Removal of Power-line Interference from ECG in Case of Non-multiple Even Sampling

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***Abstract* –This paper deals with some aspects of the subtraction procedure, which removes the power-line interference without affecting the intrinsic to ECG components. The improvement is for the cases of high even non-multiplicity between sampling rate and rated interference frequency.**

***Keywords* –ECG, power-line interference removal, subtraction procedure.**

I. INTRODUCTION

The ECG recordings are often contaminated by residual power-line interference despite the high common mode rejection ratio of the amplifiers used [1, 2] and the variety of sophisticated but conceptually traditional digital filters [3, 4], which suppress to different extent the intrinsic to ECG components around the power-line (PL) frequency. This drawback has been overcome some decades ago by the so called subtraction procedure [5]. Its principle consists of: i) applying linear phase digital filter on 'linear' ECG segments with near to zero frequency content (usually physiological baseline, low amplitude P-waves and some small parts of T-waves), ii) continuously updating and memorizing the removed phase locked interference components, and iii) subsequent subtracting the corresponding component from the signal wherever non-linear segments are encountered. Later, many improvements of the procedure have been developed to cope with PL amplitude and frequency variations including the cases of non-multiple sampling, which lead to a real (non-integer) number n of samples within one rated PL period [6-8].

The aim of this study is to enhance the accuracy of the PL interference (PLI) elimination when the truncated real number n^* is even and the multiplicity (sampling rate Φ against interference frequency F) is high.

II. THEORETICAL CONSIDERATIONS, EQUATIONS, EXPERIMENTAL RESULTS

According to the generalized structure of the subtraction procedure [7, 8], the phase locked interference B_i to be subtracted from the ongoing contaminated sample Y_i is estimated by means of an interference temporal buffer $[i\text{-}n^*, i\text{-}1]$. Its terms $B_{i-1}, B_{i-2},\ldots, B_{i-k},\ldots B_{i-n}$ represent filtered middle samples of a moving window over the contaminated sequences $X_{i-k-n-1}, X_{i-k-n-2},\ldots X_{i-k-1}$. When Φ is multiple to F, B_i takes the B_{i-n} value. Otherwise, B_i is estimated by the buffer content, which is processed by additional filter type 'moving averaging' with transfer coefficient K_{FB} for $f=F$. The corresponding equations are:

$$\frac{1}{n^*}\sum_{j=-n^*+1}^{0} B_{i+j} = B_{mid}K_{FB}\,; \tag{1}$$

$$K_{FB} = \frac{1}{n^*}\cdot\frac{\sin\dfrac{n^*\pi F}{\Phi}}{\sin\dfrac{\pi F}{\Phi}}\,; \tag{2}$$

$$B_i = B_{mid}n^*K_{FB} - \sum_{j=-n^*+1}^{-1} B_{i+j}\,. \tag{3}$$

Here $\mathbf{K} \equiv \frac{1}{n^*}\sum_{j=-n^*+1}^{0} B_{i+j}$ is the so called *K-filter* [7, 8]. It is low pass type with coefficient vector $\vec{\mathbf{K}}$ consisting of n^* terms with equal weight $1/n^*$. This filter is transformed in high pass type by subtracting from B_{mid}: $\mathbf{B} \equiv B_{mid} - \frac{1}{n^*}\sum_{j=-n^*+1}^{0} B_{i+j}$. It is called *B-filter* [7, 8] and has $1-K_{FB}$ transfer coefficient in $f = F$. The next modification denoted *B*-filter* [7, 8] is expressed by $\mathbf{B^*} \equiv \mathbf{B}/1-K_{FB}$ or $\mathbf{B^*} \equiv \dfrac{-\frac{1}{n^*}\sum_{j=-n^*+1}^{0} B_{i+j} + B_{mid}}{1-K_{FB}}$ with transfer coefficient equal to 1 at $f = F$, which is in fact Eq. 1.

