An Algorithm for Accelerated Template Localization in an Image Using Correlation Coefficient

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***Abstract* – An algorthm for accelerated template localization using computation of the correlation coefficient is proposed in the paper. Correlation coefficient is one of the most reliable matching measures used in the tasks of searching and localizing a before chosen template in a larger image. Computations are made in the space area and they are made with a set of before extracted representative points of the template. The computational complexity of the proposed algorithm is kept the same as this of the classic algorithm. The acceleration is due to the reduced number of operations. This is achieved by the "nonpromising" positions elimination. Comparisons with two elimination conditions are used for this purpose. The elimination conditions become more accurate during the calculation process. Besides it is proposed the sets of representative points to be divided into two subsets according to a before sattled feature and the points from one subset only to be involved in the calculations. The points from the second subset take part in the accurate calculation of the corerrelation coefficient in the "promising" positions only. The theoretical evaluations and the experimental results also acknowledge the priority of the proposed algorithm over the classical algorithms. Results are compared on one hand with the classic algorithm for computations over all points of the template and on the other hand it is compared with the algorithm for computations over the set of representative points.**

***Keywords* – template matching, template localization, correlation coefficient**

I. INTRODUCTION

Correlation coefficient overcomes the instability of the normalized cross-correlation to the local intensity changes. This is achieved with the including the mean intensity values of the template and the image in the computations.

Algorithms for template localization using correlation metrics, calculated in the space domain have great computational complexity – O(N.M), where N and M are sizes of the image and the template respectively. When these metrics are computed using a set of representative points, their computational complexity reduces to O(N.P), where P is a number of representative points. Despite all it remains high. This is the reason for searching faster algorithms [1], [4].

In this paper an algorithm for accelerated template localization in a greater image is proposed. It computes the correlation coefficient in the space domain. The acceleration is due to the fast dissimilarity principle and to the computations of the partial correlation coefficient.

II. SUGGESTION FOR ACCELERATED TEMPLATE LOCALIZATION WITH CORRELATION COEFFICIENT COMPUTATION

A. Description of the algorithm

It is known that correlation coefficient which is computed over the chosen set of representative points can be expressed as follows:

$$NCCE(x,y)=\frac{\sum_{q=1}^{P}\left(I_q-\bar{I}\right).\left(T_q-\bar{T}\right)}{\sqrt{\sum_{q=1}^{P}\left(I_q-\bar{I}\right)^2}.\sqrt{\sum_{q=1}^{P}\left(T_q-\bar{T}\right)^2}} \tag{1}$$

where: $\bar{T}=\sum_{q=1}^{P}T_q/p$ is the mean intensity value of the representative points of the template $q = 1, 2, \ldots, P$ and $\bar{I}=\sum_{q=1}^{P}I_q/P$ is the mean intensity value of the corresponding points of the image.

For a set of representative points

$$T=\{T_1^L(i_1,j_1),T_2^L(i_2,j_2),\ldots,T_p^L(i_p,j_p),T_{p+1}^H(i_{p+1},j_{p+1}),T_{p+2}^H(i_{p+2},j_{p+2}),\ldots,T_P^H(i_P,j_P)\}$$

the correlation coefficient at point *(x,y)* can be represented as:

$$NCCE(x,y)=\frac{\sum_{q=1}^{p}\left(I_q-\bar{I}\right).\left(T_q^L-\bar{T}\right)+\sum_{q=p+1}^{P}\left(I_q-\bar{I}\right).\left(T_q^H-\bar{T}\right)}{\sqrt{\sum_{q=1}^{p}\left(I_q-\bar{I}\right)^2+\sum_{q=p+1}^{P}\left(I_q-\bar{I}\right)^2}.\sqrt{\sum_{q=1}^{p}\left(T_q^L-\bar{T}\right)^2+\sum_{q=p+1}^{P}\left(T_q^H-\bar{T}\right)^2}} \tag{2}$$

