nonlinear model (function) classes include polynomial models, artificial neural networks, and Volterra series. For a grey-box problem one can narrow down the model class based on information known of the modeling problem.

Finding a "good" model class for a black-box problem is a very fundamental problem, for which no single all-around solution seems to exist. Typical methods used to determine model order within a given model class include cross validation and bootstrap methods (statistical methods); Minimum Description Length (MDL) (information theoretical method), Akaike Information Criterion (AIC) (ad hoc); and Structural Risk Minimization (statistical learning theory) [4]. A flowchart of a general modeling task is shown in Fig. 1.

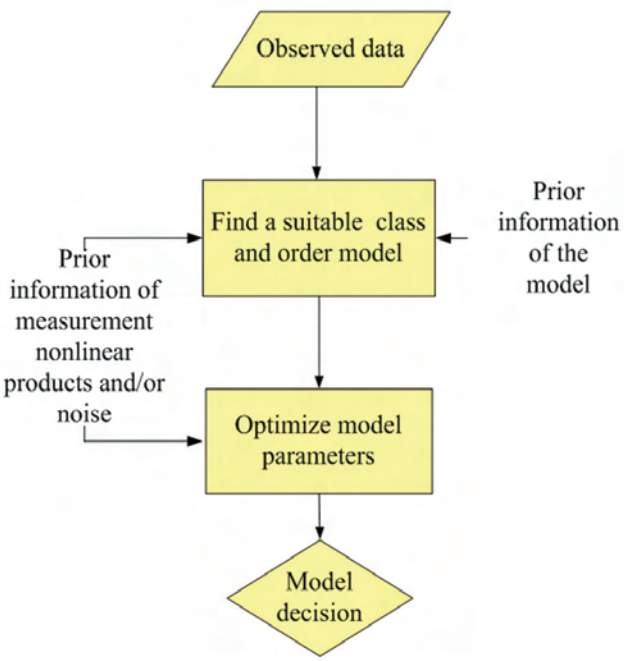

Fig.1. Algorithm for nonlinear system modeling

TABLE 1

Models	Models with memory	Models without memory
Analytical	Volterra series Nonlinear differential equations	Polynomials Simple diode
Block	Saleh model *N* - box model Recurrent neural networks	Two box Table look-up Feedforward neural networks

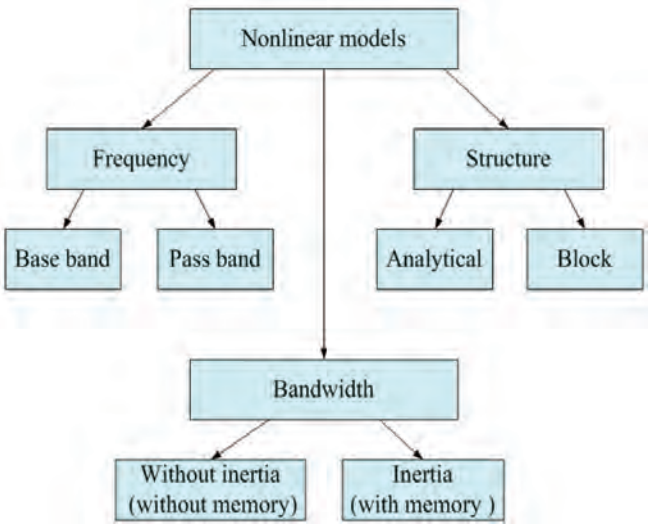

Fig.2. Nonlinear models classification

The classification of mathematical models in dependence of the different symptoms is too multiple. Thus like in a dependence of a level of complexity are isolated: *full models*, forme250 d by immediate union in general system from equations of separate components, building the CATV system; *macromodels*, witches are got by approximation of full models.

From the ways of determinating of the parameters, the models are divided on: *physical*, at whose are examine the physical regularities, inherent for the device (system), as a result of witch the structure of the equations and parameters of the models have definite physical interpreting; *formal*, whose parameters are finding experimentally – by measuring of the reaction of the system of some definite testing signals.

From made above reasoning for system modeling may be formed next classification of models (Fig. 2). Some examples for models are given in Table 1

In the further examined the models will be used the moderating of the system in the pass band, witch for the general case for modern systems for cable television is from 5 MHz to 860 MHz.

III. NONLINEAR SYSTEMS MODELING WITH VOLTERRA SERIES

Well known are many models for nonlinearities with memory, where the nonlinearity system is presented as combination from in series related linear with memory and nonlinear memoryless systems [3], [5], [6]. This model is named summary nonlinear model (SNM) and its decisions may to given with differential equations [6], [7], [8] or power series [9], [10], [11].