When n^* is odd, $n^*=2m+1$, B_{mid} coincides in time with the real $B_{i-(n-1)/2}$ and Eq. 3 becomes

$$B_i = -\sum_{j=-n^*+1}^{-\frac{n^*+1}{2}} B_{i+j} + \left(n^*K_{FB}-1\right)B_{i-(n^*-1)/2} - \sum_{j=-\frac{n^*-3}{2}}^{-1} B_{i+j}\,. \tag{4}$$

Fig. 1 represents the *B*-filter* synthesis for PLI evaluation in case of Φ=250 *Hz*, and F=48 *Hz* and F=52 *Hz*, both of them with odd multiplicity n=5. The traces are obtained in MATLAB environment by the filter vector coefficients: $\vec{\mathbf{K_B}} = [1\ \ 1\ \ 1\ \ 1\ \ 1]/5$; $\vec{\mathbf{B}} = [-1\ \ -1\ \ 4\ \ -1\ \ -1]/5$;

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$$\overrightarrow{\mathbf{B}^*} = \begin{bmatrix} -1/5 & -1/5 & 4/5 & -1/5 & -1/5 \end{bmatrix} / \left(1 - K_{FB}\right).$$

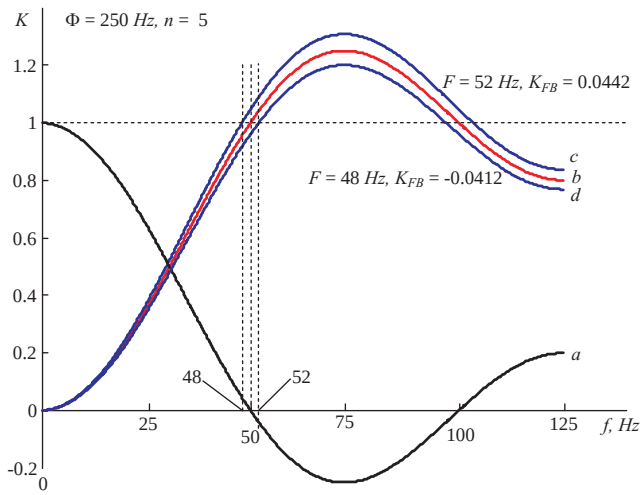

Fig. 1. *B*-filter* synthesis: *a*- basic *K-filter; b- B-filter; c* and *d-B*-filters* for F = 48 *Hz* and F = 52 *Hz* (Φ = 250 *Hz*)

The removal of PLI with F=52 *Hz* is shown in Fig. 2. The error does not exceed ±25 μ*V*.

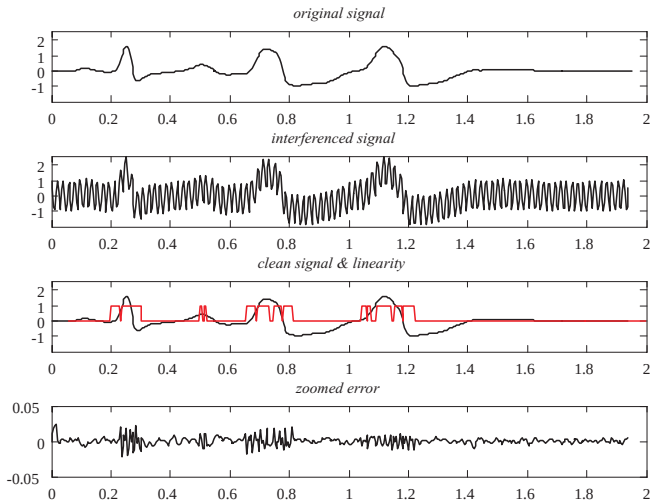

Fig. 2. Φ = 250 *Hz*, F = 52 *Hz*, odd multiplicity n = 5

If n^* is even, $n^*=2m$, B_{mid} is virtual and does not coincide with a real buffer sample being between $B_{i-n/2}$ and $B_{i-n/2+1}$, which are spaced at $\tau=1/2\Phi$ as shown in Fig. 3.

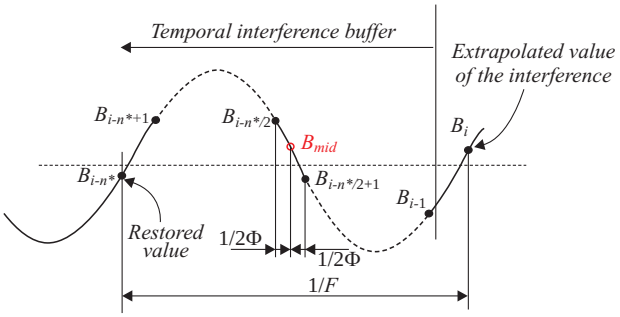

Fig. 3. Content of the temporal buffer in even multiplicity

Since the buffer has no constant component, the next Eqs. are derived assuming the B_{mid} phase is zero.

