Adaptive Notch Filters for Image Processing

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Abstract – **In this paper we develop a very simple 2-D adaptive notch filter design. Second-order all-pass circuits are used to implement the structure of our filter. They guarantee the stability and ensure low sensitivity of the developed structure. Computationally efficient adaptive algorithm is employed for the adjustment of filter coefficients controlling the notch frequency. As a result our filter has lower computational complexity compared to the existing approaches. We study the error surface of our adaptive filter and show that there is only one global minimum which guarantees the convergence. The designed 2-D adaptive notch filter can be successfully applied for image denoising when on-line processing is essential.**

Keywords – **Adaptive filters, Image processing, Adaptive algorithms, Digital filters, Noise suppression.**

I. Introduction

Adaptive notch filtering is a useful tool for narrowband interference suppression which has been used intensively for many years in different signal processing applications [1]. Examples can be easily found in the field of telecommunications, radar and sonar signals, adaptive control and sensor systems. During the years a lot of methods for design of one-dimensional adaptive notch filters have been proposed [2-5]. As a rule they employ simplified structures which allow low computational complexity and application of efficient adaptive algorithms [6, 7]. By contrast, two-dimensional (2-D) adaptive notch filter design, which is crucial for some image processing applications, is still an immature area deserving much more research efforts and development. Although several approaches have been proposed [8, 9] their computational complexity is relatively high, which is a result of the too complicated algorithms used for filter parameter adjustment.

In this paper we develop an approach based on very simple second-order all-pass structures. First we show how one-dimensional adaptive notch filters can be designed. Then we construct 2-D adaptive notch filters for image denoising. Finally, some experiments are conducted to evaluate the performance of our method.

II. All-pass Based Adaptive Notch Filters

A realization based on second-order Gray-Markel lattice circuit is used [10] – Fig. 1. Using that circuit it becomes possible to implement a second-order adaptive notch filter illustrated in Fig. 2.

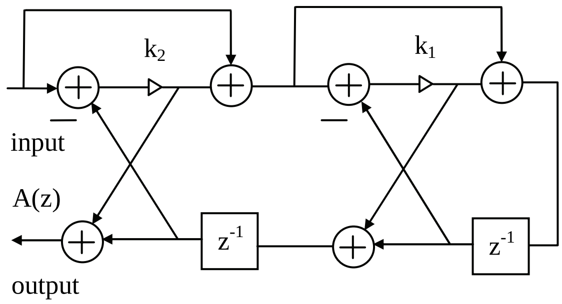

Fig. 1. Second-order lattice Gray-Markel circuit realizing all-pass function A(z)

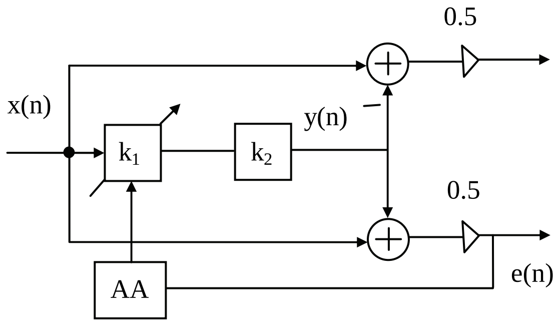

Fig. 2. Second-order adaptive notch filter

This implementation has two very important advantages: first extremely low passband sensitivity that means resistance to quantization effects, second independent control of central frequency and filter bandwidth.

Thus if the all-pass function A(z) is

$$A(z) = \frac{k_2 + k_1(1+k_2)z^{-1} + z^{-2}}{1 + k_1(1+k_2)z^{-1} + k_2 z^{-2}} \qquad (1)$$

then k_1 controls the central frequency ω_0 while k_2 is related to the bandwidth (BW) via

$$k_1 = -\cos\omega_0 \qquad (2)$$

$$k_2 = \frac{1-\tan(BW/2)}{1+\tan(BW/2)}\,. \qquad (3)$$

BW is directly connected to the distance from the pole to the unity-circle and transforming the structure in Fig.1 into an adaptive filter, it is possible to fix the bandwidth and implement an adaptive IIR filter free of stability problems.

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Adapting k_1 the central frequency can be shifted around the unity-circle.

For the adjustment of filter coefficient a Least Mean Squares (LMS) algorithm is applied as follows

$$k_1(n+1) = k_1(n) - \mu e(n)[x(n-1) - y(n-1)] \qquad (4)$$

where e(n) is the error signal and μ is the step size controlling the convergence speed.

In order to ensure the stability of the adaptive algorithm (AA) the range of the step size μ should be set according to

$$0 < \mu < \frac{K}{L\sigma^2}. \qquad (5)$$

In this case L is the filter order, σ^2 is the power of the signal [x(n-1) – y(n-1)] and K is a constant depending on the statistical characteristics of the input signal. In most of the practical situations K is approximately equal to 0.1 [11].

