Calculation of GIS 110 kV Insulating Bushig Apllying Hybrid BEM-FEM Method

Hamid Zildžo[1] and Halid Matoruga[2]

Abstract **– This paper elaborates procedure of geometry optimising 110kV SF6 gas insulated bushing with regards to dielectric toils. Iterative Dirichlet-Neumann sequential procedure was applied in frame of modern hybrid BEM/FEM method.**

Keywords **– Boundary Element Method, Finite Element Method, Method of successive underrelaxation, Galerkin procedure.**

I. Introduction

Finite element method (FEM) is suitable for calculating domains with a number of insulating mediums but with final boundaries. Boundary Element Method (BEM) is suitable for calculating domains with one insulating medium but with final or infinite boundaries. In this paper, hybrid-coupled method of BEM-FEM boundary and final elements is applied which contains best characteristics from both FEM and BEM methods.

II. Mathematical Model

Let us elaborate separately the basic principles of Galerkins procedure of weight residue in FEM method, direct BEM method and iterative sequential Dirichle-Neumanns hybrid BEM-FEM method. With FEM, Laplaces partial differential equation is solved, and with BEM, integral equation of electrostatic field is solved. Potentials on BEM-FEM boundary are iteratively calculated by applying the method of successive sub-relaxation.

III. Finite Element Method

In the method of final elements, domain of observed physical system in procedure of so called bisection of continuum is divided on final number of parts of certain geometry, which are called final elements. Laplace's partial differential equation of electrostatic field is given with:

$$\frac{\partial}{\partial x}\left(\varepsilon\frac{\partial\varphi}{\partial x}\right)+\frac{\partial}{\partial y}\left(\varepsilon\frac{\partial\varphi}{\partial y}\right)=0 \qquad (1)$$

After applying the Galerkins method of weight residue we can record the solution of electrical potentials distribution in form of linear algebraic equations system:

$$[H]^{FEM}\cdot\{\varphi\}^{FEM}=\{Q\}^{FEM} \qquad (2)$$

where:

$[H]^{FEM}$ - Two-dimensional matrix of coefficients that's general clause is given with:

$$h_{ij}^{FEM}=\sum_{e=1}^{n_e}\left[\varepsilon\int_{S_\Delta^e}\left(\frac{\partial N_i^e}{\partial x}\frac{\partial N_j^e}{\partial x}+\frac{\partial N_i^e}{\partial y}\frac{\partial N_j^e}{\partial y}\right)dS\right]$$

$$(i=1,2,..,n_f;\ j=1,2,...,n_f) \qquad (3)$$

$\{\Phi\}^{FEM}$ - Column vector matrix of unknown potentials in articulations of one final element of rank n_f x 1.

$\{Q\}^{FEM}$ - Column vector matrix of free clauses containing Neumann's boundary conditions, which's general clause is given with:

$$q_i^{FEM}=\sum_{e=1}^{n_e}\left[\sum_{j=1}^{n_f}\left(\int_{S_\Delta^e}N_i^e\cdot N_j^e\cdot\frac{\partial\varphi_j^{FEM}}{\partial n}dS\right)\right] \qquad (4)$$

N_i^e -Functions of configurations which enable that unknown function of potentials is approximated as follows:

$$\varphi=\sum_{j=1}^{n_f}N_j^e\cdot\varphi_j^e \qquad (5)$$

$\frac{\partial\varphi_j^{FEM}}{\partial n}$ - Neumann's boundary condition.

IV. Direct Method of Boundary Elements

Mathematical model of direct method of boundary elements is based on Greens symmetrical identity and equations of continuity with which boundary conditions on boundaries between domains with different mediums are entered. Let us observe two special cases of 3-D electrostatic field calculations, with the case when observation point Q is located inside of calculating domain V and second case when point Q is located on domains boundary. General formula for observing potentials within, on the boundary and outside of calculation domain is given with the following formulation:

$$C(Q)\cdot\varphi(Q)+\int_S T(P,Q)\cdot\varphi(P)\cdot dS_P=\int_S G(P,Q)\cdot\frac{\partial\varphi(P)}{\partial n_P}\cdot dS_P \qquad (6)$$