$$\left| \begin{array}{l} B_{i-n^*/2} = A\sin\omega(t - \frac{1}{2\Phi}) = A\sin\omega t.\cos\frac{\omega}{2\Phi} - A\cos\omega t.\sin\frac{\omega}{2\Phi} \\ B_{mid} = A\sin\omega t \\ B_{i-n^*/2+1} = A\sin\omega(t + \frac{1}{2\Phi}) = A\sin\omega t.\cos\frac{\omega}{2\Phi} + A\cos\omega t.\sin\frac{\omega}{2\Phi} \end{array} \right. . \quad (5)$$

The following expression is obtained substituting B_{mid} for $A\sin\omega t$ in the sum of the first and third Eqs.:

$$B_{mid} = \frac{B_{i-n^*/2} + B_{i-n^*/2+1}}{2S_C}, \quad S_C = \cos\frac{\pi F}{\Phi}. \quad (6)$$

Eq. 1 becomes $\frac{B_{i-n^*/2} + B_{i-n^*/2+1}}{2S_C} K_{FB} = \frac{1}{n^*} \sum_{j=-n^*+1}^{0} B_{i+j}$, resulting in the following expressions for extrapolated value B_i, *B*-filter* and *B-filter*:

$$\begin{aligned} B_i = &-\sum_{j=-n^*+1}^{i-n^*/2-1} B_{i+j} + B_{i-n^*/2}\left(\frac{n^*}{2}\frac{K_{FB}}{S_c} - 1\right) + \\ &+ B_{i-n^*/2+1}\left(\frac{n^*}{2}\frac{K_{FB}}{S_c} - 1\right) - \sum_{j=-n^*/2+2}^{-1} B_{i+j} \end{aligned} . \quad (7)$$

$$\mathbf{B}^* \equiv \frac{B_{i-n^*/2} + B_{i-n^*/2+1}}{2S_C} = \mathbf{B} \big/ 1 - K_{FB}. \quad (8)$$

$$\begin{aligned} \mathbf{B} \equiv &-\frac{1}{n^*}\sum_{j=-n^*+1}^{-n^*/2-1} B_{i+j} + \left(\frac{1}{2S_C} - \frac{1}{n^*}\right) B_{i-n^*/2} + \\ &+ \left(\frac{1}{2S_C} - \frac{1}{n^*}\right) B_{i-n^*/2+1} - \frac{1}{n^*}\sum_{j=-n^*/2+2}^{0} B_{i+j} \end{aligned} . \quad (9)$$

Fig. 4 represents the *B*-filter* synthesis for PLI evaluation according to Eq. 7 in case of Φ = 250 *Hz* and F = 60 *Hz* (even multiplicity $n^* = 4$. The filter vector coefficients are: $\overrightarrow{\mathbf{K_B}} = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} / 4$;

$\vec{\mathbf{B}} = \begin{bmatrix} -1 & 2/S_C - 1 & 2/S_C - 1 & -1 \end{bmatrix} / 4$, $S_C = 0{,}729$;

$\overrightarrow{\mathbf{B}^*} = \begin{bmatrix} -1 & \frac{2}{S_C} - 1 & \frac{2}{S_C} - 1 & -1 \end{bmatrix} \frac{1 - K_{FB}}{4}$, $K_{FB} = 0{,}0458$.

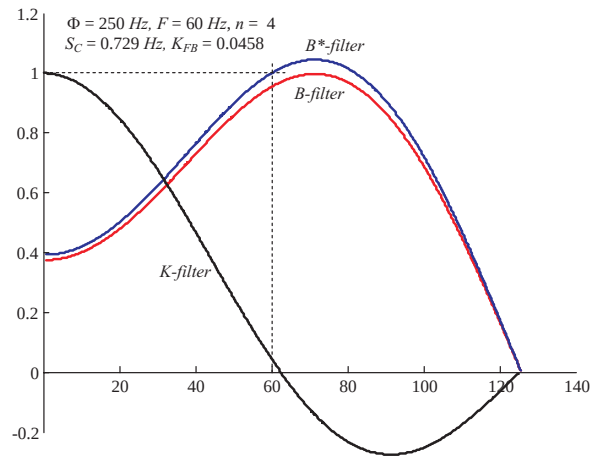

Fig. 4. *B*-filter* synthesis for F = 60 *Hz* and Φ = 250 *Hz*

Experimental results with the synthesized *B*-filters* of Fig. 4 can be observed in Fig. 5. The traces are as shown in Fig. 2.