III. Error Surface Analysis

We start the examination of error surface considering the characteristics of the developed fixed notch filter, at this stage neglecting the adaptation. In Fig. 3 magnitude and phase response is shown while Fig. 4 presents pole-zero plot. The figures show the characteristics of a narrowband filter with notch frequency at 0.4 π where the phase response undertakes a 180° shift. That frequency is determined by coefficient k_1 (Eq.2). The second coefficient namely k_2 controls the position of the poles (see Fig.4) and thus the filter bandwidth (Eq.3). The notch frequency is not related to k_2 and on the other hand filter bandwidth has no relation to k_1. That provides the possibility for absolutely independent adjustment of filter parameters which is a very important feature when considering the design of adaptive notch filters.

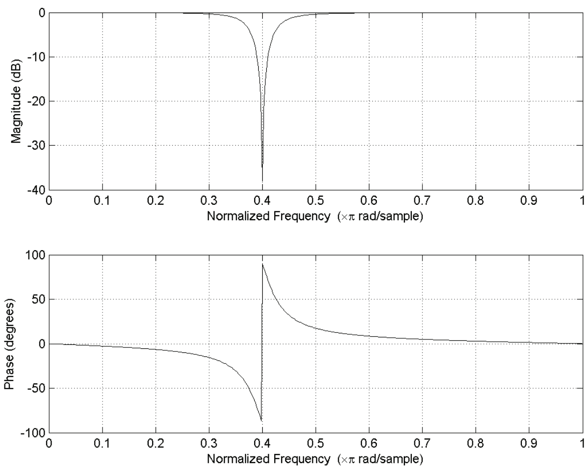

Fig. 3. Magnitude and phase response

Next we examine the characteristics of the cost function. Here we use the mean squared error (MSE)

$$MSE = E[e(n)^2] \qquad (6)$$

as measure for the performance of our filter. Fig. 5 plots MSE as function of k_1. Several cases are presented for different values of k_2. Examining the error surface we can come to two conclusions. First there is only one global minimum and it is possible to reach it using the simple LMS algorithm. Second making the BW narrower (in Fig. 5 the narrowest BW is for k_2= 0.954) we smooth the error surface and that results in a slower adaptation rate.

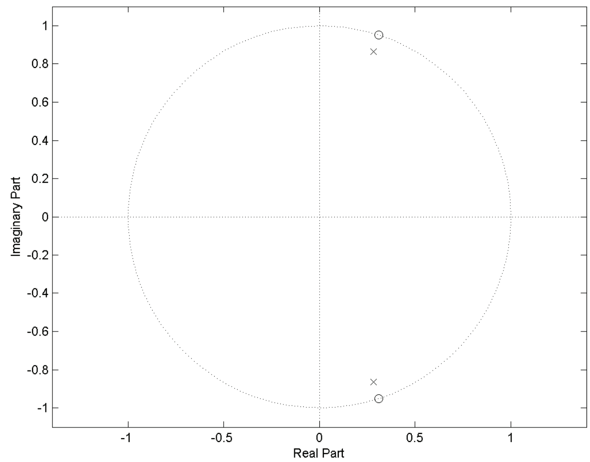

Fig. 4. Pole-zero plot

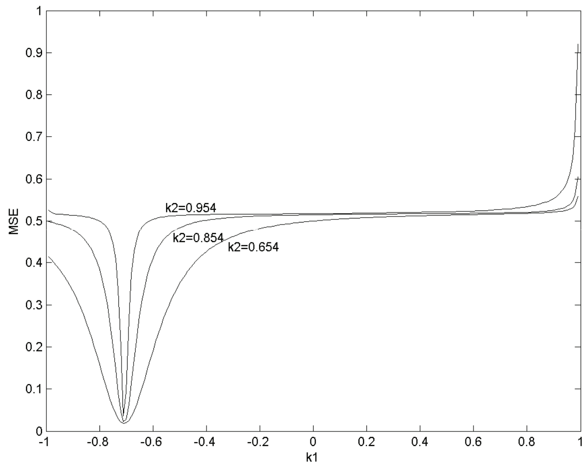

Fig. 5. Cost function

IV. Image Denoising

In Fig. 6 we present the design of our 2-D adaptive notch filter used for image denoising. Two all-pass functions defined via

$$A_1(z_1) = \frac{k_{21} + k_{11}(1+k_{21})z_1^{-1} + z_1^{-2}}{1 + k_{11}(1+k_{21})z_1^{-1} + k_{21}z_1^{-2}} \qquad (7)$$

$$A_2(z_2) = \frac{k_{22} + k_{12}(1+k_{22})z_2^{-1} + z_2^{-2}}{1 + k_{12}(1+k_{22})z_2^{-1} + k_{22}z_2^{-2}} \qquad (8)$$

are employed. Here z_1 and z_2 represent the delay in the two directions of an image (row-wise and column-wise). Cascading these all-pass structures we form the transfer function of a 2-D notch filter. The adaptation is implemented by using two error signals $e_1(m,n)$ and $e_2(m,n)$. Fig. 7 plots the error surface of MSE for our 2-D adaptive notch filter. Two minima are presented which correspond to the notch frequency determined by the coefficients k_{11} and k_{12}. As with the one-dimensional case BW is controlled by k_{21} and k_{22}. We apply LMS algorithm to find the optimal solution.

