Complex Input Signal Quantization Noise Analysis for Orthogonal Second-Order IIR Digital Filter Sections

Zlatka Nikolova

Abstract – **In this paper a new method for the estimation of complex output noise variance due to input signal quantization is proposed. The method is applied on very low-sensitivity second-order orthogonal complex IIR filter sections. They are used for the design of higher order narrow-band cascade realizations, which are preferred in many telecommunication applications and are normally implemented with fixed-point arithmetic. It is shown experimentally that the sensitivity of the orthogonal complex structure has a profound impact on its output noise level. The proposed method is applicable to any filter structure and can be used to study the complex signal quantization effects in general.**

Keywords – **complex orthogonal digital filters, sensitivity, quantization errors, noise analysis.**

I. Introduction

Finite word-length (quantization) effects are important fraction of the parasitic effects group. Initially, all quantizaion effects have been united together into a single error analysis, but the most useful approach is they to be divided into two categories, requiring different analysis techniques:

- *coefficient quantization* – errors in representing filter coefficients as finite fixed-point numbers;
- *signal quantization* – errors due to the finite-precision arithmetic operations of addition, multiplications and storage.

The filter coefficients are quantized once only and remain constant in the filter implementation. Coefficient quantization effects on filter characteristics perturb them from their ideal forms. If they no longer meet the specifications, the quantization design must be optimized by allocating more bits or choosing more proper filter realization. The structure of the digital filter has a significant effect on its sensitivity to coefficient quantization.

Signal quantization, on the other hand, due to truncation or rounding, is usually best viewed as a random process and can be modeled as producing additive white noise sources in the filter. The effect of signal quantization is to add an error or noise signal to the ideal output of the digital filter, which is composite of one or more of the following error sources: the quantization error of the filter input signal; the errors resulting from the rounding or truncation of multiplication products within the filter; and quantization of the output to refer bits for input to a digital-to-analog converter or another system. Again, as for coefficient quantization, the filter structure affects considerably signal-quantizaion noise levels.

In this work the attention is restricted to the noise analysis due to input signal quantization. In case of real digital filters there are various good techniques developed long ago [1] [2].

[1] Zlatka Nikolova is with the Dept. of Telecommunications, Technical University of Sofia, Bulgaria, e-mail: zvv@tu-sofia.bg

In the last years complex coefficients digital filters are gaining popularity, but their quantization noise theory is still not well developed. Small amount of publications touch the problems barely [3] [4]. Only specific problems are considered so far, and no general technique for quantization noise estimation is proposed.

In this work a new method for complex analytic input signal quantization noise analysis is offered and applied to a very low-sensitivity orthogonal complex second-order section. It is shown experimentally that the low coefficient sensitivity of the circuit escort low output noise variance due to the complex input signal quantization.

II. Complex Input Quantization Noise Analysis

The input signal quantization is equivalent to a set of uniformly distributed noise samples $e(n)$ added to the actual input signal $x(n)$. In case of fixed-point representation with rounding the quantization noise power (variance) of the random variable $e(n)$ is:

$$\sigma_e^2 = \frac{\delta^2}{12} = \frac{2^{-2B}}{12}, \tag{1}$$

B is the word-length in bits and δ is the quantization step size.

When the quantized input signal $x(n)$ is complex, the originated noise source $e(n)$ must be complex too. Then, the complex output signal $y(n)$ will be mixed with complex output noise $v(n)$. The noise model of complex input signal quantization is shown in Fig. 1.

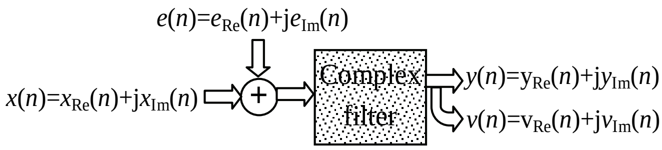


Fig.1: Noise model due to complex input signal quantization

Analytic signals are processed by a special class complex digital filters named "orthogonal", which transfer functions can be presented by its real and imaginary parts as follows:

$$H(-jz) = H_{\mathrm{Re}}(z) + jH_{\mathrm{Im}}(z). \tag{2}$$

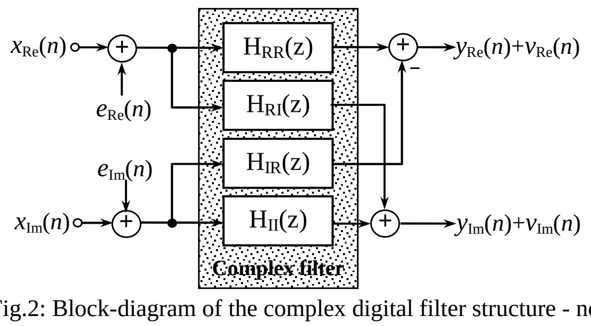


Fig.2: Block-diagram of the complex digital filter structure - noise representation

