Genetic Algorithms in Solving Multi-objective Optimization Problems

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Abstract: - **The paper suggests discusses the development of the scientific research in the area of the genetic algorithms application in solving multi-objective optimization problems. Some of the most wide-spread algorithms, developed in the last years have been considered. The main directions, connected with the use of these algorithms, have been presented and the growth of the research work in this area (publications) during the last decade**

Keywords – **evolutionary multi-objective optimization, multi-objective evolutionary algorithms, genetic algorithms**

I. Introduction

Genetic algorithms are a method for search based on the selection of the best species in the population in analogy to the theory of evolution of Ch. Darwin.

From the point of view of the information change, the evolutionary search is a sequential transformation of a single fuzzy (imprecise) set of some solutions into another one. The transform itself can be named a searching algorithm or a genetic algorithm (GA).The GA is not simply a random search, but an efficient usage of information in the evolutionary process [6], [11].

The main goal of GA-s is twofold:

- abstract and formal explanation of the adaptation processes in evolutionary systems;
- modelling natural evolutionary processes for efficient solution of determined class of optimization and other problems.

During the last years a new paradigm is applied to solve optimization problems GA-based and modifications of GA. GA realize searching a balance between efficiency and quality of solutions at the expense of selecting the strongest alternative solution from undetermined and fuzzy solutions [11].

Usually in the multi-objective optimization problems several criteria (objective functions) are optimized simultaneously in a set of feasible alternatives. In the general case there does not exist an alternative (solution), which is optimal for all the solutions. But there exists a set of alternatives (solutions), characterized by the following property: each improvement of the value of one of the criteria leads to the deterioration of the value of at least one of the other criteria. A set of alternative solutions is obtained, each of the alternatives in this set could be a solution of the multiobjective problem.

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The notion of optimality was originally introduced by F. Edgeworth in 1881 and later generalized by V. Pareto in 1896. It is called Edgeworth-Pareto optimum or, simply, Pareto optimum. In words, this definition says that a solution to an MOP is Pareto optimal if there exists no other feasible solution which would decrease some criterion without causing a simultaneous increase in at least one other criterion. It should not be difficult to realize that the use of this concept almost always gives not a single solution but a set of them, which is called the Pareto optimal set. The vectors of the decision variables corresponding to the solutions included in the Pareto optimal set are called nondominated. The plot of the objective functions whose nondominated vectors are in the Pareto optimal set is called the Pareto front.

II. Starting of Research

The first to have designed an multi-objective evolutionary algorithm (MOEA) during the mid-1980s is D. Schaffer [27]. Schaffer's approach, called Vector Evaluated Genetic Algorithm (VEGA) consists of a simple genetic algorithm with a modified selection mechanism. At each generation, a number of sub-populations were generated by performing proportional selection according to each objective function in turn. These sub-populations would then be shuffled together to obtain a new population, on which the GA would apply the crossover and mutation operators in the usual way. VEGA had a number of problems, from which the main one had to do with its inability to retain solutions with acceptable performance, perhaps above average, but not outstanding for any of the objective functions. These solutions were perhaps good candidates for becoming nondominated solutions, but could not survive under the selection scheme of this approach.

Later researchers adopted for several years other naive approaches. The most popular were the linear aggregating functions, which consist in adding all the objective functions into a single value which is directly adopted as the fitness of an evolutionary algorithm [5]. Lexicographic ordering was another interesting choice. In this case, a single objective (which is considered the most important) is chosen and optimized without considering any of the others. Then, the second objective is optimized but without decreasing the quality of the solution obtained for the first objective. This process is repeated for all the remaining objectives [9].

Despite al these early efforts, the direct incorporation of the concept of Pareto optimality into an evolutionary algorithm was first hinted by David E. Goldberg in his book on genetic algorithms [10]. He suggested the use of nondominated ranking and selection to move a population toward the Pareto front in a multi-objective optimization problem. The basic idea is to find the set of solutions in the population that are Pareto nondominated by the rest of the population. These solutions are then assigned the highest rank and eliminated from further contention. Another set of Pareto nondominated solutions is determined from the remaining population and are assigned the next highest rank.This process continues until all the population is suitably ranked. Goldberg also suggested the use of some kind of niching technique to keep the GA from converging to a single point on the front.

