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Application of Kalman Filtering Technique to Increase the Probability of Faults Detection in Test Equipment

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***Abstract* - One approach for application of Kalman filtering theory in test equipment has been considered. The main goal is to estimate the state (good or faulty) of this test equipment in case of failure in the measurement channel. We simulated this failure at given step of working of the channel. The covariance matrix P(k|k) is shown when the failure has been occurred.**

***Keywords* - Kalman filtering theory, covariance, failure model, precise test equipment.**

I. INTRODUCTION

Let's consider the following measurement channel. This channel can be described by using state-space method [1]. It is also a linear discrete-time system with two equations describing the dynamics of the system Eq.(1.1) and the formation of the data accessible to the measurement Eq.(1.2):

$$\mathbf{x}(k+1) = \Phi(k+1, k)\mathbf{x}(k) + \mathbf{G}(k+1, k)\mathbf{w}(k) \quad (1.1)$$

$$\mathbf{y}(k) = \mathbf{H}(k)\mathbf{x}(k) + \mathbf{F}(k)\mathbf{v}(k), \quad (1.2)$$

where:
- k – time index,
- $\mathbf{w}(k)$- zero-mean white noise(process noise),
- $\mathbf{x}(k)$- internal state,
- $\mathbf{y}(k)$- observed signal,
- $\mathbf{v}(k)$- additive noise in observed signal.

Also $\Phi(k+1, k)$, $\mathbf{G}(k+1, k)$, $\mathbf{H}(k)$, $F(k)$ are matrices regarding to $\mathbf{x}(k)$, $\mathbf{w}(k)$, $\mathbf{y}(k)$, $\mathbf{v}(k)$.
In this case we consider the following model of the system (shown in Fig.1).

II. PRESENTATION

Since the observed signal y consists of additive noise $\mathbf{v}$, in test equipment it is used optimal estimation approach based on the theory of Kalman filtering in order to reduce the uncertainty in the measurement.

This estimation method is strong in several aspects- it can estimate previous, present and even future states of the system. The equation describing the estimated value $\hat{x}(k/k)$ is obtained by the expression:

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$$\begin{aligned}\hat{x}(k/k) &= \Phi(k,k-1)\hat{x}(k-1/k-1) + \\ &+ K(k)[y(k) - H(k)\Phi(k,k-1)\times \\ &\times \hat{x}(k-1/k-1)] = \\ &= \hat{x}(k/k-1) + K(k)\hat{z}(k/k-1)\end{aligned} \quad (2.1)$$

where: K(k)- matrix coefficient of gain,
$\hat{z}(k/k-1)$ - innovation of the process,

$$\begin{aligned}K(k) &= P(k/k)H^T(k)R^{-1}(k) \\ &= P(k/k-1)\times H^T(k)[H(k)\times \\ &\times P(k/k-1)H^T(k) + R(k)]^{-1}\end{aligned} \quad (2.2)$$

The covariance matrix of the filtering error $P(k/k)$ consisting of the variances of different parameters is [2]:

$$\begin{aligned}P(k/k) &= P(k/k-1) - P(k/k-1)H^T(k)\times \\ &\times [H(k)P(k/k-1)H^T(k) + \\ &+ R(k)]^{-1}H(k)P(k/k-1).\end{aligned} \quad (2.3)$$

The main property of this filtering is that after each step of working the variance P [k|k] of each parameter is going to reduce its initial value
P [0|0] and the uncertainty tends to be zero. This is shown in Fig. 2. Also it is shown the state **x** and the observed signal y .

III. PRACTICAL CONSIDERATION AND APPLICATION

During normal functioning of the system (without failures), the variance P(k|k) is minimum (after completing the transition processes).
We will consider the following important case in practice when the failure in the system is related to increasing of measurement noise $\mathbf{v}(k)$ in the equation of observation Eq.(1.2):

$$\mathbf{y}(k) = \mathbf{H}(k)\mathbf{x}(k) + \mathbf{F}(k)\mathbf{v}(k)$$

A failure event has been simulated at given step in order to show the real process. Then the covariance matrix of the filtering error $P(k/k)$ Eq.(2.3) consisting of the variances of different parameters will increase due to increasing of measurement noise $\mathbf{v}(k)$. It is shown in Fig. 3.

