Boundary Conditions when using Quadrature Modulations in Coaxial Cable Television Networks

Kiril Koitchev[1], Krasen Angelov[2] and Stanimir Sadinov[3]

***Abstract* – Estimation of potential noise stability is made concerning downstream and upstream channels when using quadrature modulations of the type QPSK and QAM along coaxial CATV networks. It is assumed that, within the channel, there is white Gaussean noise of zero mathematical expectation and one-side power spectrum density of power. Error probability is calculated during transmission of individual symbols. Recommendations are made concerning the optimal choice in using a certain type of modulation with regard to cable network quality and the selected method of coding.**

***Keywords* – CATV, QAM, M-PSK, BER, DOCSIS**

I. INTRODUCTION

In digital TV broadcasting along coaxial lines by the standard DVB (Digital Video Broadcasting) the error probability *BER* (Bit Error Rate) at the receiver input should not exceed 10^{-11} [1, 8]. A larger value of *BER* will decrease the quality of picture since the signal is continuously being transmitted and, therefore, there will be no opportunity for correction of errors.

Should the CATV network be used for transmission of digital traffic (LAN, Internet and others) the situation then is quite different. In this particular case, if data are transmitted with error, this error could be detected through the incorrect control sum total of the digital stream. If IP protocol is used, then in case of error generation on channel level it can be corrected at transmission level or at the level of the package transfer route. Such mechanism allows successful operation at values for *BER* of $10^{-8} – 10^{-6}$.

Frequencies distribution according to DOCSIS standard is shown on Fig. 1. The downstream channel occupies the frequency band from *87* to *862 MHz*. Data transmission is done along a channel of width *6MHz* (or *8MHz* for Europe – EuroDOCSIS). The used type of modulation can be 64-QAM or 256-QAM [1, 2]. Symbols transmission rate depends on the type of cable and on amplitude-frequency response of the filter in the downstream channel and amounts to *30,342Mbit/s* and *42,88Mbit/s*, for 64-QAM and 256-QAM respectively.

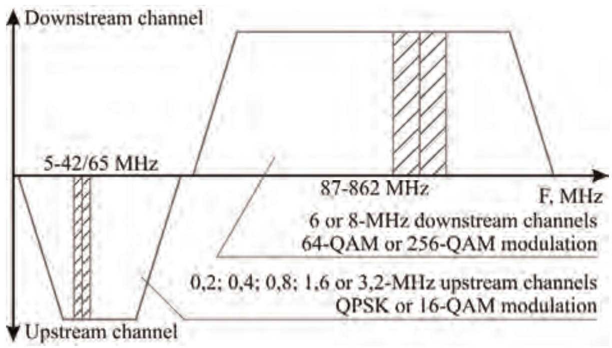

Fig. 1. Distribution of frequencies of downstream and upstream channel in accordance with DOCSIS

The upstream channel operates within the frequency band from *5* to *42 MHz* (from *5* to *62 MHz* for Europe). This range can be divided into channels of widths *0,2*; *0,4*; *0,8*; *1,6* and *3,2 MHz*. In the upstream channel it is possible to use either QPSK or 16-QAM modulation. In case we use a modulation of the type QPSK, the maximum possible rates will vary from *0,32* to *5,12 Mbit/s*, and with 16-QAM – from *0,64* to *12,24 Mbit/s*.

II. ESTIMATION OF NOISE STABILITY IN THE DOWNSTREAM AND UPSTREAM CHANNEL

Potential noise stability within the downstream and upstream channel will be calculated first. It is assumed that in it there is a white Gaussian noise of zero mathematical expectation and one-side power spectrum density of the power N_0. At first the error probability of an individual case will be calculated followed by an estimation of the *BER* magnitude.

In the common case the error probability P_a of an individual symbol is [3, 4]

$$P_a = P\left[\bigcup (A_j \geq A_i)\right],$$

where

$$A_i = \int_0^T \left| N(t) + S_i(t) \right| S_i(t)dt - 0{,}5E_j \ . \qquad (1)$$

In the above formula $N(t)$ stands for fluctuation disturbance; $S_i(t)$ – the symbol being transmitted; E_j – the energy of the symbol.