Below we offer nonlinear model on advance broadband communication system, which is modeling with Volterra series. This model is named single-frequency dependent (SFD) model (Fig.3). He is consist n numbers of parallel connected branches, each of them contains nonlinear memoryless and linear system with memory from respective order. That helps for the calculating of the spectrum of output signal of the system from each one order ($\leq$ n). The most important priority of the proposed models is the relative simplicity, with witch the kernels (transfer functions of Volterra) can be measured and applied in the analysis of the nonlinear distortions.

Other priorities of the model are:

- Meanwhile compact reading of the inertial and nonlinear characteristics of the tract;
- The block presenting of the transforming of the input influence allow simplifying of the problem about dividing of the nonlinear products from different order. As a result of that studying of the different types nonlinear distortions became possible, instead of studying the process as a whole one.

Although the indicated model may not capture the full aspects of a nonlinear CATV system, its primary usefulness perhaps resides in the associated measurements and computations required for system identification.

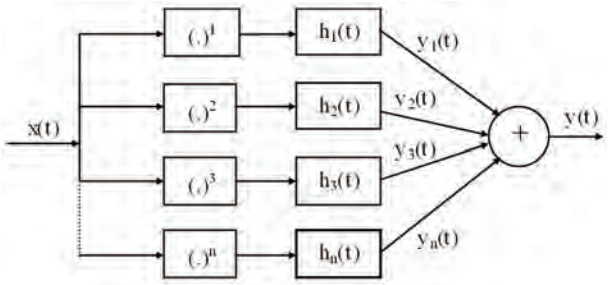

Fig.3. Single-frequency dependent model

Volterra theory is a generalization of the linear convolution integral approach often applied to linear, time-invariant systems. The theory states that any time-invariant, nonlinear system can be modeled as an infinite sum of multidimensional convolution integrals of increasing order. This is represented symbolically by the series of integrals

$$y(t) = h_0 + \int_0^{+\infty} h_1(\tau_1)x(t-\tau_1)d\tau_1$$

$$+ \int_0^{+\infty}\int_0^{+\infty} h_2(\tau_1,\tau_2)x(t-\tau_1)x(t-\tau_2)d\tau_1 d\tau_2$$

$$+ \int_0^{+\infty}\int_0^{+\infty}\int_0^{+\infty} h_3(\tau_1,\tau_2,\tau_3)x(t-\tau_1)x(t-\tau_2)x(t-\tau_3)d\tau_1 d\tau_2 d\tau_3$$

$$+\cdots+$$

$$+ \underbrace{\int_0^{+\infty}\cdots\int_0^{+\infty}}_{n} h_n(\tau_1,\tau_2,...,\tau_n)x(t-\tau_1)x(t-\tau_2)...$$

$$...x(t-\tau_n)d\tau_1 d\tau_2 ... d\tau_n \ , \qquad (2)$$

which are known as the Volterra series. Here, *x(t)* represents the system input while *y(t)* represents the system response. Volterra theory is based on dynamic data, and as such the average values of all input and response data sets are removed. Each of the convolution integrals contains a kernel, either linear (h_1) or nonlinear ($h_2,...,h_n$), which represents the behavior of the system. Knowledge of these kernels allows the prediction of a system's response to any arbitrary input, and as such is critical to nonlinear Volterra modeling.

The application of the Volterra series requires careful consideration of various factors that can impact on the successful calculation of transfer functions. They are useful tools in observing the behavior of nonlinear systems. Transfer functions convert an input signal into an output signal, and for nonlinear systems can be calculated separately for each order of response using the Volterra series. The Volterra series quantifies the linear and nonlinear (e.g. quadratic, cubic etc.) responses separately for systems with either Gaussian or non-Gaussian inputs, and is particularly useful when calculating transfer functions from experimental data, as the calculations can be performed using a discrete frequency-domain format.

For theoretical examination of the nonlinear distortions we choose (in CATV system spectrum) one channel, who under condition we mark with s and call it test (useful) channel. In each of other channels are transferred unmodulated signals with value of a picture carrier frequency in agreement with European standard CENELEC EN 50083 (B/G, D/K). All picture carrier frequencies are CW. In this case we may to write next equation for output system signal (Fig.3):

$$y(t) = y_1(t) + y_2(t) + y_3(t) + ... + y_s(t) + ... + y_n(t) \qquad (3)$$

According to the (2) in the one-dimensional case, such as the output signal of the system, (3) became:

$$y(t) = \int_{-\infty}^{+\infty} h_1(\tau)x(t-\tau)d\tau + \int_{-\infty}^{+\infty} h_2(\tau)x^2(t-\tau)d\tau$$