[1]Hamid Zildžo is with the Electrical Engeneering Faculty Zmaja od Bosne bb 71000 Sarajevo, Bosna i Hercegovina
E-mail: hamid.zildzo@etf.unsa.ba
[2]Halid Matoruga is with the Electrical Engeneering Faculty Zmaja od Bosne bb 71000 Sarajevo, Bosna i Hercegovina
E-mail: hmatoroga@etf.unsa.ba, matorogahalid@yahoo.com

where:
$G(P,Q)$ - Greens function,
$T(P,Q)$ - Extraction of Greens function in perpendicular domain on boundary surface,
φ i $\frac{\partial\varphi}{\partial n}$ - Calculated functions of potentials and normal field component on boundary surface,
$C(P,Q)$ - Constant which depends on observation point is given with:

$$C(Q)=\begin{cases} 1 & \text{inside domain V (Poisson formula)} \\ \frac{1}{2} & \text{on smoothly boundary in } 2-\text{D and } 3-\text{D domain} \\ \frac{\gamma_{3-D}}{4\pi} & \text{on winding boundary in } 3-\text{D domain} \\ \frac{\gamma_{2-D}}{4\pi} & \text{on winding boundary in } 3-\text{D domain} \\ 0 & \text{out domain V} \end{cases} \tag{7}$$

After application of collocation procedure in point in method of weight residue on Eq. 6 we will get the solution in form of matrix system:

$$[H]^{BEM}\cdot\{\varphi\}^{BEM}=[G]^{BEM}\cdot\left\{\frac{\partial\varphi}{\partial n}\right\}^{BEM} \tag{8}$$

where:
$[H]^{BEM}$ – Two dimensional matrix of coefficient that's general clause is given with:

$$h_{i,j}^{BEM}=\sum_{e=1}^{n_e}\int_S N_j^e\cdot T_{i,j}^e\cdot dS_p+\delta_{i,j}\cdot C_i$$
$$(i=1,2,..,n_e\ j=1,2,...,n_e)\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots \tag{9}$$

$[G]^{BEM}$ – Two dimensional matrix of coefficient that's general clause is given with:

$$g_{i,j}^{BEM}=\sum_{e=1}^{n_e}\int_S N_j^e\cdot G_{i,j}^e\cdot dS$$
$$(i=1,2,..,n_e\ j=1,2,...,n_e) \tag{10}$$

$\{\varphi\}^{BEM}$ i $\left\{\frac{\partial\varphi}{\partial n}\right\}^{BEM}$ - Column vector matrix of variables.

On domain's boundary with one medium, in every articulation of boundary elements, the value of variable φ or $\partial\varphi/\partial n_p$ known. So in Eq. 8 calculation is conducted on boundary domain of the variable φ or $\partial\varphi/\partial n_p$ which is not given as boundary condition.

On boundary between two domains with different mediums, unknown are φ and $\partial\varphi/\partial n_p$ as well. In that case, for each domain boundary the system of Eq. 6 is written with consideration of Dirichlet's and Neumann's boundary conditions, and on boundaries between two domains with different mediums additional continuity equations are written for φ and $\partial\varphi/\partial n_p$, which are valid on these boundaries.

V. Hybrid BEM-FEM method

Let us observe example of calculation with related BEM-FEM domain, on Fig. 1.

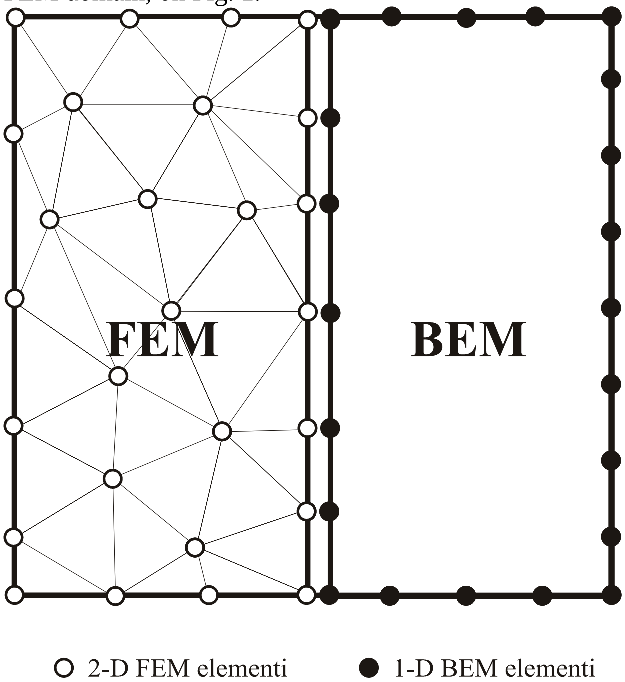

Fig. 1. Example of related BEM-FEM domain calculation

FEM domain is bisected with 2-D FEM final elements whose articulations are marked with white articulations, and BEM domain with 1-D boundary elements whose articulations are marked with black articulations. BEM-FEM boundary from FEM side is connected with 2-D final elements, and from BEM side is connected with 1-D boundary elements.