If note

$$NCCE^L(x,y)=\frac{\sum_{q=1}^{p}\left(I_q-\bar{I}\right).\left(T_q^L-\bar{T}\right)}{\sqrt{\sum_{q=1}^{p}\left(I_q-\bar{I}\right)^2+\sum_{q=p+1}^{P}\left(I_q-\bar{I}\right)^2}.\sqrt{\sum_{q=1}^{p}\left(T_q^L-\bar{T}\right)^2+\sum_{q=p+1}^{P}\left(T_q^H-\bar{T}\right)^2}} \tag{3}$$

and

$$NCCE^H(x,y)=\frac{\sum_{q=p+1}^{P}\left(I_q-\bar{I}\right).\left(T_q^H-\bar{T}\right)}{\sqrt{\sum_{q=1}^{p}\left(I_q-\bar{I}\right)^2+\sum_{q=p+1}^{P}\left(I_q-\bar{I}\right)^2}.\sqrt{\sum_{q=1}^{p}\left(T_q^L-\bar{T}\right)^2+\sum_{q=p+1}^{P}\left(T_q^H-\bar{T}\right)^2}} \tag{4}$$

than for *NCCE(x,y)* can be written:

$$NCCE(x,y)=NCCE^L(x,y)+NCCE^H(x,y) \tag{5}$$

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In the ideal case when the compared template and the current window of image coincide, the intensities at every pair of points are equal, i.e. $I_q = T_q^L$ for $q = 1, 2, \dots, p$ and $I_q = T_q^H$ for $q = p+1, p+2, \dots, P$; p and $P \neq 0$. It means that average values are also equal. Than after the substitution in (2), (3) and (4) it can be written:

$$NCCE^{Id} = \frac{\sum_{q=1}^{p}\left(T_q^L - \bar{T}\right)^2}{\sum_{q=1}^{p}\left(T_q^L - \bar{T}\right)^2 + \sum_{q=p+1}^{P}\left(T_q^H - \bar{T}\right)^2} + \frac{\sum_{q=p+1}^{P}\left(T_q^H - \bar{T}\right)^2}{\sum_{q=1}^{p}\left(T_q^L - \bar{T}\right)^2 + \sum_{q=p+1}^{P}\left(T_q^H - \bar{T}\right)^2} \tag{6}$$

It can be written in another way also:

$$NCCE^{Id} = NCCE^{L,Id} + NCCE^{H,Id} \tag{7}$$

The ideal values of partial correlation coefficients $NCCE^{Id}$, $NCCE^{L,Id}$ and $NCCE^{H,Id}$ can be computed in advance and only once.

Let $\bar{I} = \bar{T}$. Than the correlation coefficient at point *(x,y)*, computed over the subset of points T^L can be approximated with:

$$NCCE^{L,Appr}(x,y) = \frac{\sum_{q=1}^{p}\left(I_q - \bar{T}\right).\left(T_q^L - \bar{T}\right)}{\sqrt{\sum_{q=1}^{p}\left(I_q - \bar{T}\right)^2 + \sum_{q=p+1}^{P}\left(T_q^H - \bar{T}\right)^2}.\sqrt{\sum_{q=1}^{p}\left(T_q^L - \bar{T}\right)^2 + \sum_{q=p+1}^{P}\left(T_q^H - \bar{T}\right)^2}} \tag{8}$$

$\bar{T}$, $\left(T_q^L - \bar{T}\right)$, $\sum_{q=1}^{p}\left(T_q^L - \bar{T}\right)^2$ и $\sum_{q=p+1}^{P}\left(T_q^H - \bar{T}\right)^2$ are already computed. Thus the first elimination condition can be formulated:

$$\left|NCCE^{L,Id} - NCCE^{L,Appr}(x,y)\right| > (\varepsilon_{NCCE^L})_{\min_1}, \tag{9}$$

where $(\varepsilon_{NCCE^L})_{\min_1}$ is the current computed minimum value of the deviation of $NCCE^{L,Appr}(x,y)$ from the ideal value $NCCE^{L,Id}$.