For pity the author develops only theoretically his ideas and does not supply any real alteration of the procedures, described by him.

III. Extension of the Investigations

In the previous study (section 2) it was pointed out that Goldberg does not provide a real execution of his procedures in multi-objective optimization (MOP), but in fact all the variants of this algorithm, later developed, are on the basis of his theory and are influenced by them.

3.1 Nondominated Sorting Genetic Algorithm (NSGA)

This algorithm is suggested by Srinivas and Deb [29]. The NSGA is based on several layers of classifications of the individuals as suggested by Goldberg [10]. Before selection is performed, the population is ranked on the basis of nondomination: all nondominated individuals are classified into one category (with a dummy fitness value, which is proportional to the population size, to provide an equal reproductive potential for these individuals). To maintain the diversity of the population, these classified individuals are shared with their dummy fitness values. Then this group of classified individuals is ignored and another layer of nondominated individuals is considered. The process continues until all individuals in the population are classified. Since individuals in the first front have the maximum fitness value, they always get more copies than the rest of the population. The algorithm of the NSGA is not very efficient, because Pareto ranking has to be repeated over an over again. Evidently, it is possible to achieve the same goal in a more efficient way.

3.2. Niched-Pareto Genetic Algorithm (NPGA)

This algorithm is suggested by Horn, Natpliotis and Goldberg in [16]. The NPGA uses a tournament selection scheme based on Pareto dominance. The basic idea of the algorithm is quite clever: Two individuals are randomly chosen and compared against a subset from the entire population (typically, around 10% of the population). If one of them is dominated (by the individuals randomly chosen from the population) and the other is not, then the nondominated individual wins. All the other situations are considered a tie (i.e., both competitors are either dominated or nondominated). When there is a tie, the result of the tournament is decided through fitness sharing.

3.3. Multi-Objective Genetic Algorithm (MOGA)

This algorithm is suggested by Fonseca and Fleming in [7]. In MOGA, the rank of a certain individual corresponds to the number of chromosomes in the current population by which it is dominated. All nondominated individuals are assigned the highest possible fitness value (all of them get the same fitness, such that they can be sampled at the same rate), while dominated ones are penalized according to the population density of the corresponding region to which they belong (i.e., fitness sharing is used to verify how crowded is the region surrounding each individual).

Making comparative analysis of the algorithms, above pointed, it is established with no doubt, that MOGA is excelling, followed by NPGA and NSGA. The investigations in that period are characterized by simplicity of the algorithms offered and the lack of any methodology for their testing [6].

The main conclusion about the implementations of this generation of GA is, that in order one MOEA to be successful, a good mechanism for the selection of the nondominated species has to be combined with a good mechanism for variety support, which will enable the generation of MOEA[6].

IV. Elitism – the Standard Mechanism in the most Recent Evolutionary Algorithms

The wide development of MOEA in the recent years has begun after the works of Eckart Zitzler [32], due to it the elitism has become a standard mechanism in the development in this direction. In the context of multi-objective optimization, elitism usually (although not necessarily) refers to the use of an external population (also called secondary population) to retain the nondominated individuals found along the evolutionary process. The main motivation for this mechanism is the fact that a solution that is nondominated with respect to its current population is not necessarily nondominated with respect to all the populations that are produced by an evolutionary algorithm. Thus, what we need is a way of guaranteeing that the solutions that we will report to the user are nondominated with respect to every other solution that our algorithm has produced. Therefore, the most intuitive way of doing this is by storing in an external memory (or archive) all the nondominated solutions found. If a solution that wishes to enter the archive is dominated by its contents, then it is not allowed to enter. Conversely, if a solution dominates anyone stored in the file, the dominated solution must be deleted.

After the offered by Zitzler theory, most of researchers began to started to incorporate external populations in their MOEAs and the use of this mechanism (or an alternative form of elitism) became a common practice.

4.1. Strength Pareto Evolutionary Algorithm (SPEA)

This algorithm is introduced by Zitzler and Thiele in [32]. This approach was conceived as a way of integrating different MOEAs. SPEA uses an archive containing nondominated solutions previously found (the so-called external nondominated set). At each generation, nondominated individuals are copied to the external nondominated set. For each individual in this external set, a strength value is computed. This strength is similar to the ranking value of MOGA [7], since it is proportional to the number of solutions to which a certain individual dominates.