Realized by real elements a complex orthogonal structure (Fig. 2) has got two inputs - real and imaginary and a relevant output couple, producing thereby four real coefficient transfer functions two-by-two equal with ± sign:

$$H_{\mathrm{Re}}(z)=H_{RR}(z)=H_{II}(z);\quad H_{\mathrm{Im}}(z)=H_{RI}(z)=-H_{IR}(z). \qquad (3)$$

The real $x_{Re}(n)$ and imaginary $x_{Im}(n)$ parts of an analytic signal $x(n)$ are inphase and quadrature components. If their levels are much larger than the quantization step size δ the resulting quantization errors $e_{Re}(n)$ and $e_{Re}(n)$ can be modelled as additive noise sources. The following assumptions can be made:

- ♦ The quantization errors are uniformly distributed over the range (-0,5δ ÷ 0,5δ). They are stationary white noise sequences (i.e. $e(n)$ and $e(m)$ for $n \neq m$ are uncorrelated);
- ♦ The error sequence is uncorrelated with the initial signal sequence;
- ♦ $x_{Re}(n)$ and $x_{Im}(n)$ are orthogonal, i.e. sufficiently different, so that quantization errors $e_{Re}(n)$ and $e_{Im}(n)$ are uncorrelated.

Normally, the real and imaginary parts of the analytic signal are quantized the same word-length. Hence, their noise variances will be identical $\sigma^2_{e,\mathrm{Re}}=\sigma^2_{e,\mathrm{Im}}=\sigma^2_e$ and calculated by Eq. (1).

The assumption that noise signals are statistically independent from source to source leads to the implication that the quantization noise power of their sum is equal to the sum of the respective quantization noise powers. In effect, superposition can be employed and beside the structure from Fig. 2 makes obvious, the real $v_{Re}(n)$ and imaginary $v_{Im}(n)$ components of the complex output noise variance $v(n)$ will be composed respectively as follows: beside

$$\sigma^2_{v,\mathrm{Re}}=\sigma^2_{v,H_{\mathrm{Re}}}-\sigma^2_{v,H_{\mathrm{Im}}} \qquad (4)$$

$$\sigma^2_{v,\mathrm{Im}}=\sigma^2_{v,H_{\mathrm{Re}}}+\sigma^2_{v,H_{\mathrm{Im}}}. \qquad (5)$$

$\sigma^2_{v,H_{\mathrm{Re}}}$ and $\sigma^2_{v,H_{\mathrm{Im}}}$ are the corresponding output noise variances of the real $H_{\mathrm{Re}}(z)$ and imaginary $H_{\mathrm{Im}}(z)$ parts of the orthogonal complex transfer function (2):

$$\sigma^2_{v,H_{\mathrm{Re}}}=\sigma^2_e\frac{1}{2\pi j}\oint H_{\mathrm{Re}}(z)H_{\mathrm{Re}}\left(z^{-1}\right)z^{-1}dz\,, \qquad (6)$$

$$\sigma^2_{v,H_{\mathrm{Im}}}=\sigma^2_e\frac{1}{2\pi j}\oint H_{\mathrm{Im}}(z)H_{\mathrm{Im}}\left(z^{-1}\right)z^{-1}\,dz\,. \qquad (7)$$

III. COMPLEX ORTHOGONAL DIGITAL FILTER CIRCUIT DERIVATION

In order to test the method for complex output noise variance proposed in section II, it is executed on two orthogonal sections – the DF (Direct Form) - and LS2 (Low Sensitivity)-based structures. They are derived after the circuit (poles rotation) transformation in its orthogonal form:

$$z^{-1}=jz^{-1}\text{ or }z=-jz\,, \qquad (8)$$

is applied on the low-pass (LP) second-order real-prototypes [5]. The obtained orthogonal complex coefficients transfer functions have real and imaginary parts of band-pass (BP) type and doubled order. For the DF-based orthogonal complex structure shown in Fig. 3a they are as follow:

$$H^{DF}_{RR}(z)=H^{DF}_{II}(z)=H^{DF}_{\mathrm{Re}}(z)=g_0\frac{1+(-1-g_2+2g_1)z^{-2}+g_2z^{-4}}{1+\left(g_1^2-2g_2\right)z^{-2}+g_2^2z^{-4}} \qquad (9)$$