If this condition is satisfied, then the current position of the template over the image can be ignored as an "unpromising" without any additional computations and the next position can be checked.

If the elimination condition is not satisfied then the real value of the correlation coefficient $NCCE^L$ at point *(x,y)* must be computed:

$$NCCE^L(x,y) = \frac{\sum_{q=1}^{p}\left(I_q - \bar{I}\right).\left(T_q^L - \bar{T}\right)}{\sqrt{\sum_{q=1}^{p}\left(I_q - \bar{I}\right)^2 + \sum_{q=p+1}^{P}\left(I_q - \bar{I}\right)^2}.\sqrt{\sum_{q=1}^{p}\left(T_q^L - \bar{T}\right)^2 + \sum_{q=p+1}^{P}\left(T_q^H - \bar{T}\right)^2}} \tag{10}$$

Thus, the second condition for the elimination of the current position can be formulated:

$$\left|NCCE^{L,Id} - NCCE^L(x,y)\right| > (\varepsilon_{NCCE^L})_{\min_2} \tag{11}$$

In this condition $(\varepsilon_{NCCE^L})_{\min_2}$ is the current minimal difference between the ideal and the real correlation coefficient at point *(x,y)*. If this condition is satisfied, this means that the current position is not "promising", i. e. it can not correspond to the correlation maximum and thus the computations are interrupted and this position is skipped.

If the condition is not satisfied, than the real $NCCE^H$ at point *(x,y)* is computed using equation (4). For this purpose only the numerator of $NCCE^H$ has to be computed. The other parts of the equation (the denominator and the average intensity value of the image $\bar{I}$) are already computed.

The maximum value of the correlation coefficient determines the position in the image where the best coinciding between the template and the current window of the image.

The order of computations can be changed and the elimination conditions can be formed on the basis of subset T^H.

B. Sets of representative points

- Edge points

In [3] we propose a new edge definition, based on the interrupted first derivative of the intensity function. In practice at the points of interruption second derivative has local extreme values. The rule for determining if a point belongs to an edge (for one-dimensional case) is:

$$\text{If} \quad \left| \underset{i}{extr}\left(\frac{d^2 I(x)}{dx^2}\right) \right| > \theta, \quad \text{then} \quad i \in E, \tag{12}$$

where *I(x)* is the intensity function; *E* is a set of points *i*, which are edge points; θ is a before settled threshold.

Here we propose to divide such formed set *E* into two subsets, depending on the sign of the local extremum. We propose the following **definition:**

Definition 1: The set *E* consists of two subsets, which are formed according the following criteria:

$$\text{If} \quad \left(\underset{i}{extr}\left(\frac{d^2 I(x)}{dx^2}\right)\right) > 0, \quad \text{then} \quad i \in E_L \tag{13.1}$$

and

$$\text{If} \quad \left(\underset{i}{extr}\left(\frac{d^2 I(x)}{dx^2}\right)\right) < 0, \quad \text{then} \quad i \in E_H \tag{13.2}$$

The set of all edge points *E* is a sum of both subsets

$$E = E_L \cup E_H . \tag{14}$$

- Informative points, selected with the method of equipotential planes

In [2] we propose extracting the representative points to be conformed to the criterion of D-optimality. According to this criterion the most informative points lie on the protruded peripheral wrapping of the object. Thus, we propose the choice to be made by equipotential planes, which are parallel to the plane *xOy* and which cut the three-dimensional image profile (relief) on proper intensity levels. Thus, the extracted points outline the horizontal contours of the local "hollows" and "hills" from the three-dimensional image profile.