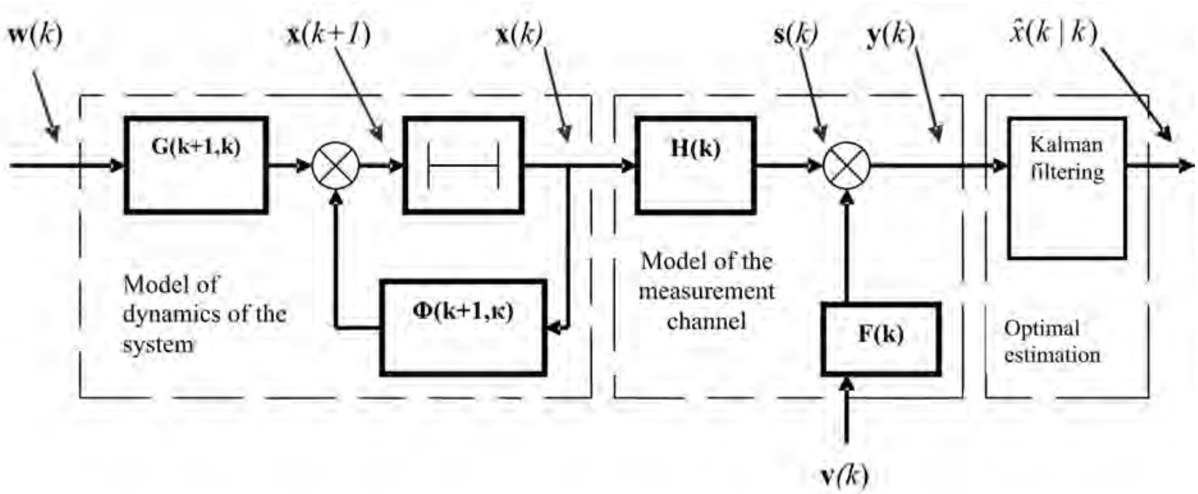

Fig. 1. Model of the dynamics of the system and model of measurement channel in “state-space” method

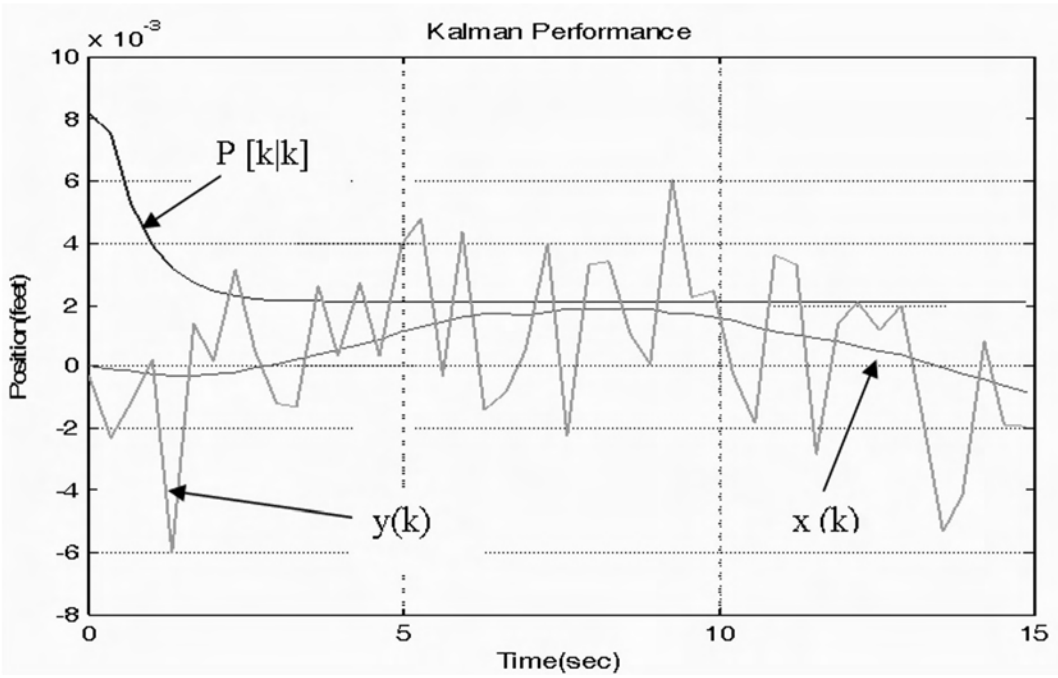

Fig. 2. The uncertainty estimation method based on Kalman filtering approach.

We can create algorithm for faults detection by using the covariance matrix of the filtering error $P(k/k)$ [3]. Thus, increasing of this error will be a sign for fault in the observed system. After creating the algorithm for faults detection at each step of working we can estimate the system when fault has been occurred.

In practice it is so important to take the right decision for failure because there are two errors:

- Error 1: α - when the object is working properly we take a decision for failure. It leads up to missed detection.
- Error 2: β - when the object is failed we take a decision for working properly. It leads up to false alarm.

