Intermodulation Composite Distortions Theoretically Research in HFC Networks

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***Abstract* – It is shown that Volterra operator techniques play a key role to study the main structural properties of the CATV/HFC system under consideration. Using the presentation of this operator in the ring of power series allows us to apply some algebraic methods for research.**

***Keywords* – Composite distortions, HFC, trunk amplifier, Volterra series.**

I. INTRODUCTION

During the exploitation of the advanced (HFC – hybrid fiber coaxial) systems for cable television (fig.1) at definite conditions in the active devices, used in the system, spring up nonlinear products. Thus at the subscribers besides the useful signal, proceed and parasitical (disturbing) signals, whose in many cases of big nonlinear prevail over the useful signal. The nonlinearity is being approximated most often with polynomial up to third rank, because of theoretical examinations by mathematical models are too successfully realizable [1], [2].

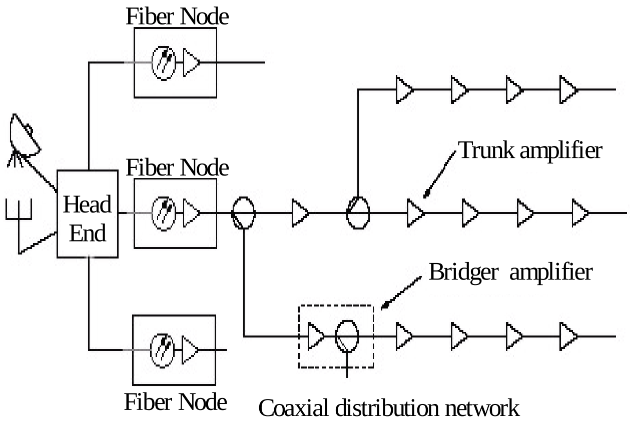

Fig.1. Advanced CATV/HFC system

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For getting of optimal results while using the Volterra series is needed profound understanding of the basic methods and way of modeling of the nonlinear systems, witch are presented as a combination of linear and nonlinear subsystems (parts, elements).

The biggest defect of the methods for analyzing by the Volterra series is that the difficulty for finding a resolution at passing of the system from a regime with "weak" nonlinearity to a regime with "strong" nonlinearity [3], [4].

There are some symptoms, which could help us to definite that the initial conditions are out of the admissible borders. For example with increasing of the signal, the intermodulation products do not get into a regime of saturation, but products from transforming of the basic frequencies are weakly saturatable [4].

II. THE RELATIONSHIPS BETWEEN COMPOSITE DISTORTIONS AND SYSTEM PARAMETERS

Below are brought out the relationships that concerns CATV system, which works on a "weakly" nonliner regime, and the polynomial with which the system is definite is from third order. The output signal of the system is from that type:

$$y(t)=\sum_{n=1}^{3}\int_{-\infty}^{+\infty}\ldots\int_{-\infty}^{+\infty}h_n(\tau_1,\ldots,\tau_n)\prod_{i=1}^{n}x(t-\tau_i)d\tau_i\ldots d\tau_n\ , \qquad (1)$$

where $h_n(\tau_i)$ is the Volterra kernel, n = 1, 2, 3 and $\tau \le t$.
Let's apply, in the accepted model, the approximation of third order when the input signal is

$$x(t)=\sum_{i=1}^{N}A_i\cos(\omega_i t+\theta_i);\ \omega_i=2\pi f_i\,. \qquad (2)$$

Let's replace with (2) in (1) and do the definite mathematical operation: presenting of $cos\omega_i t$ like exponential function, applying the direct Fourier's' transformation, etc.

In this case in the output of the system we receive

$$y(t)=\sum_{i=1}^{N}A_i\left|H_1(\omega_i)\right|cos(\omega_i t+\theta_i)$$

$$+\tfrac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}A_iA_j\left|H_2(\omega_i\pm\omega_j)\right|cos\left[(\omega_i\pm\omega_j)t+\theta_2\right]$$

$$+\tfrac{1}{4}\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N}A_iA_jA_k\left|H_3(\omega_i\pm\omega_j\pm\omega_k)\right|\cos\left[(\omega_i\pm\omega_j\pm\omega_k)t+\theta_3\right] \qquad (3)$$

We can see that the Volterra kernels $|H_n(\omega)|$ and the phase corner $\theta_n(\omega)$ are frequency dependant. The valuation of the intermodulation distortions (IMD) in CATV systems is reduced on the base of definite attitudes for the composite distortions from second and third order. For the gain G of advanced amplifiers can be accepted that it is constant in the entire frequency band [5].