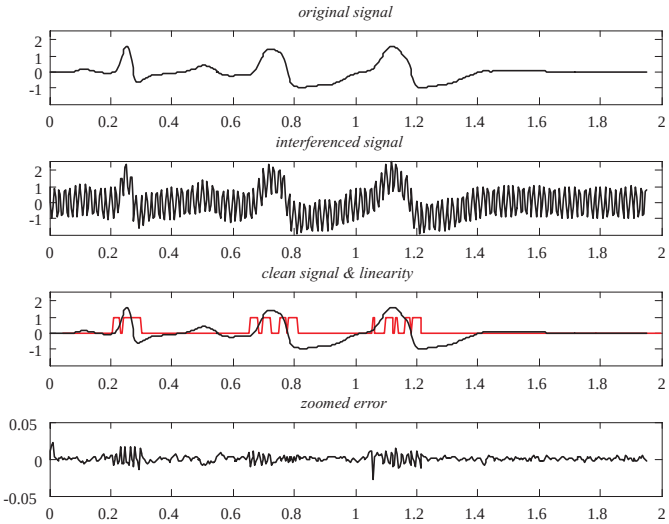

Fig. 5. Φ = 250 *Hz*, *F* = 60 *Hz*, even multiplicity *n* = 4

Similar results have been obtained with Φ = 500 *Hz* and *F* = 60 *Hz* (even multiplicity *n** = 8).

III. Investigation of the Subtraction Procedure Stability in Non-Linear ECG Segments

These investigations are challenged by colleagues of the Cairo University, who experimented PLI removal in case of even multiplicity with very high truncated real number *n**, using even B-filter identical to the *K-filter* according to publication [8]. It was found that the procedure is prone to autoexcitation if the relative weight of the sample to be compensated in the *B-filter* equation. is in presence of low frequency components within the temporal buffer. For a longer non-linear segment the *B-filter* transforms into IIR filter. This process was study in the following way:

1. Low frequency signal epoch of 4 *Hz*, 200 μ*V* amplitude and 2 *s* duration is synthesized.
2. Synthesized PLI with 200 μ*V* amplitude is added.
3. The subtraction procedure is applied with preset flag for linear segment during the first half of the epoch, this flag being for non-linear segment until the epoch end.
4. The error committed (difference between the free of interference signal and the processed one) is analyzed.

The next figures illustrate the results obtained. All abscissas are in *s*, the ordinates are scaled in *m*V. The *K-filter* is marked as K_B*-filter* everywhere it is used to build the *B*-filter* according to [8].

Experiment 1 (Fig. 6): B_i is calculated using Eq. 7. The applied *K-filter* is $\vec{\mathbf{K}} = [1/2 \quad 1 \quad 1 \quad 1 \quad 1/2]/4$. Another filters are the same for the shown in Fig. 4 synthesis. The traces are the same as in Fig. 5. No autoexcitation can be observed, the filter is stable for infinite duration of the non-linear signal (see the forth trace). Further, Figs. 7-11 consist of filtered signal and zoomed error graphic only.

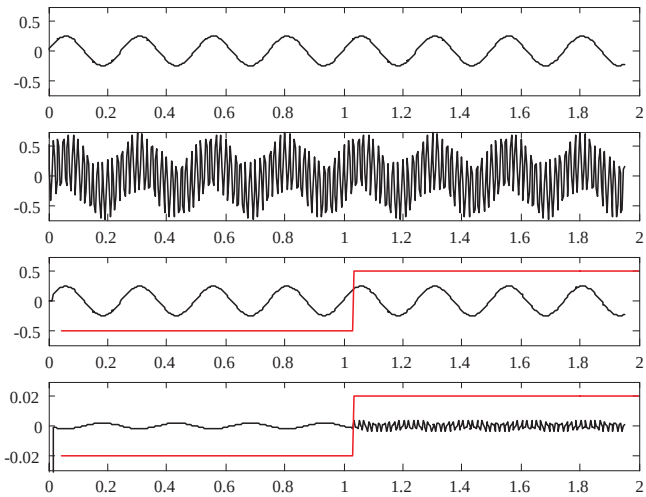

Fig. 6. Φ = 250 *Hz*, *F* = 60 *Hz*, even multiplicity *n*=4, B_i is calculated from Eq. 7.