There are direct and iterative algorithms for connecting the methods of boundary and final elements.

With direct approach, forming of linear system of algebra equations is conducted with the aid of exponent (2) in FEM domain and exponent (8) in BEM domain, and equations of continuity are added on BEM-FEM boundary. This procedure has a large flaw because it is necessary to solve large full equations system.

For the sake of memory saving it is recommended to use some of iterative procedures, and the best known are:
- Robbins's relaxation algorithm for connecting.
- Neumann – Neumann's algorithm for connecting.
- Advanced Dirichlet – Neumann's algorithm.
- Advanced sequential Dirichlet – Neumann's algorithm.

With iterative BEM-FEM procedures, separated solving of two separated matrix system of linear equations is conducted separately for BEM and separately for FEM domain, and the results of potentials' distribution or flux on BEM-FEM boundary is iteratively calculated by applying successive sub-relaxation method.

VI. ADVANCED SEQUENTIAL DIRICHLET–NEUMANN'S BEM-FEM ALGORITHM

In this paper advanced [4]sequential Dirichlet-Neumann's BEM-FEM algorithm will be used which consists of following steps .

- Dividing the calculating domain on BEM and FEM domains
- Defining of starting values of potentials on boundary of BEM- FEM
- Starting the iterative cycle which lasts until satisfying the convergence terms:

DO n=0, 1, 2… to convergence

- Solving the field in BEM domain:

On BEM domain boundary other than on BEM-FEM boundary itself, Dirichlet or Neumann's boundary terms are inflicted. Accordingly, in matrix system (8) in matrix $[H]^{BEM}$ and $[G]^{BEM}$ we can write BEM-BEM contributions from pure BEM boundary and BEM-FEM contributions from BEM-FEM boundary.

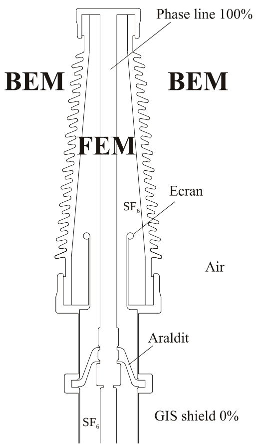

Fig. 2. Example of calculation of 110 kV insulating bushing in SF6 GIS station

$$[H]^{BEM} \begin{Bmatrix} \{\varphi\}_{n+1}^{BEM-BEM} \\ \{\varphi\}_{n+1}^{BEM-FEM} \end{Bmatrix} = [G]^{BEM} \begin{Bmatrix} \left\{\dfrac{\partial\varphi}{\partial n}\right\}_{n+1}^{BEM-BEM} \\ \left\{\dfrac{\partial\varphi}{\partial n}\right\}_{n+1}^{BEM-FEM} \end{Bmatrix} \quad (11)$$

In the system (11) it is necessary to consider Dirichlet's and Neumann's boundary terms including potential values on BEM-FEM boundary from previous iteration step, as well. Solving the system (11) will give us the values of normal component of fields on BEM-FEM boundary $\left\{\dfrac{\partial\varphi}{\partial n}\right\}_{n+1}^{BEM-FEM}$

- In this step, solving of continuity equation on BEM-FEM boundary is conducted:

$$\begin{aligned} \varepsilon^{FEM}\left\{\frac{\partial\varphi}{\partial n}\right\}_{n+1}^{FEM-BEM} &= -\varepsilon^{BEM}\left\{\frac{\partial\varphi}{\partial n}\right\}_{n+1}^{BEM-FEM} \\ \left\{\frac{\partial\varphi}{\partial n}\right\}_{n+1}^{FEM-BEM} &= -\frac{\varepsilon^{BEM}}{\varepsilon^{FEM}}\left\{\frac{\partial\varphi}{\partial n}\right\}_{n+1}^{BEM-FEM} \end{aligned} \quad (12)$$

As a result we will get Neumann's boundary terms which are relevant in perpendicular direction on FEM side of BEM-FEM boundary .