$$H^{DF}_{RI}(z)=-H^{DF}_{IR}(z)=H^{DF}_{\mathrm{Im}}(z)=g_0z^{-1}\frac{(2-g_1)+(g_1-2g_2)z^{-2}}{1+\left(g_1^2-2g_2\right)z^{-2}+g_2^2z^{-4}}; \qquad (10)$$

whereas for the LS2-based orthogonal section (Fig.3b) they are:

$$H^{LS2}_{RR}(z)=H^{LS2}_{II}(z)=H^{LS2}_{\mathrm{Re}}(z)=$$
$$=0,5a\frac{1+(4a+3b-6)z^{-2}+(1-b)z^{-4}}{1+\left[(2a+b-2)^2-2(1-b)\right]z^{-2}+(1-b)^2z^{-4}}; \qquad (11)$$

$$H^{LS2}_{RI}(z)=-H^{LS2}_{IR}(z)=H^{LS2}_{\mathrm{Im}}(z)=$$
$$=0,5az^{-1}\frac{(4-2a-b)+(2a+3b-4)z^{-2}}{1+\left[(2a+b-2)^2-2(1-b)\right]z^{-2}+(1-b)^2z^{-4}}. \qquad (12)$$

The orthogonal complex filter structures are with canonical number of elements and preserve some properties of their real prototypes. In order to verify this deduction with respect to the input signal quantization noise assessment, the orthogonal BP filters are turned into the narrow-band realizations which are the most often used in practice. It is shown experimentally that the narrow-band BP LS2-based structure has many times lower coefficient sensitivity than the DF-based orthogonal section in a very short word-length setting [6].

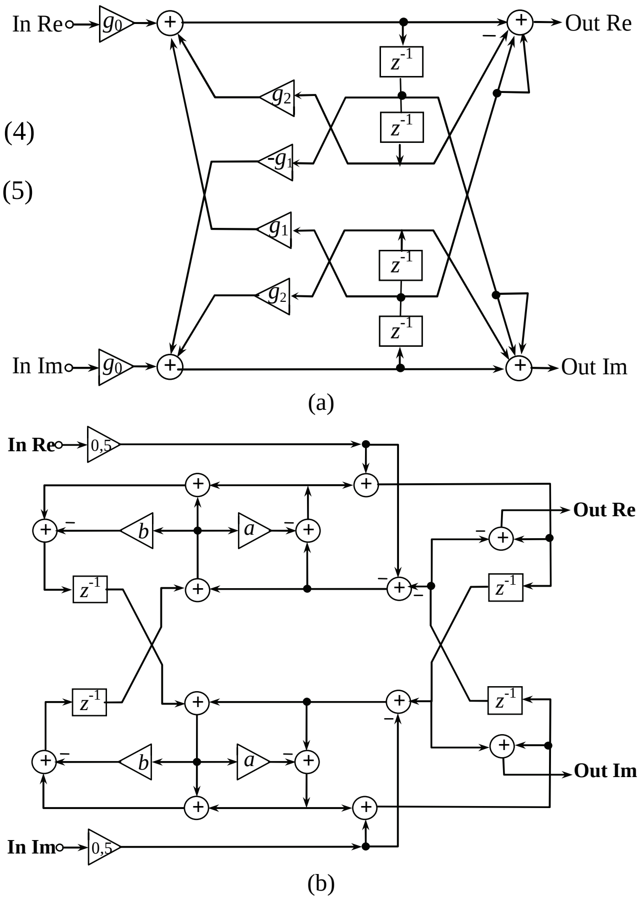


Fig.3: BP orthogonal structure based on (a) DF; (b) LS2 second-order sections

IV. Noise Analysis of Complex Output Signal Quantization Errors

In this section both real and orthogonal structures are investigated in regard to the output errors after analytic input signal quantization.

Initially the real input signal is quantised with a different word-length. The output noise variance for the LS2 and DF real sections is calculated for same pole disposition providing narrow-band LP realizations. Experimental results for input signal quantization from 3 to 8 bits are shown in Fig. 4. Apparently, the low-sensitivity LS2 section output noise variance is significantly lower than this of the DF-section when the input signal is limited to 3 bits only. The numerical results in Table 1 show that the difference comes to be more insignificant as the word-length grows up.

