# Ordered Complex Hadamard Transform of Images 

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#### Abstract

A method for new ordered Complex Hadamard Transform of halftone images is presented. The "basis" ordered images for 2D CHT are calculated and distribution of spectral coefficients is obtained and evaluation of coefficients distribution in complex spectrum space for test images is made. Keywords - digital signal processing, orthogonal transforms, complex Hadamard transform, image processing.


## I. Introduction

The discrete Walsh Hadamard Transform (WHT) is a fairly simple orthogonal transform [1] and has found applications in data compression, spectral analysis, pattern recognition and watermarking involving image transmission, storage and security. The idea of using complex, rather than integer transforms matrices for spectral processing and analysis has been shown in [2], [3] and [4]. From the Complex Hadamard Transform (CHT), several complex decisions diagrams are derived.

In this paper arranging of spectral coefficients of general Complex Hadamard matrix are investigated, which provide better concentration of energy in low-frequency area of 2D image spectrum. The 'basis' ordered images for 2D CHT are obtained by using a developed MATLAB program simulation. The similar results with well-known Walsh Hadamard Transform are received, what shows that CHT can be used by the same way in more complicated analysis and processing [5], [6], [7].

## II. Mathematical Description

The coefficients of Complex Hadamard Transform matrix $\left[\mathrm{CH}_{N}\right]$ with dimension $N$ by $N$ can be represented by the following equations [4]:

$$
\left\lvert\, \begin{align*}
& c(u, v)=j^{u v} s(u, v)  \tag{1}\\
& c^{*}(u, v)=(-j)^{u v} s(u, v)
\end{align*}\right.,
$$

where: $N=2^{n}, j=\sqrt{-1}, \mathrm{u}, \mathrm{v}=0,1, \ldots 2^{\mathrm{n}}-1$ and

$$
s(u, v)= \begin{cases}1 & \text { for } n=2  \tag{2}\\ & \sum_{(-1)^{r=3}}^{n}\left\lfloor u / 2^{r-1}\right\rfloor\left\lfloor_{v / 2^{r-1}}\right\rfloor \\ \text { for } n=3,4,5 \ldots . .\end{cases}
$$

[^0]is the sign function. Here $\lfloor$.$\rfloor is an operator, which represents$ the integer part of the result, obtained after the division.

The complex Hadamard matrix can be presented in logarithmic form [6], [7]:

$$
\begin{equation*}
\left[C H_{N}\right]=e^{j\left[\Phi_{N}\right]} \tag{3}
\end{equation*}
$$

where $\left[\Phi_{N}\right]$ is a phase matrix of CHT.
From the equation (3) the CHT basis matrix of order $2^{n}$ can be calculated for $n=2$ and $u, v=1,2,3,4$ by the following way:

$$
\left[\mathrm{CH}_{4}\right]=\left[\begin{array}{cccc}
1 & 1 & 1 & 1  \tag{4}\\
1 & j & -1 & -j \\
1 & -1 & 1 & -1 \\
1 & -j & -1 & j
\end{array}\right] \rightarrow 0 \rightarrow 3, \begin{aligned}
& \rightarrow 2
\end{aligned},\left[\Phi_{4}\right]=\frac{\pi}{2}\left[\begin{array}{cccc}
0 & 0 & 0 & 0 \\
0 & 1 & 2 & 3 \\
0 & 2 & 0 & 2 \\
0 & 3 & 2 & 1
\end{array}\right]
$$

Each number in the right column toward the CHT matrix $\left[\mathrm{CH}_{4}\right]$ is obtained as a sum of sign changes between the elements of each row. As a sample for $3^{\text {rd }}$ row of the matrix is implemented:

$$
\sum_{i=1}^{3} \operatorname{sign}_{(i)}=1+1+1=3
$$

The ordered complex Hadamard matrix can be received as rearrange of row numbers in increasing order. In result the ordered CHT matrix of order 4 and the corresponding phase matrices are:
$\left[\mathrm{CH}_{4}^{\prime}\right]=\left[\begin{array}{cccc}1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1\end{array}\right] \rightarrow 0 \rightarrow 2,\left[\Phi_{4}^{\prime}\right]=\frac{\pi}{2}\left[\begin{array}{llll}0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 \\ 0 & 3 & 2 & 1 \\ 0 & 2 & 0 & 2\end{array}\right]$.
This approach for rearranging of CHT matrix can be summarized for order $N>4$.