Here we propose to divide such formed sets into two subsets, depending on their belonging to local "hollow" or "hill", according to the following **definition:**

Definition 2: Set of points *EP* consists of two subsets, which are formed according to the following criteria:

$$\text{If}\begin{pmatrix}[(PI(i,j)\le Pt_{max})\cap(PI(i+1,j)>Pt_{max})]\cup\\ \cup[(PI(i,j)\ge Pt_{max})\cap(PI(i+1,j)<Pt_{max})]\end{pmatrix},\text{then}(i,j)\in EP_H \quad (15.1)$$

$$\text{If}\begin{pmatrix}[(PI(i,j)\le Pt_{min})\cap(PI(i+1,j)>Pt_{min})]\cup\\ \cup[(PI(i,j)\ge Pt_{min})\cap(PI(i+1,j)<Pt_{min})]\end{pmatrix},\text{then}(i,j)\in EP_L \quad (15.2)$$

where: $PI(i,j)$ is the intensity at the point (i,j); Pt_{min}, Pt_{max} are intensity values, which determine the distance between the *xOy* plane and the planes, cutting low and high parts of the relief ("hollows" and "hills"); EP_L, EP_H are sets of points, outlining "hollows" and "hills".

The set of all informative points *EP* is an union of both subsets:

$$EP = EP_L \cup EP_H \quad (16)$$

C. Advantages of the proposed algorithm

The proposed algorithm is based on the "fast dissimilarity principle". It means that less data are needed to make a conclusion that two object (in our case images) are different than they are equivalent.

When the template and the method for representative point extraction are properly chosen, than the representative points can be situated over the hall image but not only over the bounded part of it.

During the process of computations the elimination conditions are made more precise.

III. EVALUATION OF THE PROPOSED ALGORITHM

A. Analytical evaluation

The computational complexity of the proposed algorithm is equal to the complexity of the classical algorithm for correlation coefficient computation and it is *O(N.P).* The acceleration is due to the reducing the number of operations, because of the "unpromising" positions elimination. The power of acceleration is greatly dependent on the data and has not an absolute evaluation.

According to Nakov P. and P. Dobrikov [5] "… elementary operation is this one, which is executed for a constant time, independently on the size of data. Elementary operations in common case are for example addition, multiplication etc.".

In the algorithms for correlation coefficient computation the most numerous operations are addition and multiplication (of real numbers) and their number is approximately the same. The number of more complicated operations like division and square root is significantly less than the number of the rest operations. This is the reason they are accepted as equivalent with the rest.

Thus the number of elementary operations in the classic algorithm for correlation coefficient computation over the all points of the template is:

$$NEO(NCCE)_{Cl}^{All} = 6.N.M + 4.N \quad (17)$$

where $N = n_1$ x n_2 is the size of the image and $M = m_1$ x m_2 is the size of the template.

The number of elementary operations in the classic algorithm for correlation coefficient computation over the set of representative points is:

$$NEO(NCCE)_{Cl}^{P} = 6.N.P + 4.N \quad (18)$$

where P is the number of representative points of the template.

The number of elementary operations in the proposed accelerated algorithm is:

$$NEO(NCCE)_{Accs} = N.(5p+5) + S_1[4.P + 2.p + 5] + S_2[3.(P-p)+2] \quad (19)$$

where S_1 is the number of points, which remain after the first elimination condition and S_2 is the number of points, which remain after the second elimination condition.

The relative acceleration can be computed using Eq. (20):

$$\frac{NEO(NCCE)_{Cl}}{NEO(NCCE)_{Accs}}. \quad (20)$$

Here $NEO(NCCE)_{Cl}$ is $NEO(NCCE)_{Cl}^{All}$ or $NEO(NCCE)_{Cl}^{P}$.

In Fig. 1 the acceleration (in times) is shown. The size of the image is 128 x 128 pixels; the size of the template is 64 x 64 pixels. Set of representative points consists of 200 points, and it is divided on two subsets, every one of them contains 100 points. It is accepted that 1.25% to 0.25% of all possible positions remain as "promising" after the first elimination condition.