4.2. Strength Pareto Evolutionary Algorithm 2 (SPEA2)

This algorithm, introduced by Zitzler and Thiele in [31] has three main differences with respect to its predecessor:

(1) it incorporates a fine-grained fitness assignment strategy which takes into account for each individual the number of individuals that dominate it and the number of individuals by which it is dominated;

(2) it uses a nearest neighbor density estimation technique which guides the search more efficiently;

(3) it has an enhanced archive truncation method that guarantees the preservation of boundary solutions.

4.3. Pareto Archived Evolution Strategy (PAES)

This algorithm is introduced by Knowles and Corne in [21]. PAES consists of a (1 + 1) evolution strategy (i.e., a single parent that generates a single offspring) in combination with a historical archive that records the nondominated solutions previously found. This archive is used as a reference set against which each mutated individual is being compared. Such a historical archive is the elitist mechanism adopted in PAES. However, an interesting aspect of this algorithm is the procedure used to maintain diversity which consists of a crowding procedure that divides objective space in a recursive manner. Each solution is placed in a certain grid location based on the values of its objectives (which are used as its "coordinates" or "geographical location"). A map of such grid is maintained, indicating the number of solutions that reside in each grid location. Since the procedure is adaptive, no extra parameters are required (except for the number of divisions of the objective space). This adaptive grid (or variations of it) has been adopted by several modern MOEAs [4].

V. Applications

The advance in the research of MOEA ensures them their widening application. In order to give a general fancy for the type of applications, they could be classified in four main directions [1], [2]: science, engineering, industry and various other directions (miscellaneous applications). Some specific areas inside any of these directions are discussed below.

- Scientific applications [1], [13], [17], [22], [23], [26]– of chemical, analysis of spectroscopy, medical image reconstruction [22], computer aided diagnosis, machine-learning in high-dimensional data, the analysis of promoters in biological sequences in the problem to deal with [26] and all.

- Engineering applications [1], [6], [12], [14] – electrical, hydraulic, structural, aeronautical, robotics and control and all.

- Industrial applications [1], [19], [24], [25] – design, manufacture, scheduling, management and all.

- Miscellaneous applications [1], [28], [15]– problem of attribute selection in data mining, decisions support system, finance, optimization a forecast model, forest management and all.

The strong interest for MOEA in so many different disciplines reinforces the idea that there will be new possibilities for solving still more real-life problems.

VI. Conclusions

After the attempt for a short survey it could be noted, that the scientific research in the area considered are directed towards different aspects, but one of the major aspects is the efficiency, which is regarded at algorithmic level and at data structure level [2], [17], [20]. A variety of measures for implementation quality is suggested. It allow a quantitative (rather than only qualitative), comparison of results [30], [8], [32]. Zitzler et al. [30] stated that, when assessing performance of an MOEA, one was interested in measuring three things:

- Maximize the number of elements of the Pareto optimal set found.

- Minimize the distance of the Pareto front produced by our algorithm with respect to the global Pareto front (assuming we know its location).

- Maximize the spread of solutions found, so that we can have a distribution of vectors as smooth and uniform as possible.

- Concurrently with the research on performance measures, other researchers were designing test functions.

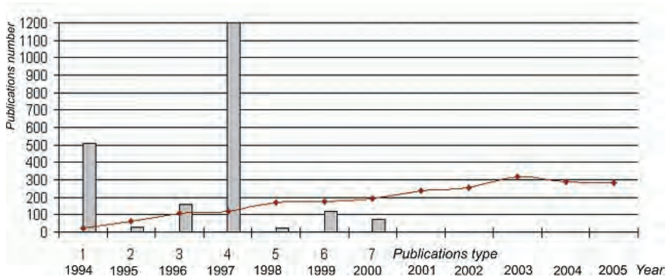

Fig.1 Distribution of the publications

For the enhancing development of the scientific investigations in thus direction (MOEA) the basic proof is the continuously increasing number of references and applications in the last ten years. In a paper of his Coello [3] represents approximate graphics of the publications according their type. Fig.1 represents the distribution of the publications, depending on the issue – 1-journal papers, 2 - books, 3- book chapters, 4-conference papers, 5- master's theses, 6- Ph.D. theses, 7 - technical reports and the distribution of the publications in years.

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