$$+ \int_{-\infty}^{+\infty} h_3(\tau)x^3(t-\tau)d\tau + ... + \int_{-\infty}^{+\infty} h_s(\tau)x^s(t-\tau)d\tau + ...$$

$$+ \int_{-\infty}^{+\infty} h_n(\tau)x^n(t-\tau)d\tau. \qquad (4)$$

In correspondence with the accepted upper conditions on the input of the system we apply group signal, witch represents a group of unmodulated signals

$$x(t) = \sum_{i=1}^{N} A_i \cos(\omega_i t + \theta_i); \ \ \omega_i = 2\pi f_i; \ i \neq s. \qquad (5)$$

Let's replace with (5) in (4) and do the definite mathematical operation: presenting of $cos\omega_i t$ like exponential function, applying the direct Fourier's' transformation, etc. Let group together the obtained by the calculating addends in attitude with the frequencies (summary and differenced). In this case we will accept that the amplitudes of the signals for every picture carrier frequencies are same (just like the request of the CENELEC EN 50083 for every advanced CATV system). The last results from based frequencies, harmonically, two-component and three-component beet are presented in Table 2, (N=3, n=3). The Volterra kernels in the frequency domain from first, second and third order are obtained from following equations:

$$|H_1(\omega_i)| = \left|\int_{-\infty}^{+\infty} h_1(\tau)\exp(\pm j\omega_i\tau)d\tau\right|; \qquad (6)$$

$$|H_2(\omega_b)| = \left|\int_{-\infty}^{+\infty} h_2(\tau)\exp(\pm j\omega_b\tau)d\tau\right|; \qquad (7)$$

$$|H_3(\omega_b)| = \left|\int_{-\infty}^{+\infty} h_3(\tau)\exp(\pm j\omega_b\tau)d\tau\right|, \qquad (8)$$

where:

$\omega_b = \omega_i \pm \omega_j$ in case of second order beat;

$\omega_b = 2\omega_i \pm \omega_j$ in case of two-component beat;

$\omega_b = \omega_i \pm \omega_j \pm \omega_k$ in case of three-component beat

and $i \neq j \neq k = 1, 2, 3;$

$\theta_1 = \theta_1(\pm\omega_i)$; $\theta_2 = \theta_2(\pm\omega_b)$; $\theta_3 = \theta_{23}(\pm\omega_b)$.

TABLE 2

$y(t) = \sum_{i=1}^{3} y_i(t)$	Σ terms
y_1	$A_1 \lvert H_1(\omega_1)\rvert \cos[\omega_1 t + \theta_1]$ $A_2 \lvert H_1(\omega_2)\rvert \cos[\omega_2 t + \theta_1]$ $A_3 \lvert H_1(\omega_3)\rvert \cos[\omega_3 t + \theta_1]$
y_2	$\frac{1}{2}(A_1^2 + A_2^2 + A_3^2)\lvert H_2(0)\rvert$ $\frac{1}{2} A_1^2 \lvert H_2(2\omega_1)\rvert \cos[(2\omega_1)t + \theta_2]$ $\frac{1}{2} A_2^2 \lvert H_2(2\omega_2)\rvert \cos[(2\omega_2)t + \theta_2]$ $\frac{1}{2} A_3^2 \lvert H_2(2\omega_3)\rvert \cos[(2\omega_3)t + \theta_2]$ $A_1 A_2 \lvert H_2(\omega_1 \pm \omega_2)\rvert \cos[(\omega_1 \pm \omega_2)t + \theta_2]$ $A_1 A_3 \lvert H_2(\omega_1 \pm \omega_3)\rvert \cos[(\omega_1 \pm \omega_3)t + \theta_2]$ $A_2 A_3 \lvert H_2(\omega_2 \pm \omega_3)\rvert \cos[(\omega_2 \pm \omega_3)t + \theta_2]$
y_3	$\frac{3}{4}(A_1^3 + 2A_1 A_2^2 + 2A_1 A_3^2)\lvert H_3(\omega_1)\rvert \cos[2\omega_1 t + \theta_3]$ $\frac{3}{4}(A_2^3 + 2A_1^2 A_2 + 2A_2 A_3^2)\lvert H_3(\omega_2)\rvert \cos[2\omega_2 t + \theta_3]$ $\frac{3}{4}(A_3^3 + 2A_1^2 A_3 + 2A_2^2 A_3)\lvert H_3(\omega_3)\rvert \cos[2\omega_3 t + \theta_3]$ $\frac{1}{4} A_1^3 \lvert H_3(3\omega_1)\rvert \cos[(3\omega_1)t + \theta_3]$ $\frac{1}{4} A_2^3 \lvert H_3(3\omega_2)\rvert \cos[(3\omega_2)t + \theta_3]$ $\frac{1}{4} A_3^3 \lvert H_3(3\omega_3)\rvert \cos[(3\omega_3)t + \theta_3]$ $\frac{3}{4} A_1^2 A_2 \lvert H_3(2\omega_1 \pm \omega_2)\rvert \cos[(2\omega_1 \pm \omega_2)t + \theta_3]$ $\frac{3}{4} A_1^2 A_3 \lvert H_3(2\omega_1 \pm \omega_3)\rvert \cos[(2\omega_1 \pm \omega_3)t + \theta_3]$ $\frac{3}{4} A_2^2 A_1 \lvert H_3(2\omega_2 \pm \omega_1)\rvert \cos[(2\omega_2 \pm \omega_1)t + \theta_3]$ $\frac{3}{4} A_2^2 A_3 \lvert H_3(2\omega_2 \pm \omega_3)\rvert \cos[(2\omega_2 \pm \omega_3)t + \theta_3]$ $\frac{3}{4} A_3^2 A_1 \lvert H_3(2\omega_3 \pm \omega_1)\rvert \cos[(2\omega_3 \pm \omega_1)t + \theta_3]$ $\frac{3}{4} A_3^2 A_2 \lvert H_3(2\omega_3 \pm \omega_2)\rvert \cos[(2\omega_3 \pm \omega_2)t + \theta_3]$ $\frac{3}{2} A_1 A_2 A_3 \lvert H_3(\omega_1 \pm \omega_2 \pm \omega_3)\rvert \cos[(\omega_1 \pm \omega_2 \pm \omega_3)t + \theta_3]$