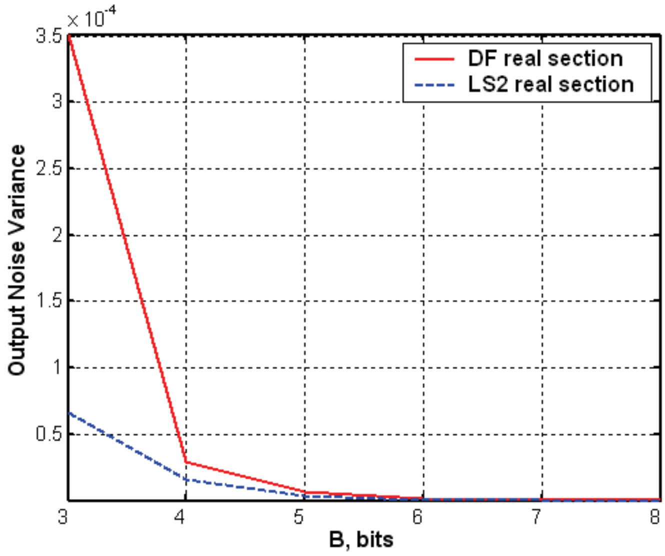


Fig.4: Output noise variance as a function of input signal quantization for LS2 and DF real sections

Table 1

Input signal word-length in bits	Output noise variances of the real sections
	DF-based ($x\ 10^{-2}$)
3	0.03510957123100
4	0.00291925961900
5	0.00068310479600
6	0.00016369047700
7	0.00004088935200
8	0.00001021082700
	LS2-based ($x\ 10^{-3}$)
3	0.06610812663435
4	0.01648302703813
5	0.00411908868389
6	0.00102955782586
7	0.00025738522125
8	0.00006434590146

Applying the method proposed in section II, a complex input signal quantization noise analysis is performed. Some experimental results for complex output noise variances for the LS2 and DF orthogonal sections in different complex input signal word-length environment are presented in Table 2.

Table 2

Input signal word-length in bits	Complex output noise variances of the orthogonal complex sections
	DF-based ($x\ 10^{-3}$)
3	0.07371185555950 + j 0.07311495898425
4	0.01852191939397 + j 0.05813840921642
5	0.00463195749233 + j 0.01567003760465
6	0.00115823432349 + j 0.00409778278276
7	0.00028956349204 + j 0.00102688073474
8	0.00007239109170 + j 0.00025704567665
	LS2-based ($x\ 10^{-4}$)
3	0.04627424248761 + j 0.16527031658588
4	0.01157974082248 + j 0.04120756759532
5	0.00289529076001 + j 0.01029772170972
6	0.00072391264233 + j 0.00257389456465
7	0.00018097816584 + j 0.00064346305313
8	0.00004524454240 + j 0.00016086475366

In order to compare the obtained complex output signal noise variances their complex modulus are graphically presented in Fig. 5. Obviously, the low-sensitivity LS2-based orthogonal complex section demonstrates more than two times lower output noise in case of 3 bits input signal quantization. Let's note that the shorter word-length quantization of the input signal means lower power consumption and faster computation process. For low-sensitivity circuits the resistance to quantization effects provides better signal-to-noise ratio (SNR), i.e. higher quality digital signal processing.