A. The estimate of the composite distortions from second order

For estimate of IMD from second order is determinate the ratio of double beat *CSO* (*composite second order*), that is definite as attitude between the level of the basic signal and that, got as a result of totals and differences beating from second order (expressed in dB), i.e.

$$CSO = 20\lg \frac{A|H_1(\omega_b)|}{A^2|H_2(\omega_b)|} \qquad (4)$$

where we assume that:

- It is chosen a test channel with ω_r, $r = 1, 2, 3... N$;
- All levels are even $A_i = A_j = A_k = A$;
- It is accepted a frequency beating $\omega_b = \omega_i \pm \omega_j$, $\omega_b = \omega_i \pm \omega_k$, $\omega_b = \omega_j \pm \omega_k$, while $i \neq j \neq k$ and $i,j,k = 1,2,3\ldots$;
- Volterra kernels $|H_2(\omega_b)|$ are middle value, because of frequency dependence of the device inertia;
- The gain $G = |H_1(\omega_b)|$ is constant for whole spectrum of CATV system.

Then it could be written

$$CSO = 20\lg \frac{G}{|H_2(\omega_b)|} - 20\lg A. \qquad (5)$$

When is known the output level of the system (the amplifier) A_{out} and the gain G, as it is concerned that

$$A_{out} = GA, \qquad (5a)$$

from expression (5) at a normal Volterra kernel

$$CSO = 40\lg G - 20\lg|H_2(\omega_b)| - 20\lg A_{out}. \qquad (6)$$

The first addend in the right part of expression (5) represents reciprocal value of the normalized Volterra kernel from second order or

$$H_{2G} = \frac{|H_2(\omega_b)|}{G}. \qquad (7)$$

In association with (5) and (6) for CSO, expressed by the normalized Volterra kernel can be written

$$CSO = -20\lg H_{2G} - 20\lg A. \qquad (8)$$

When A_{out} and G for CSO are known, expressed with the normalized kernel, it is obtaining

$$CSO = 20\lg G - 20\lg H_{2G} - 20\lg A_{out}. \qquad (9)$$

For system with M numbers of cascade related identical amplifiers

$$CSO_{(M)} = CSO_{(1)} - 10\lg M. \qquad (10)$$

B. The estimate of the composite distortions from third order

At similar way it is possible to be definite the ratio of beating from third order CTB (*composite triple beat*), like an attitude between the level of a basic signal and that, got as a result of total and difference beats from third order (expressed in dB), i.e.

$$CTB = 20\lg \frac{A|H_1(\omega_b)|}{\frac{3}{4}A^3|H_3(\omega_b)|}, \qquad (11)$$

where we assume that, different from this presented in definition of CSO:

- Beating is from type $\omega_b = 2\omega_i \pm \omega_j$, $\omega_b = 2\omega_i \pm \omega_k$, $\omega_b = 2\omega_j \pm \omega_k$;
- Volterra kernels $|H_3(\omega_b)|$ are middle value, because of frequency dependence of the inertial devices.

Then it could be written

$$CTB = 20\lg\frac{4}{3} + 20\lg \frac{G}{|H_3(\omega_b)|} - 40\lg A. \qquad (12)$$

When is known the output level of the system (the amplifier) A_{out} and the coefficient of delivering G, and using (5a), for CSO at two-component beat is validity the following formula

$$CTB = 60\lg G - 20\lg|H_3(\omega_b)| - 40\lg A_{out} + 2{,}5. \qquad (13)$$

The second addend in the right part of expression (12) represents reciprocal value of the normalized Volterra kernel from third order or

$$H_{3G} = \frac{|H_3(\omega_b)|}{G}. \qquad (14)$$

In association with (12) and (14) for CTB it gets the following appearance:

$$CTB = -20\lg H_{3G} - 40\lg A + 2{,}5. \qquad (15)$$

For familiar A_{out} and G for CTB, expressed with the normalized kernel we note down:

$$CTB = 40\lg G - 20\lg H_{3G} - 40\lg A_{out} + 2{,}5\,. \qquad (16)$$

When the beating from third order is from the following type $\omega_b = \omega_i \pm \omega_j \pm \omega_k$, expression (12) takes the following look:

$$CTB = 20\lg\frac{2}{3} + 20\lg G - 20\lg\left|H_3(\omega_b)\right| - 40\lg A\,, \qquad (17)$$

and expression (13) looks like that:

$$CTB = 60\lg G - 20\lg\left|H_3(\omega_b)\right| - 40\lg A_{out} - 3{,}52\,. \qquad (18)$$