As a sample, the basis ordered Complex Hadamard matrix of order 8 is:

$$
\left[C H_{8}^{\prime}\right]=\left[\begin{array}{cccccccc}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\
1 & j & -1 & -j & -1 & -j & 1 & j \\
1 & j & -1 & -j & 1 & j & -1 & -j \\
1 & -j & -1 & j & 1 & -j & -1 & j \\
1 & -j & -1 & j & -1 & j & 1 & -j \\
1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1
\end{array}\right] \rightarrow 0 \rightarrow 4 . \begin{aligned}
& \rightarrow 2 \\
& \rightarrow 7 \\
& \rightarrow 7 \\
& \rightarrow 7
\end{aligned} .
$$

In conclusion, the forward and inverse ordered CHT for two-dimensional signals (images) can be generalized in matrix form as follows:

$$
\left\lvert\, \begin{align*}
& {[Y]=\left[C H_{N}^{\prime}\right][X]\left[C H_{N}^{\prime}\right]} \\
& {[X]=\frac{1}{N^{2}}\left[C H_{N}^{\prime}\right][Y]\left[C H_{N}^{\prime}\right]} \tag{6}
\end{align*}\right.
$$

where the input image is represented by the matrix $[X]$ with the size $N \mathrm{x} N$ and the result is a spatial spectrum matrix $[Y]$ with the same size.

## III. Experimental Results

The 2D CHT can be expressed in different way by the equation [4],[5]:

$$
\begin{equation*}
[X]=\sum_{k=0}^{N-1} \sum_{l=0}^{N-1} y_{k l}\left[T_{k l}\right] \tag{7}
\end{equation*}
$$

where $\left[T_{k l}\right]$ is the matrix of "basis" image with consecutive number ( $k, l$ ). This expression can be presented as image decomposition $[X]$ in order on $N^{2}$ "basis" ordered images with weighted coefficients $y_{k l}$. Using equations (5) and (7) these images of order 4 are simulated on MATLAB and are shown in Fig.1.


Fig.1. "Basis" function for 2D CHT with ordered matrix of size $\mathrm{N}=4$.

In this figure the values $(+1)$ are colored with white level, the values $(-1)$ - with dark gray level, the values $(+j)-$ with white gray level and the values ( -j ) - with black level. It is show that the received "basis" images look like the ordered "basis" images of real Hadamard Transform (HT) [5].

On Fig. 2 and Fig. 3 are shown the "basis" ordered images for 2D CHT of size $8 \times 8$ and $16 x 16$ respectively, received in the same way.

For the analyses of spectral distribution between the coefficients of 2D ordered CHT test image "PEPPERS", with size $256 x 256$, 256 gray levels and $64 \times 64,256$ gray levels are used.


Fig.2. "Basis" function for 2D CHT with ordered matrix of size N=8.


Fig.3. "Basis" function for 2D CHT with ordered matrix of size $\mathrm{N}=16$

On Fig.4a and Fig. 4b 2D amplitude frequency spectrums for each one are present.


Fig.4a. 2D CHT spectrum for image of size $64 \times 64$.


Fig.4b. 2D CHT spectrum for image of size $256 \times 256$.
From this figures the high concentration of image energy in the low-frequency coefficients of the spectrum can be seen.

On the Fig. 5 the averaged PSNR results from experimental investigation of coding of amplitude and phase spectrums of four test images "LENNA", "BABOON", "CAMERAMAN" and "PAPPERS" (Fig.6), received by the 2D ordered CHT and 2D real HT with size $16 \times 16$ are shown.


Fig. 5. Dependence of PSNR (dB) from saved coefficients of ordered CHT and HT

The improvement of PSNR for ordered CHT with respect to HT for 16 saved spectral coefficients from each block, with size $16 \times 16$, is about 2.5 dB . Therefore, the ordered CHT have some advantages, concerning the real ordered HT by the compression of images, trough the filtration of image transformation.

## IV. Conclusion

A class of new ordered Complex Hadamard Transform for images is presented. The general principles of complex matrices construction of high order for 1D and 2D transforms are given. The basic properties of ordered CHT are discussed. The obtained amplitude spectrums for ordered CHT and ordered HT are practically identical and show that both can be used in similar applications.

The presented ordered Complex Hadamard Transform can be used in digital signal processing for spectral analysis, pattern recognition, digital watermarking, coding and transmission of one-dimensional and two-dimensional signals.

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Fig.6. Test images "LENNA", "BABOON", "CAMERAMAN" and "PAPPERS" with size $512 \times 512$ pixels and 256 gray levels.


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