Experiment 2 (Fig. 7): An even K_B*-filter* is applied which is the same as a *K-filter* for even multiplicity. B_i is calculated by Eq. 7:

$$B_i = -B_{i-n^*} - 2\sum_{j=-n^*+1}^{-n^*/2-1} B_{i+j} - \\ -2\left(1-n^*K_{FB}\right)B_{i-n^*/2} - 2\sum_{j=-n^*/2+1}^{-1} B_{i+j} \quad . \qquad (10)$$

The filter vectors are: $\vec{\mathbf{K}} = [1/2 \quad 1 \quad 1 \quad 1 \quad 1/2]/4$; $\vec{\mathbf{K_B}} = [1/2 \quad 1 \quad 1 \quad 1 \quad 1/2]/4$; $\vec{\mathbf{B^*}} = [-1/2 \quad -1 \quad 3 \quad -1 \quad -1/2]/4/(1-K_{FB})$, K_{FB}= 0,0334.

Autoexcitation can be observed almost immediately after the nonlinear segment begins. That is due to residual components in the temporal buffer. Additional experiment was made with free of low frequency input signal (the lower graphic). The autoexcitation occurs 300 *ms* later because of computing error accumulation.

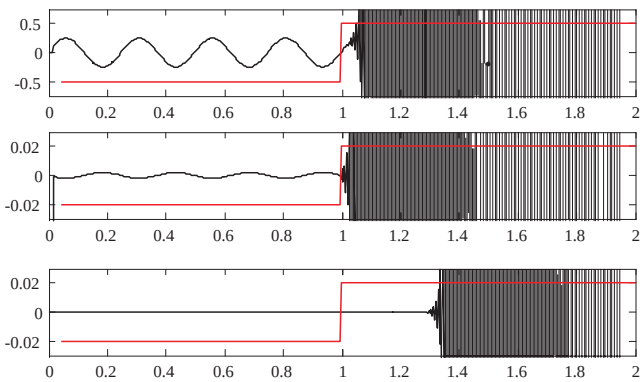

Fig. 7. Φ = 500 *Hz*, *F* = 60 *Hz*, even multiplicity *n* = 4

Experiment 3 (Fig. 8): Reduced by 2 even *B*-filter* is applied, other data as in ***Experiment 2***. The filter vectors are: $\vec{\mathbf{K}} = [1/2 \quad 1 \quad 1 \quad 1 \quad 1/2]/4$; $\vec{\mathbf{K_B}} = [1/2 \quad 0 \quad 1 \quad 0 \quad 1/2]/2$; $\vec{\mathbf{B^*}} = [-1/2 \quad 0 \quad 1 \quad 0 \quad -1/2]/2/(1-K_{FB})$, K_{FB} = 0,0039.

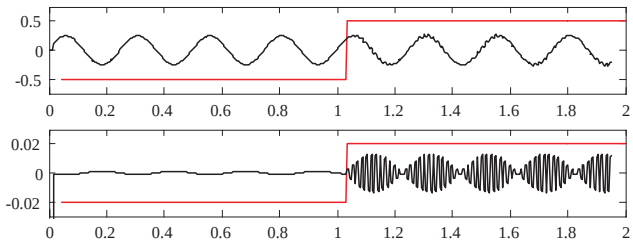

Fig. 8. Reduced by 2 even *B*-filter*

Experiment 4 (Fig. 9): The same as ***Experiment 2*** but with reduced by 2 even *K-filter* for linear segments Filter vectors: $\vec{\mathbf{K}} = [0{,}5 \quad 0 \quad 1 \quad 0 \quad 0{,}5]/2$; $\vec{\mathbf{K_B}} = [0{,}5 \quad 0 \quad 1 \quad 0 \quad 0{,}5]/2$; $\vec{\mathbf{B*}} = [-0{,}5 \quad 0 \quad 1 \quad 0 \quad -0{,}5]/2/(1-K_{FB})$, $K_{FB} = 0{,}0039$.