The estimation of this probability is difficult since in the use of quadrature modulation of the type QPSK and QAM, the amplitude of symbols is not identical. The task can be simplified if we assume that there will be an error in the

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reception of the symbol only when noise power exceeds the half of the difference between the symbol being transmitted and the symbol which is closest to it. This is the so called equivalent power of the two symbols P_E [2, 6].

By employing this approach in equally probable symbol transmission through Gray code and 4-PSK modulation we get

$$P_{E\,4PSK} = \frac{1}{2}\left[P_E(a_0) + P_E(a_1)\right] = \frac{1}{2}(1+1)P_a\,, \qquad (2)$$

where a stands for the basis coordinates with regard to axis "a", P_a – the error probability related to the solution based in relation to the same axis.

By means of analogy it is possible to write equations for 8-PSK and 16-PSK, respectively:

$$P_{E\,8PSK} = \frac{1}{3}\left[P_E(a_0) + P_E(a_1) + P_E(a_2)\right] \approx \frac{1}{3}(1+1+2)P_a\,, \quad (3)$$

$$\begin{aligned} P_{E\,16PSK} &= \frac{1}{4}\left[P_E(a_0) + P_E(a_1) + P_E(a_2) + P_E(a_3)\right] \approx \\ &\approx \frac{1}{4}(1+1+2+4)P_a \end{aligned} \quad . \quad (4)$$

Summarizing for M-multiple PSK modulation we get

$$\begin{aligned} P_{E\,MPSK} &= \frac{1}{\log_2 M}\left(1+1+2+4+\ldots+2^{\log_2 M-2}\right)P_a \approx \\ &\approx \frac{M}{2\log_2 M}P_a \end{aligned} \quad . \quad (5)$$

Error probability P_a can be expressed as [6, 7]

$$P_a = \frac{4}{M}\sum_{i=1}^{M/4}\left[\sqrt{\frac{2E_b\log_2 M}{N_0}}\sin\frac{(2i-1)\pi}{M}\right], \qquad (6)$$

where $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} e^{-\frac{t^2}{2}}dt$ is Gaussian function, N_0 – the spectrum density of noise.

Taking Eq. (5) into account, we get for Eq. (6)

$$P_{E\,MPSK} \approx \frac{2}{\log_2 M}\sum_{i=1}^{M/4}Q\left[\sqrt{\frac{2E_b\log_2 M}{N_0}}\sin\frac{(2i-1)\pi}{M}\right], \quad (7)$$

In case of $M \geq 4$, then the ratio $E_b/N_0 >> 1$, and the first member of the sum will be dominating thus $i = 1$. Following that Eq. (7) gets simplified to the kind

$$P_{E\,MPSK} \approx \frac{2}{\log_2 M}Q\left[\sqrt{\frac{2E_b\log_2 M}{N_0}}\sin\frac{\pi}{M}\right]. \qquad (8)$$

Eq. (8) can be regarded as approximation characteristics for MPSK modulated signal, *BER* being taken into account.

Fig. 2 shows the analysis results in using Eq. (8) and simulation by the Monte Carlo method.

In case of equal probability symbols *BER* for 16-QAM can be rendered by the expression by way of analogy with Eq. (2)

$$P_{E\,16QAM} \approx \frac{1}{2}(1+2)P_Q\,, \qquad (9)$$

where P_Q stands for error probability related to the solution with regard to the Q axis. Thus, for the general case of M-QAM modulation

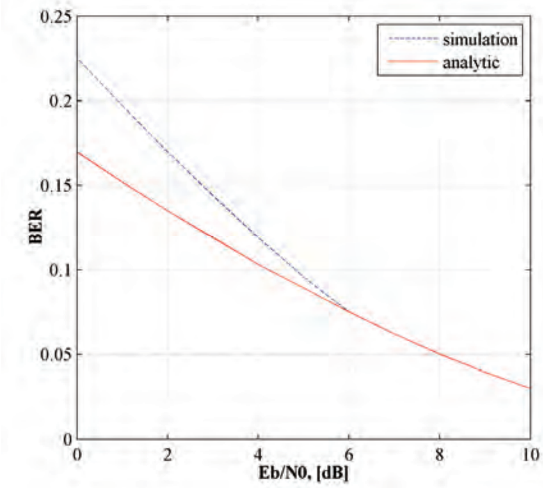

Fig. 2. Monte Carlo simulated and estimated from Eq. (8) *BER* as a function of E_b/N_0 for M-PSK modulated signal