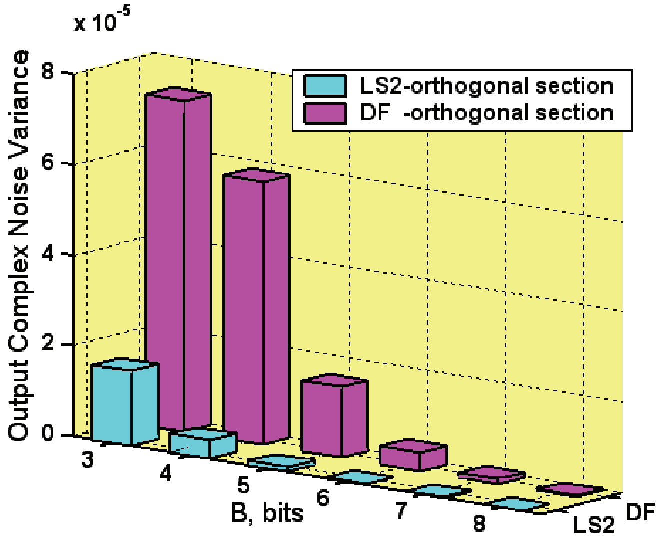
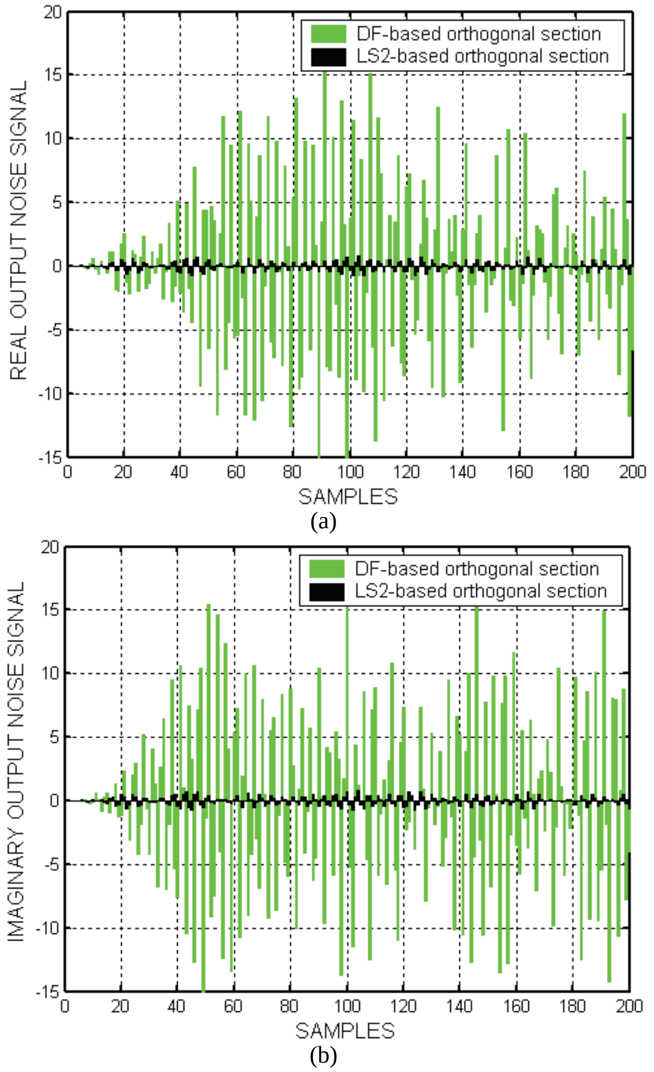


Fig.5: Output noise variances after analytic input signal quantization for LS2 and DF -based orthogonal complex sections

V. EXPERIMENTS

The examined narrowband orthogonal second-order filter sections are tested for limited word-length analytic signal processing. The performed quantized complex input signal is a mixture of white noise and analytic sinusoidal signal. The uniformly distributed white noise samples correspond to the word-length of the input complex signal after its quantization.

In Fig.6a the real part of the complex noise reached to the real output is shown for both DF- and LS2-based orthogonal complex structures. The imaginary output noise signals are presented in Fig. 6b. Apparently, the complex noise at the outputs due to the quantization of the analytic complex input signal is considerably more for the DF-based section than for the LS2-based.



(a)

(b)

Fig.6: The output noise signals after input quantization to 3 bits for LS2 and DF - based orthogonal complex sections (a) real output; (b) imaginary output.

The output SNR for the LS2-orthogonal section is about 1,5 times higher in comparison to the DF-based circuit. To achieve the same good result LS2 section demonstrates for 3 bits word-length input signal quantization, the DF orthogonal filter must be employed in minimum 6 bits environment.

It is clear that the level of the output noise as a result of the input signal quantization and the sensitivity of the system are in a direct relative amount. Therefore, very low-sensitivity complex filter derivation is important to achieve a better noise resistance, improved complex signal filtering and higher quality digital signal processing.

VI. CONCLUSIONS

In this paper a new method for complex noise analysis is proposed. The resulting error signals at the outputs of orthogonal complex second-order digital filter sections after input signal quantization are examined. The proposed method is general enough to be applied for complex filter sections of higher order. After relevant alterations it could be effectively applied for all other types of finite word effects estimation in complex coefficient systems like errors from quantization of multiplication products within the filter.

The expectation that the real prototype properties will be inherited by its complex filter counterpart was confirmed once again with respect to the noise analysis after input signal quantization. It was shown experimentally that both real and orthogonal complex LS2-based filter sections beside very low coefficient sensitivity demonstrate low output noise variance due to input analytic signal quantization. The DF-based real and orthogonal complex circuits keep the same mutual performance even if they have many times higher output noise variance than LS2-based.

Low-sensitivity of complex filters in very limited word-length circumstances for signals and coefficients quantization makes available low computational complexity and provides better quality of the filtering process.

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