There is a big difference between them. In practice error 1 is more important than error 2 (it is more dangerous).

Let’s consider an information system with 100 parallel, independent working channels with probability of failure $Q = 1 - e^{-\lambda.t}$, where: $\lambda = 10^{-3}$, $t = 10^{3}$ h (Fig.1). During their working we can not repair them. After completing the mission the system is subject to repair. We find the failed channels and after that the system starts working again with 100 channels. If during the maintenance we find all failed channels and after that we recover them, then mean value of channels working properly is *N*.

Let now during the maintenance the error 1 (missed detection) is $\alpha = 0{,}4$ [1]. Since the mean value of failed components for a time t from N independent working (parallel) channels is:

$$\bar{k} = N.Q(t) \qquad (3.1)$$

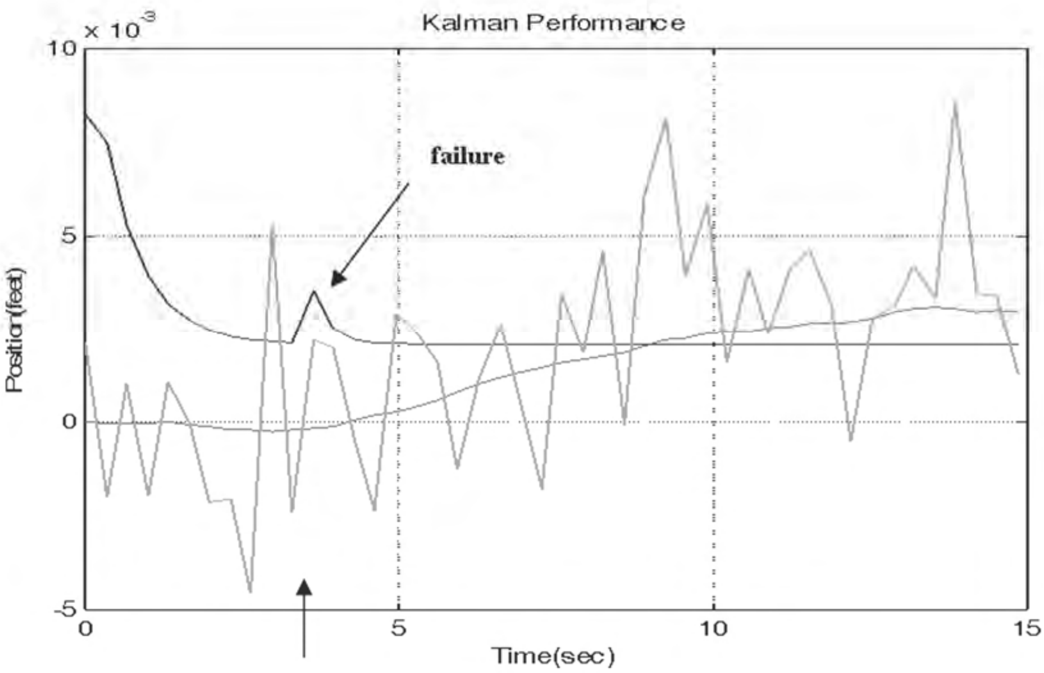

Fig. 3. Failure of the system at given step affecting on P(k|k).

Then after the first cycle of maintenance we have:

$$\bar{k}_1(1000) = 100.Q(1000) =$$
$$= 100.(1-e^{-1}) = 100.0{,}63 = 63$$

In this case of 63 failed components the following ones can not be found: *63.0,4= 25***.**

After completing the second cycle of maintenance the failed components will be $\bar{k}_2 = 47$ and then total number failed channels will be *(47+25)=72*.

That is, the failed channels which will not be found are now: *72.0,4=29* and the third cycle starts working with 71 channels. After completing the third cycle we will have 45 failed channels. The total number failed channels will be *(45+29)=74* and finally we will have *74.0,4=30 (not found channels)*.

The fourth cycle starts working with 70 channels in good state. This is the steady-state value of the channels working properly.

IV. Conclusions

Therefore, due to incorrect monitoring in test equipment the information system will work up to 70 % of its capacity.

One approach for decision-making in case of faults detection has been proposed. This one includes estimation method based on Kalman filtering theory and it considers failure model related to increased value of noise in the observed signal. The main goal is to reduce the error 1: α because it is more dangerous in practice. The benefit is we will have more precise test equipment**.**

That is, in practice it is so important to use methods taking right decision in order to detect faults in the system. It is so actually in such systems as avionics, control systems related to technological processes, radiolocation systems, etc.

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