$$\begin{aligned} P_{E\,MQAM} &\approx \frac{1}{\log_2 M}\left(1+2+4+\ldots+2^{\log_2\sqrt{M}-1}\right)P_Q = \\ &= \frac{2}{\log_2 M}\left(\sqrt{M}-1\right)P_Q \end{aligned} \quad . \quad (10)$$

Taking account of the constellation diagram for M-QAM in employing Gray code, it follows that

$$P_Q = \frac{2}{\sqrt{M}}\sum_{i=1}^{\frac{\sqrt{M}}{2}}Q\left[\frac{(2i-1)d}{\sqrt{\frac{N_0}{2}}}\right], \qquad (11)$$

and d can be calculated from

$$E_b\log_2\sqrt{M} = \frac{M-1}{3}d^2\,. \qquad (12)$$

Then Eq. (12) is reduced to

$$P_{E\,MQAM} \approx \frac{4}{\log_2 M}\left(1-\frac{1}{\sqrt{M}}\right)\sum_{i=1}^{\frac{\sqrt{M}}{2}}Q\left[(2i-1)\sqrt{\frac{3E_b\log_2 M}{(M-1)N_0}}\right]. \ (13)$$

In this way Eq. (13) appears to be an expression of approximation for M-QAM and as is in Eq. (8) there can be drafted the characteristics shown on Fig. 2.

Since the signals transmitted along the line appear to be simple ones then it follows that it is possible to determine the necessary relation E_b/N_0 for every type of modulation, provided the error probability is adequately assigned. Assuming that Gray code is used, (in this case each symbol will be distinguished from the next closest one by *1bit* only in order to ensure minimum error), it is possible to calculate the magnitude of *BER* as a result of dividing the symbol error by *8* for 256-QAM; by *6* for 64-QAM; by *4* for 16-QAM and, finally by 2 for QPSK. For these cases the graphs of *BER* dependence from E_b/N_0 ratio are shown on Fig. 3.

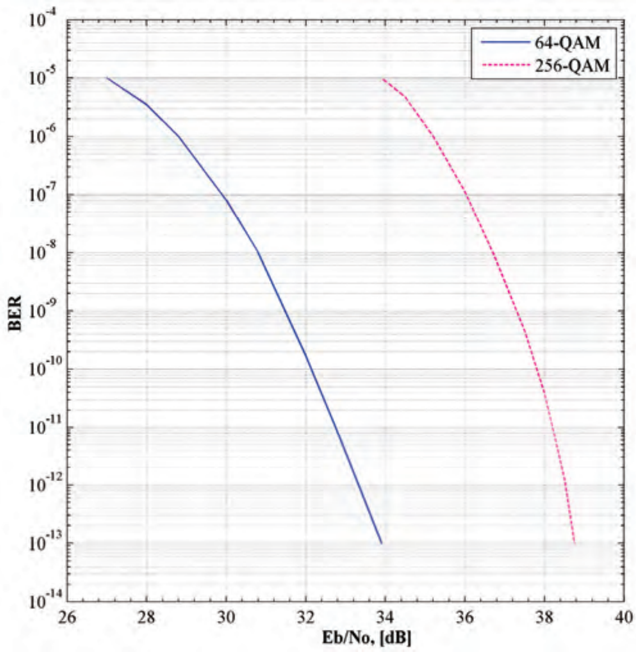

Fig. 3. $BER = f(E_b/N_0)$ for 64-QAM and 256-QAM with Gray code using

Table I contains the required values for E_b/N_0 with error probability 10^{-11}, 10^{-8} and 10^{-6} during data transmission along the downstream channel of CATV.

