Optimal Control Using Neural Networks

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Abstract – **In the present paper a literature review concerning the problem of optimal control synthesis using neural networks is presented.**

Key words – **neural networks, optimal control**

I. INTRODUCTION

During last ten years a large number of papers treating the application of neural networks for optimal control synthesis for plants, whose dynamics is described by linear and nonlinear ordinary and partial differential equations are published. It is determined by the main property of the neural networks to approximate any linear and non-linear function. The most widely used neural networks are feedforward neural networks. Another structure which is also applied is the recurrent neural network. A tendency to start using a broader range of structures emerges which is related to developing of new structures.

II. OPTIMAL CONTROL SYNTHESIS USING NEURAL NETORKS

In [6] a survey of the possibilities of using neural networks for modeling, identification and control of the systems is presented. Here only the optimal decision control and model prediction control will be mentioned. In the case of model prediction control the plant is modeled through a neural network. By using the neural network model the future plant response are predicted over a specified time horizon. On the basis of predicted future response a specified performance index is minimized to obtain the optimal control. In optimal decision control case the state space is partitioned into separate regions, in which the control action is assumed constant. The control surface is realized through a training procedure of the neural networks.

In [4] model predictive optimal controller for nonlinear discrete systems is considered. A block for system input state realization is used, which transforms the system into quadratic system with equal number of inputs and states in order to develop optima receding horizon controller which leads to decreasing the amount of the calculations in comparison to traditional optimal controllers. A nonlinear feedback law is derived where a neural network in feedback loop is used to generate of optimal input action.

The generated input approximates the solution which is minimal in respect of quadratic cost function with parameters controlling the final states, the value and variation of the input action for the control purposes. An analysis of the local stability and robustness of the controller is presented.

In [11] the problem of determining optimal controls for nonlinear dynamical systems by using neural networks is considered. Through a few examples the possibilities of the neural networks for on-line solution of the optimal control problems are demonstrated.

In the major part of the publications [2, 3, 10, 12, 13, 14, 15, 16, 17, 18, 21, 22] the optimal control problem is derived by using the dynamic programming method and the obtained solutions are approximated by using neural networks. Although the presented new approaches bear different names – adaptive critic methodology [2, 10, 12, 13, 14, 15, 16], neuro-dynamic programming [3], neural dynamic optimization [20, 21] – they do not differ from each other substantially. Their main advantage is that so called “curse of dimensionality” problem is solved.

The adaptive critic methodology is introduced in [2]. The control law for linear or nonlinear system is determined through consecutive adapting of two neural networks – action network and critic network. The action network captures the relationship between the state and control and the critic neural network captures the relationship between the state and co-state. Through this methodology the control law is determined for a large set of initial conditions. It is not necessary the control law to be determined analytically. The neural network, which is used (multilayered perceptron) do not need external training; it is necessary the functional form of the control law to be known preliminarily. In [10] the necessary conditions for solutions obtained through the adaptive critic methodology to converge are presented and it is shown that the obtained solution is optimal. In [12, 15] the a.m. methodology is developed for distributed parameter systems. In [13] the method is applied for optimal control synthesis for distributed parameter systems, whose dynamics is described by coupled nonlinear partial differential equations. In [14] the proper orthogonal decomposition concept is used for reducing of distributed parameter system to lumped parameter system of law order. The optimal control problem is solved in time through applying the adaptive critic algorithm. Then the control solution is given in the spatial domain by using the same orthogonal functions. In [16] the adaptive critic algorithm is elaborated. It is shown that the necessity of action neural networks drops off.

In [17] three adaptive critic methods, which are used for designing of neural controllers – heuristic dynamic programming, dual heuristic programming and globalized dual heuristic programming are described. Two modifications of the globalized dual heuristic programming as well as generalized training procedure are suggested. The developed approaches do not differ substantially from the methodology

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suggested in [2]. In both approaches two neural networks are used for approximating the solution for the optimal control – action and critic neural networks. The only difference is that a recurrent neural network is used instead of multilayered perceptron. In [18] the approach is developed for discreet distributed parameter systems. Moreover the algorithm is elaborated and the necessity of action neural networks as in [16] drops off.

In [20, 21] neural dynamic optimization is presented as a method for synthesis of optimal feedback control for nonlinear MIMO systems. The main characteristic of neural dynamic optimization is that the solution for the optimal feedback, whose existence is proved through dynamic programming method is approximated by using neural networks. In [20] the background and motivation for development of neural dynamic optimization is described and in [21] the neural dynamic optimization theory is presented. One major drawback of this approach is the big memory requirements although this requirement is not so severe compared to the classical dynamic programming method.

Another methodology having for a theoretical basis the dynamic programming and using neural networks for approximation is so called neuro-dynamic programming According to the definition given in [3], neuro-dynamic programming “enables the to learn how to make good decisions by observing their own behavior, and use built-in mechanisms for improving their actions through a reinforcement mechanism”. This methodology is used not only for solving the optimal control problems but for a broader class of problems.