The composite distortion from third order for three-component beat, expressed with the normalized kernel, has the following look:

$$CTB = -\,20\lg H_{3G} - 40\lg A - 3{,}52 \qquad (19)$$

$$CTB = 40\lg G - 20\lg H_{3G} - 40\lg A_{out} - 3{,}52\,. \qquad (20)$$

For system with M numbers of cascade related identical amplifiers

$$CTB_{(M)} = CTB_{(1)} - 20\lg M\,. \qquad (21)$$

III. CALCULATING OF THE COMPOSITE INTERMODULATION DISTORTIONS

Here we will show the application of the worked out higher up analytic expressions for determination of the composite distortions for a real cable amplifier VX97B/G. It's a product of the German WISI company and it has been used in the trunk lines. His working frequency outflanking is from 47 to 850 MHz, gain G = 35 dB, and maximal output level A_{out} = 121 dBμV [6] (for two programs). Volterra kernels are determinate according to the European standard CENELEC EN 50083 (Appendix C) and the method is proposed in [7]. The results for the Volterra kernels from second and third order (for two- and three-component beat) are presented in Table 1.

From expressions (6), (13), (18) and the technical data for trunk amplifiers (output level and gain) we took out the simplified formulas, with witch we will calculate the composite distortions in dependence from the working frequency outflanking of the cable distribution network (47 to 520 MHz).

1) Composite distortions from second order

$$CSO = -\,51 - 20\lg\left|H_2(\omega_b)\right| \qquad (22)$$

2) Composite distortions from third order (three-component beat)

$$CTB^{(3)} = -140{,}52 - 20\lg\left|H_3^{(3)}(\omega_b)\right| \qquad (23)$$

TABLE 1

Volterra kernels / f MHz	Second order	Third order	
		$f_i \pm f_j \pm f_k$	$2f_i \pm f_j$
	H_2	$H_3^{(3)}$	$H_3^{(2)}$
49,75	2,04E-06	9,00E-11	1,80E-10
119,25	2,12E-06	8,69E-11	1,74E-10
175,25	2,06E-06	8,10E-11	1,62E-10
191,25	2,00E-06	8,70E-11	1,74E-10
207,25	2,62E-06	8,90E-11	1,78E-10
223,25	2,31E-06	9,06E-11	1,81E-10
231,25	1,48E-06	8,30E-11	1,66E-10
247,25	2,31E-06	8,63E-11	1,73E-10
263,25	2,21E-06	8,75E-11	1,75E-10
287,25	2,04E-06	7,99E-11	1,60E-10
311.25	2,31E-06	8,63E-11	1,73E-10
327.25	2,06E-06	8,81E-11	1,76E-10
343.25	2,02E-06	7,00E-11	1,40E-10
359.25	1,92E-06	7,43E-11	1,49E-10
375.25	2,01E-06	6,71E-11	1,34E-10
391.25	2,31E-06	6,51E-11	1,30E-10
407.25	2,21E-06	7,90E-11	1,58E-10
479.25	1,92E-06	7,43E-11	1,49E-10
495.25	1,68E-06	7,71E-11	1,54E-10
511.25	1,62E-06	7,81E-11	1,56E-10

3) Composite distortions from third order (two-component beat)

$$CTB^{(2)} = -134{,}5 - 20\lg\left|H_3^{(2)}(\omega_b)\right| \qquad (24)$$

Numbered results are given in Table 2, and the graphic presentation is shown on fig.2.

On fig.3 is presented the nomogram for determination of CSO and CATV for f = 175.25 MHz, and A_{out} is changing from 90 dBμV to 120 dBμV. G gets three values: 15 dB, 25 dB and 35 dB. The results are given after replacement with the respective values in expressions (6) and (13), while the simplified formulas (25), (26) are graphical interpreted by MAPLE 6.

$$CSO = 2*G - A_{out} + 113{,}72\,; \qquad (25)$$

$$CTB = 3*G - 2*A_{out} + 198{,}31\,. \qquad (26)$$

IV. CONCLUSION

The presented theoretical results for the composite intermodulation distortions, successfully can be applied and in the practice. That was demonstrated for one concrete trunk amplifier, but they can be also used for other – subtrunk and house amplifiers.

The presented values in Table 2 and on fig.2 with big accuracy corresponds with the pointed out in [6], witch confirms the obtained results from the theoretically drawing formulas.