- Solving the field in FEM domain:

Solving the field in FEM domain is conducted in this step. On FEM domain boundary except on the BEM-FEM boundary itself, Dirichlet's or Neumann's boundary terms are given. Accordingly, in matrix system (2) in matrix $[H]^{FEM}$ we can write FEM-FEM contributions from pure FEM boundary and contributions from FEM-BEM from FEM-BEM boundary:

$$[H]^{FEM} \cdot \begin{Bmatrix} \{\varphi\}_{n+1}^{FEM-FEM} \\ \{\varphi\}_{n+1}^{FEM-BEM} \end{Bmatrix} = \begin{Bmatrix} \{Q\}_{n+1}^{FEM-FEM} \\ \{Q\}_{n+1}^{FEM-BEM} \end{Bmatrix} \quad (13)$$

where the matrix elements $\{Q\}_{n+1}^{FEM}$ are calculated by applying (4) and (12) formulation. As a result we will get potentials $\{\varphi\}_{n}^{FEM-BEM}$ on FEM-BEM boundary.

- Correction of calculated potentials on BEM-FEM boundary:

In this step correction of calculated potentials that are calculated on FEM-BEM boundary in previous step is conducted. Correction is conducted by applying the method of successive sub-relaxations:

$$\{\varphi\}_{n+1}^{BEM-FEM} = (1-\theta)\cdot\{\varphi\}_{n}^{BEM-FEM} + \theta\cdot\{\varphi\}_{n+1}^{FEM-BEM} \quad (14)$$

Sub-relaxation factor θ is given in interval from 0 to 1.

- Checking the convergence of iterative cycle and stopping when suitable punctuality is achieved.

VII. EXAMPLE OF CALCULATION

Fig. 2 presents example of calculation of the electrostatic field on 110kV SF_6 GIS of insulating bushing. Bushing is usually located on top of SF_6 gas insulated station (GIS) and through it an air 110kV phase line is introduced into SF_6 GIS bus bar. It is very important to correctly optimise the geometry of this bushing considering the dielectric tensions.

Copper bus bar goes trough porcelain insulator and goes into the GIS bus bar. Interior of porcelain insulator and GIS bus bar is filled with SF_6 gas. Dielectric permitivity of porcelain is ε_r=5, and SF_6 gas ε_r=1. Araldit support insulator supports bus bar and it has ε_r=4. Permitivity of outside air is ε_r=1. Phase line is located on up thrown 100 % potential, and cover of GIS bus bar is grounded with 0% potential. Various insulating mediums SF_6 gas, araldit and porcelain with final boundaries are bisected with 2 – D final elements. Surrounding air whose boundaries are reaching the infinity represents BEM domain, and is bisected with 1 – D boundary elements.

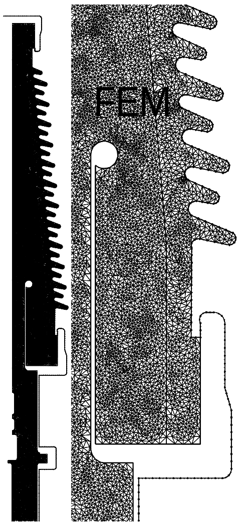

Fig. 3. Generated network of final and boundary elements

Fig. 4 shows the results of calculated distribution of dielectric tensions in observed bushing. Results are derived by applying BEM-FEM computer programme

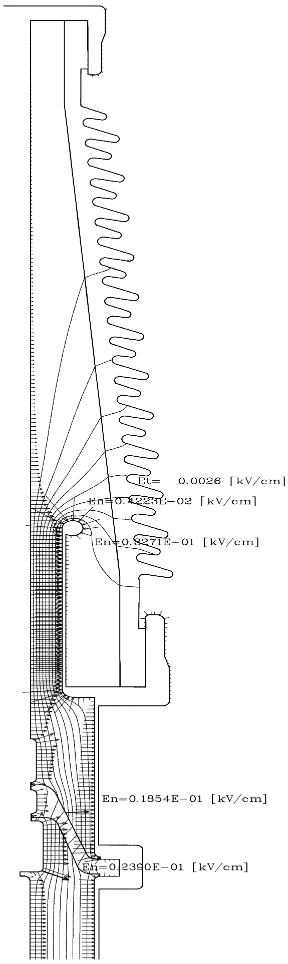

Fig. 4. Calculation results of maximum tangential and normal components of electric field bushing

VIII. CONCLUSION

In this paper modern approach to optimising geometry of conductor bushing is shown regarding the dielectric tensions. Infinite boundaries of air are taken into consideration with establishing BEM-FEM boundary and on outer bushing surface itself. Series of calculations was made and final version of bushing is shown. In final version auxiliary display is built in which serves to stretch optimally the line force fields over the surface of porcelain insulator to satisfy maximum allowed values of normal and tangential components of electric field in all insulating mediums, individually.

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