TABLE I
REQUIRED VALUES OF E_B/N_0 WITH ASSIGNED *BER* FOR THE DOWNSTREAM CHANNEL FOR 64-QAM AND 256-QAM MODULATION

BER	64-QAM, [dB]	256-QAM, [dB]
10^{-6}	29	35
10^{-8}	30,7	36,7
10^{-11}	32,4	38,4

When transmitting data along computer networks the sufficient value of *BER* is approximately 10^{-8}. In case of using coaxial networks it is necessary to select E_b/N_0 ratio of *36,7dB* and more for 256-QAM modulation since it ensures maximum transmission rate of *52 – 55 Mbit/s* with *8MHz* (according to EuroDOCSIS). For the sake of comparison it is worth noting that for transmission of digital TV signals using the Read-Solomon code according DVB-C standard it is required that the ratio E_b/N_0 is *32dB* with 256-QAM.

In case of using the upstream channel it should be borne in mind that it is a case of data transmission from numerous subscribers. Thus, the channel quality is defined in connection with *PER* (Package Error Rate) [4]. Table II shows the values of signal-to-noise ratio for $PER = 10^{-6}$, 10^{-8} and 10^{-11} with various modulations for the upstream channel.

TABLE II
REQUIRED VALUES OF E_b/N_0 WITH ASSIGNED *PER* FOR UPSTREAM CHANNEL FOR QPSK, M-PSK AND 16-QAM MODULATION

PER	QPSK, [dB]	36, 16, 10 QPSK, [dB]	16-QAM, [dB]
10^{-6}	16,5	7,2	23,5
10^{-8}	18,0	8,4	25,0
10^{-11}	19,5	9,5	26,5

By knowing *BER* (or *PER*) and in relation to E_b/N_0 ratio it is possible to determine the rates of the digital traffic along the transmission channel. Fig. 4 shows the relationship between the rate of the digital traffic *V*, according to DOCSIS 1.1/2.0, depending on E_b/N_0 for channel width of *3,2MHz* and error probability (*BER*) of 10^{-8}.

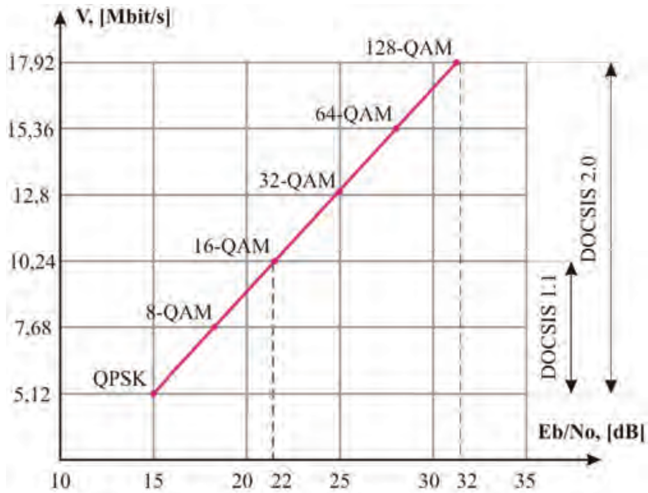

Fig. 4. $V = f(E_b/N_0)$ for upstream channel in accordance with DOCSIS 1.1/2.0 when using *3,2MHz* channel bandwidth and $BER = 10^{-8}$

III. CONCLUSION

The optimum modulation format used should be selected on the basis of the quality parameters of the CATV and the selected way of coding. Signal power can be increased to help improve signal-to-noise ratio. This in turn may cause some other parasitic effects such as the penetration of the signal at the input of the TV set, disturbances due to the presence of second and third harmonic in the signal along the upstream channel data transmission, etc.

Signal-to-noise ratio can be improved at the design and construction stage of networks set-up by using higher quality components such as coaxial cables with triple screening, passive dividers and deflectors of maximum screening (not less than *120dB*), or through the network topology, for example, by employing HFPC technology – a hybrid network featuring passive coaxial part also referred to as “fiber to the home”. This particular type of technology appears to be the most attractive one both in terms of signal quality performance and network transmission capacity.

ACKNOWLEDGEMENT

This paper has been sponsored under the auspices of the “Planning of Multimedia Telecommunications Networks with Traffic and QoS Management” project – a part of the “SCIENTIFIC INVESTIGATIONS” Fund, contract ВУ-ТН-105/2005.

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