TABLE 2

f *MHz*	Intermodulation composite distortions		
	CSO *dB*	*CTB*(3) *dB*	*CTB*(2) *dB*
49,75	62,81	60,40	60,41
119,25	62,47	60,70	60,72
175,25	62,72	61,31	61,30
191,25	62,98	60,69	60,67
207,25	60,63	60,49	60,42
223,25	61,73	60,34	60,30
231,25	65,59	61,10	61,13
247,25	61,73	60,76	60,74
263,25	62,11	60,64	60,60
287,25	62,81	61,43	61,48
311.25	61,73	60,76	60,78
327.25	62,72	60,58	60,60
343.25	62,89	62,58	62,60
359.25	63,33	62,06	62,10
375.25	62,94	62,94	62,91
391.25	61,73	63,21	63,26
407.25	62,11	61,53	61,57
479.25	63,33	62,06	62,10
495.25	64,49	61,74	61,77
511.25	64,81	61,63	61,68

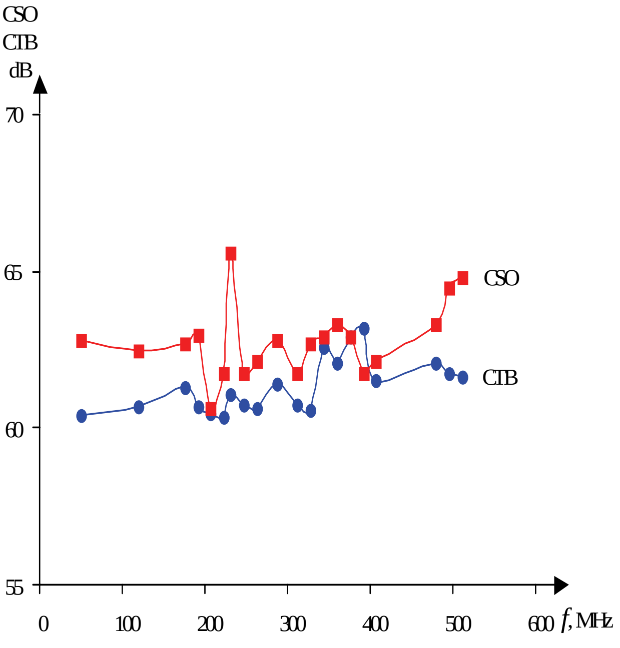

Fig.2. Graphic presentation of the composite distortions from the working frequencies according to Appendix C

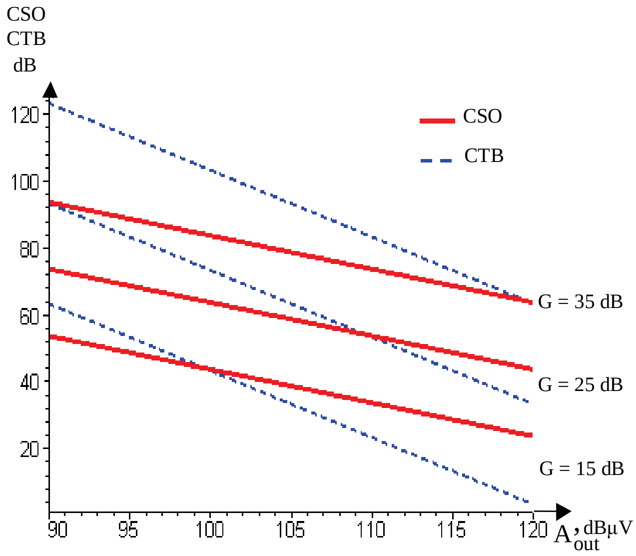

Fig.3. The nomogram for determination of CSO and CATV in dependence from the output/input level and gain on the trunk amplifier VX97B/G

From the nomogram it can be reported values for the composite distortions at different values for the gain and output level, respective for the input level. Let's remind that by exploitation of the CATV systems, the trunk amplifiers work with definite gain. For some reasons, if it gets bigger level on the input, the level in the output also will grow up, while CSO and CTB will change, too. These changes can be reviewed from the nomogram on fig.3 in case when there are three values for G, but they could be multiplicated in dependence of needs of the concrete cable operator.

REFERENCES

[1] J. Steel, A. Parker, D. Skellern, "Characterization of cable amplifiers for broadband network applications", Electronics Department, Macquarie University, NSW 2109, Australia, pp. 39-45, 2001.
[2] W. Ciciora, *Cable Television in the United States - An Overview*. Cable TV Labs. Inc., Louisville, 1999.
[3] D. Gingold, "Integrated digital services for cable networks. Academic Dissertation", MIT, Sept. 1996.
[4] S.A. Maas, "Analysis and optimization of nonlinear microwave circuits by Volterra series". Microwave J., vol.33, No.4, pp. 245, April l990.
[5] Ultra Low Noise and Distortion High-Speed Amplifier for 16-Bit Systems. www.analog.com, pp. 1-12, Dec. 2001.
[6] WISI. "A link to the future". Catalogue'2001.
[7] O. Panagiev, "Analysis and reduction on nonlinear distortions for signals in the broadband cable communication systems". Dissertation, Technical university of Sofia, 2006.
[8] European standard CENELEC "EN 50083", 1998.