IV. CONCLUSION

In each term in (4) from higher order are contained harmonic and composite nonlinear products from higher order. Thus by examination of the nonlinear distortions from a particular order, it is not necessary to be exanimate the terms from lower order. But the opposite is compulsory, to be much more exact the obtained results.

Even only with three input signals (N=3) you can see, that the undesired vibrating in the output of the system are more then 30 (for n=3), and in one modern system for cable television number of the transferred channels is much bigger (about 100). From the aforesaid, it is not difficult to see that the problem with analyzing of the nonlinear distortions is too complicated and ambiguity solved, even in using of a modern fast speed computers.

REFERENCES

[1] J. Steel, A. Parker, D. Skellern," Characterization of cable amplifiers for broadband network applications. Electronics Department", Macquarie University, NSW 2109, Australia, pp. 39-45, 2001.

[2] V. Germanov,"Calculating the CSO/CTB Spectrum of CATV Amplifiers and optical Receivers", IEEE Trans. on Broadcasting, vol.44 No.3 pp. 363-366, Sep.1998.

[3] J. Sjoberg, Q. Zang, L. Ljung, A. Benveniste and etc. "Nonlinear Black-box modeling in System Identification: a Unified Overview", Automatica, vol.31, pp.1961-1724, 1995.

[4] V. Cherkassky, F. Mulier, *Learning from Data*. John Wiley, 1998.

[5] M. C. Jeruchim, P. Balaban and K. S. Shanmungan,*Simulation of Communications System: Modeling, Methodology, and Techniques*. Second edition, Kluwer Academic/Plenum Publishers, New York, 2000.

[6] G. J. Foshini, A.A.M. Saleh, "Overcoming Optical Amplifier Intermodulation Distortion Using Coding in Multichannel Communications Systems", IEEE Trans.on commun., vol. 38, No.2, pp. 187-191, Feb.1990.

[7] I. W. Sanberg, "On Volterra expansions for time-varying nonlinear systems", IEEE Trans. on Circuits and Systems, vol.CAS-30, No.2, pp. 61-67, Feb. 1983.

[8] Y. Wan, T. Dodd and R. Harrison. "Infinite degree Volterra series estimation". The University of Sheffield, S1 3JD, UK, 2003.

[9] J. C. Pedro, N.B. Garvalho, R.C. Madureira, "Nonlinear distortion model for VCO-PLL FM transmission system", IEEE Trans. on Circuits and Systems, vol.46, No.3, pp. 348-352, Mar. 1999.

[10] M. Weiss, C. Evans, D. Rees, "Identification of nonlinear cascade systems using paired multisine signals". Proc. IEEE Inst. and measur. tech. conf., Canada, pp. 765-770, 1997.

[11] S. A. Maas, "Analysis and optimization of nonlinear microwave circuits by Volterra series", Microwave J., vol.33, No.4, pp. 245, April 1990.