It is seen that the acceleration in comparison with the classic algorithm over all points is significant – over 47 times. The acceleration of the proposed algorithm is not so great in comparison with classic algorithm of consecutive comparison over the set of representative points, but it is over 2.3 times.

B. Evaluation of the experimental results

Series of experiments have been carried out. Parameters of experiments are: size of the image - 128 x 128 pixels, size of the template - 64 x 64 pixels, the set of representative points contains 200 points and the admissible deviation from the right position - $\varepsilon = \pm 1$ pixel. Table 1 contains results of 200 experiments.

The received results are strongly dependent on data. That is the reason that in the table there are shown average, minimal and maximal values for any of parameters.

It can be concluded, that the elimination conditions are very strong – over 90% of all possible position drop out during the first elimination. During the second elimination new about 48% of the rest positions drop out. Thus the full correlation coefficient is computed for maximum 0.56% of all possible

positions. The typical number is about 0.27% of all possible position of the template over the image.

The acceleration of the computations is also significant in comparison with the classical algorithm over all points of the template as well as in comparison with the classical algorithm, which computes correlation coefficient over the set of representative points.

TABLE I
EXPERIMENTAL RESULTS FOR NCCE, COMPUTED OVER DIFFERENT SETS OF REPRESENTATIVE POINTS

Parameter	**Representative points** (Basic subset of 100 points)		
Part of eliminated positions	After the first elimination condition	Minimum number	98,93%
		Maximum number	99,81%
		Average number	99,44%
	After the second elimination condition (from the remaining positions)	Minimum number	0%
		Maximum number	84,85%
		Average number	48,67%
Accele-ration (Reducing the operations) in times	In comparison with the classic algorithm over all points	Minimum	48,09
		Maximum	57,66
		Average	50,65
	In comparison with the classic algorithm over set of points	Minimum	2,36
		Maximum	2,82
		Average	2,48

The reliability of the proposed algorithm (Table 2) is evaluated regarding the possibility of skipping the real position of the searched template. Series of 2500 experiments have been made. Parameters of experiments are: size of the image - 128 x 128 pixels, size of the template - 64 x 64 pixels, the set of representative points contains 200 points and the admissible deviation from the right position - $\varepsilon = \pm 1$ pixel, levels of Gausian noise 10% and 20% and different numbers of representative points.

The received results (which are shown in Table 2) allow concluding that:

- The proposed algorithm for accelerated template localization using correlation coefficient as a measure of similarity does not decrease significantly the reliability of the measure in comparison with the classical algorithms in the conditions of the examined levels of noise and numbers of representative points.
- There are not any unsuccessful localizations in any of series.
- In contrast to other known algorithms using "fast dissimilarity principle", like SSDA algorithm of Castillo [1] and PAIRS algorithm of Lundberg [4], which use comparisons between arbitrarily chosen points, the here proposed algorithm uses points which represent the images in the best way, according to criteria, which are used for their extraction.

TABLE II
EXPERIMENTAL RESULTS – PARAMETER "EXACTNESS OF LOCALIZATION"

Algorithm	**10% noise**				**20% noise**			
	Points	**Successive localizations [%]**	**σX*10^{-7}**	**σY*10^{-7}**	**Points**	**Successive localizations [%]**	**σX*10^{-7}**	**σY*10^{-7}**
Accele-rated	60	100	0	5,06	100	100	0	8,31
Classic		100	0	3,23		100	0	7,67
Accele-rated	80	100	0	3,92	125	100	0	2,26
Classic		100	0	2,98		100	0	1,84
Accele-rated	100	100	0	1,60	150	100	0	3,57
Classic		100	0	0,61		100	0	0,52

IV. CONCLUSION

It must be pointed out that the evaluations of the relative acceleration are not one to one connected with the evaluations of the run-time. It is due the fact that the different elementary operations have different times for their execution.

To conclude it can be generalized that the proposed algorithm shows good enough results about the examined parameters, to be used in practice for solving different task of the area of computer vision and in particular – for solving the task for template localization.

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