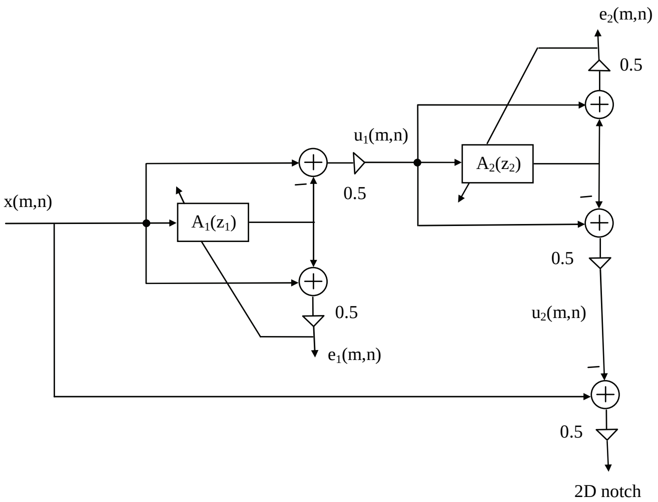

Fig. 6. 2-D adaptive notch filter

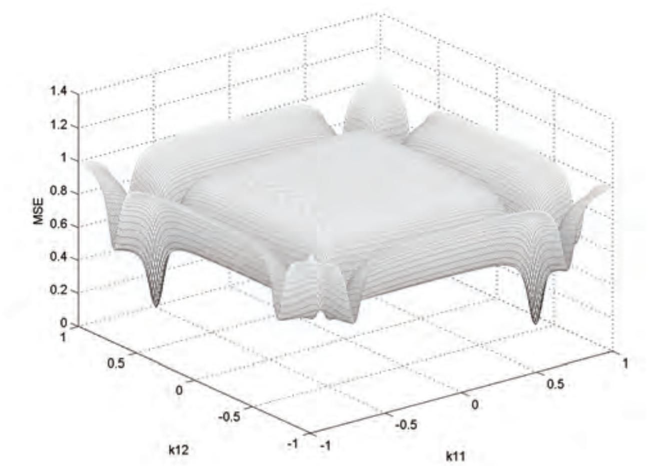

Fig. 7. Error surface

V. EXPERIMENTS

To evaluate the performance of our design we set up an experiment where a 2-D periodical interference in the form of

$$s(m,n) = 0.2\sin(0.3\pi m + 0.4\pi n) \quad (9)$$

is presented. In practice this kind of disturbances are often due to the power supply systems or cross-interference in image transmission channels. Fig. 8 presents the original 256 x 256 image. In Fig. 9 the noisy image is shown. Finally, Fig. 10 displays the image after noise suppression. It can be seen that the interference is removed and the original image is restored. To study the behavior of our adaptive filter we draw the learning curves of coefficients k_{11} and k_{12} (Fig. 11). Obviously the adaptive process converges to the optimal solution. In addition the filter exhibits very high adaptation rate (about 1000 iterations).

Fig. 8. Original image

Fig. 9. Noisy image

Fig. 10. Image after filtration

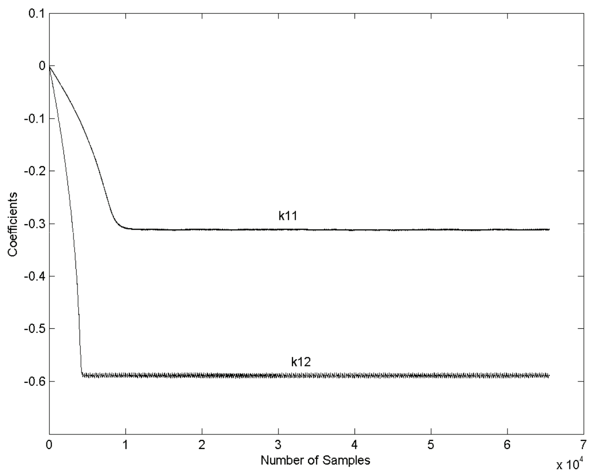

Fig. 11. Learning curves

VI. Conclusions

In this paper we develop a very simple 2-D adaptive notch filter design. Based on all-pass structures and employing computationally efficient adaptive algorithm the presented method can be successfully applied for image denoising when on-line processing is essential. The main advantages of our approach can be summarized as follows:

- low computational complexity – we adjust only two coefficients and use the simple LMS algorithm;
- stability is easily monitored for the employed second-order all-pass structures;
- error surface has global minimum which guarantees the convergence.

Using the developed method it is possible to construct more complex filter structures in cascade or parallel form for multiple 2-D periodical interference removal which is a challenging topic for further studies in the field of 2-D adaptive notch filter design.

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