In [9] a recurrent neural network is introduced for the N-stage optimal control problem. The first step of the presented approach is reformulating the N-stage optimal control problem and then the gradient method is used for deriving the dynamics equation of the recurrent neural network. Although the approach enables obtaining real-time solutions it possesses two drawbacks. First, the rigorous mathematical analysis for the stability of the neural network lacks. Second, a neural network which combines the structure of the N-stage optimal control problem and a faster optimization method needs to be explored.

In [22] an approach for synthesis of optimal control for nonlinear systems, which incorporates the N-stage optimal control problem as well as least square support vector machines approach for mapping the state space into action space. SVM with radial basis function kernel are used. The solution is characterized by a set of nonlinear equations. An alternative formulation as a constrained nonlinear optimization problem in less unknowns is given, together with a method for imposing local stability in the LS – SVM control scheme. Advantages of LS – SVM control are that no number of hidden units has to be determined for the controller and that no centers have to be specified for the Gaussian kernels when applying Mercer’s condition. The curse of dimensionality is avoided in comparison with defining a regular grid for the centers in classical radial basis function networks. This is at expense of taking the trajectory of state variables as additional unknowns in the optimization problem, while classical neural network approaches typically lead to parametric optimization problems. In the SVM methodology the number of unknowns equals the training data, while in the primal space the number of unknowns can be infinite dimensional. A drawback of this approach is the large number of the unknowns.

In [5] the problem of multistage optimal control is considered. The problem is solved by using wavelet neural networks (WNN) as its capability for learning and generalization of functions are bigger. The control law is approximated by using WNN. The Langragian is constructed in order to from optimal control problem to come to optimization problem. A weight is introduced to regulate the balance between control system and its good performance by using WNN for mapping of the function from the state space into action space after which the optimal control is achieved. The value of the weight has effect on the simulation result.

In [19] an interactive fuzzy satisfying method for the solution of a multiobjective optimal for a linear distributed parameter system governed by heat conduction equation is suggested. In order to reduce the control problem to an approximate multiobjective linear programming problem a numerical integration formula is used and the suitable auxiliary variables are introduced. By considering the vague nature of the human judgment, the decision maker is assumed to have fuzzy goals for the objective functions. Having elicited the corresponding linear membership functions through the interaction with the decision maker, if the decision maker specifies the reference membership values, the corresponding Pareto optimal solution can be obtained by solving the minimax problems. Then a linear programming based interactive fuzzy satisfying method for deriving a satisfying solution for the decision maker efficiently from a Pareto optimal solution set is presented.

In [23] an approach for optimal control synthesis, in which a fuzzy neural network is used as a controller through simulation of the process of the controlled system is suggested.

In [1] the designing of a neural networks based regulator for nonlinear plants is considered. Both state and output feedback regulators with deterministic and stochastic disturbances have been investigated. A multilayered feedforward neural network has been employed as the nonlinear controller. The training of neural network utilizes the concept of so called “block partial derivatives”. The suggested approach may also be used for optimal control synthesis for plant with state and control constraints. In [7] a neural network based algorithm for a discreet constrained optimal control synthesis for nonlinear systems is presented.

In [25] a recurrent learning algorithm for optimal control synthesis for continuous dynamic systems is suggested. The designed controllers are in the form of unfolded recurrent neural networks. The proposed learning algorithm is characterized by its double–forward–recurrent–loops structure for solving both the temporal recurrent and the structure recurrent problems. The first problem is resulted from the nature of general optimal control problems, where the objective functions are often related (evaluated at) to some specific instead of all time steps or system states only. This causes missing learning signals at some time steps or system states. The second problem is due to the high-order discretization of the continuous systems by the Runge-Kutta method that is performed to increase the control accuracy. The discretization transforms the system into several identical subnetworks interconnected together, like a recurrent neural network expanded in the time axis. Two recurrent learning algorithms with different convergence properties are derived: the first- and second-order learning algorithms. The stability and the robustness of the designed controllers have to be studied in details.

In [24] a multilayered recurrent neural network is suggested for synthesizing linear quadratic optimal control systems by solving the algebraic matrix Riccati equation in real time. The suggested recurrent neural network consists of four bidirectionally connected layers. It is shown to be capable of solving the algebraic matrix Riccati equations, which enables synthesizing linear quadratic control systems in real time.

In [8] a new alternative for finding of the optimal control for discrete systems, which is based on using the continuous neural network of Hopfield (CNNH) is developed. The quadratic cost function is transformed into energy function of CNNH and the control is the output vector of CNNH. As CNNH works in parallel and in real time, the method may meet all the requirements for control in real time.

III. Conclusion

As a conclusion of the survey of the publication considering the optimal control synthesis problem for distributed parameter systems of parabolic type and also those discussing the utilizing neural networks in the optimal control synthesis problems it may be noted the following

- Neural networks enable optimal control synthesis in real time.;
- Important characteristics of the neural networks is their property to be universal function approximators but on the other hand the good approximation is hindered from the possibility of getting trapped in a local minimum;
- The problem for neural networks application for synthesis of optimal control for distributed parameter systems is not investigated entirely (as far as it is known to author only one approach is suggested – adaptive critic [12-16]);
- It is not pointed out definetely in the publications how stable is the suggested controller performance.

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