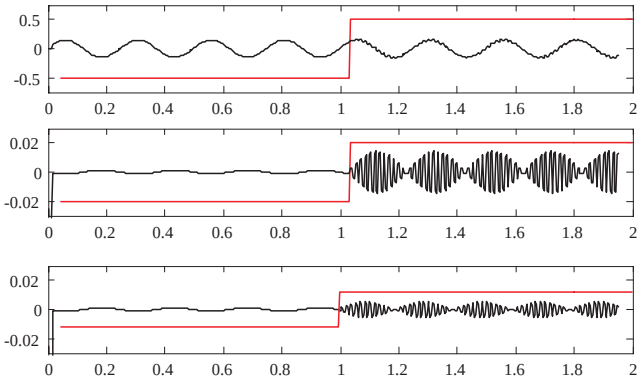

Fig. 9. Reduced by 2 *B*-filter* and reduced by 2 even *K-filter*.

The lower graphic is obtained setting the flag for non-linear segment when the sinusoid is going through zero (minimum non-linearity). The error is considerably reduced.

Experiment 5 (Fig. 10): The *B*-filter* is synthesized by odd K_B*-filter*. n^* is computed using $n^* = 2 \times floor\left(\frac{\Phi}{2F}\right) + 1$, where *floor*(*a*) is MATLAB function generating the lower integer of *a*. Filter vectors: $\vec{\mathbf{K}} = [1/2 \quad 1 \quad 1 \quad 1 \quad 1/2]/4$; $\vec{\mathbf{K_B}} = [1 \quad 1 \quad 1 \quad 1 \quad 1]/5$; $\vec{\mathbf{B*}} = [-1 \quad -1 \quad 4 \quad -1 \quad -1]/5/(1-K_{FB})$, $K_{FB} = -0{,}1717$.

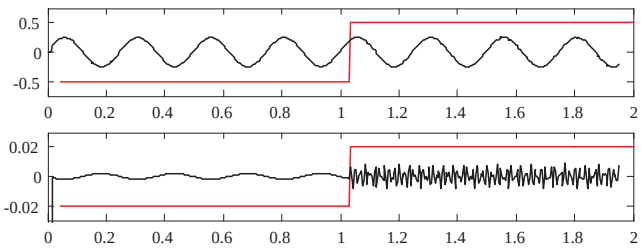

Fig. 10. Even *B*-filter* based on odd K_B*-filter*.

Experiment 6 (Fig. 11): The same as ***Experiment 5*** but with even *K-filter* for linear segments. Filter vectors: $\vec{\mathbf{K}} = [1 \quad 1 \quad 1 \quad 1 \quad 1]/5$; $\vec{\mathbf{K_B}} = [1 \quad 1 \quad 1 \quad 1 \quad 1]/5$; $\vec{\mathbf{B*}} = [-1 \quad -1 \quad 4 \quad -1 \quad -1]/5/(1-K_{FB})$, $K_{FB} = -0{,}1717$.

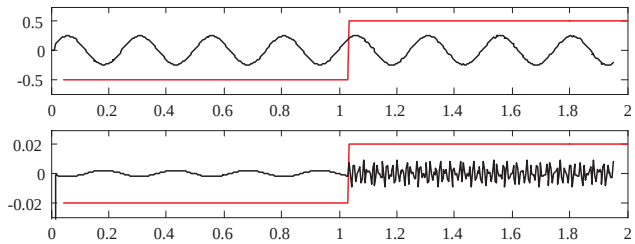

Fig. 11. Reduced by 2 even *B*-filter* and even *K-filter* for linear segments.

IV. Discussion and Conclusion

The presented experiments are a significant excerpt only of all carried out. They lead to the following conclusions:

1. Except for the even *B-filter* used in ***Experiment 2***, all other filters are stable for infinite duration of the non-linear ECG signal.
2. The approximation procedure is negligibly influenced by the *K-filter* type intended for PLI removal from the linear ECG segments. This may be seen comparing the couples ‘***Experiment 3 – Experiment 4***’ and ‘***Experiment 5 – Experiment 6***’
3. The approximation error considerably depends on the residual low frequency components within the temporal buffer (see ***Experiment 4***).
4. Out of all analyzed *B-filters*, the approximation procedure using Eq. 7 is that one that is most efficient for PLI removal from the non-linear ECG segments in case of even multiplicity.

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