# Recovering Optical Image Transferred Through Atmospheric Turbulence 

Kalin L. Dimitrov ${ }^{1}$


#### Abstract

In this paper we study model-based restoration of long exposure space-to-ground images. Such images may show strong degradation due to non-linear atmospheric turbulence effects and therefore, any efficient restoration method must take into account modeling of the turbulence. The paper deals with the restoration of one dimensional image with least squares methods applied to our turbulence model.


Keywords - Atmospheric turbulence, Least-squares methods, Restoration of images

## I.Introduction

The resolution of images is determined by the processes of diffraction. Recently, practical algorithms have emerged that are capable of recovering spatial frequency detail that lies beyond the diffraction limit of an image sensor [1]. In this paper we will briefly review some application of our previous works [2,3]. We will present a least-squares method based algorithm that achieves reconstruction of diffraction-limited one dimensional images [4,5]. Examples of simulations of the algorithm will be given, along with a discussion of how the algorithm can be integrated in software applications.

## II. Solution of the inverse observing PROBLEM

## A. Turbulence model

We will use as base analytical results for average optical intensity from our previous works [2,3]:

$$
\begin{aligned}
& \langle I(x)\rangle=\left\langle I_{0}(x)\right\rangle \exp \left(-\gamma^{2}\right)+\frac{a L_{m}}{\sqrt{2} f} \times \\
& \quad \times \sqrt{\frac{\frac{2}{\eta}}{1+\frac{2}{\eta}\left(1+r^{2}\right)}\left[1-\exp \left(\gamma^{2}\right)\right] \times} .
\end{aligned}
$$

[^0]\[

$$
\begin{equation*}
\times \exp \left[-2 \frac{\frac{2}{\eta}\left(1+r^{2}\right)}{1+\frac{2}{\eta}\left(1+r^{2}\right)} \frac{x^{2}}{x_{0}^{2}}\right] \tag{1}
\end{equation*}
$$

\]

where

$$
\begin{equation*}
\left\langle I_{0}(x)\right\rangle=\frac{a L_{m}}{\sqrt{2} f} \sqrt{\frac{1}{1+r^{2}}} \exp \left(-2 \frac{x^{2}}{x_{0}^{2}}\right) \tag{2}
\end{equation*}
$$

and

$$
r=\frac{2 z}{k a b}, x_{0}=\sqrt{1+r^{2}} x_{b}, x_{b}=\frac{f b}{z}
$$

$a$ - aperture scale, $b$ - investigation object scale, $L_{m}$ maximal brightness, $f$ - focal length, $Z$ - distance to object.

In most of the cases $r^{2} \ll 1$, so we assume

$$
\begin{equation*}
\sqrt{1 /\left(1+r^{2}\right)} \approx 1 \tag{3}
\end{equation*}
$$

Using (1), (2) and (3) and making some normalization we derive

$$
\begin{array}{r}
\langle I(x)\rangle_{n}=\exp \left(-2 \frac{x^{2}}{x_{0}^{2}}\right) \exp \left(-\gamma^{2}\right)+ \\
+\sqrt{\frac{\frac{2}{\eta}}{1+\frac{2}{\eta}}\left[1-\exp \left(\gamma^{2}\right)\right] \exp \left[-2 \frac{\frac{2}{\eta}}{1+\frac{2}{\eta}} \frac{x^{2}}{x_{0}^{2}}\right]} \tag{4}
\end{array}
$$

We accept that we have information about atmospheric turbulence (for example from SODAR, SCIDAR [6] or LIDAR etc.). This mean that functions $\gamma=\sqrt{\pi} C_{n}^{2} k^{2}\langle l\rangle^{5 / 3} Z$
and
$\eta=\ln \left[\left(\exp \left(-\gamma^{2}\right)-1\right)\left(\exp \left(\gamma^{2} / e^{2}\right)-1\right)\right]$ are known.

## B. Formulation of analysis

The formulation is shown on fig.1.


Fig. 1. Formulation of analysis

Our main goal is to find scale $x_{0}$ (see (4)). If we know $x_{0}$ we have possibility simply to calculate $\left\langle I_{0}\right\rangle$ from (2) i.e. initial image distribution.

## C. Least square solution

The method of least squares provides a means of estimating the values of coefficients in an equation. Typically, the estimates are based upon some sample of data. The idea of least squares is to minimize the total amount of error due to these estimates. Note that the technique of least squares is different from linear regression, though they have similar objectives [5].

We implement least squares method in way

$$
\begin{equation*}
\sum_{i}^{n}\left[y_{i}-\varphi\left(x_{i} ; a\right)\right]^{2}=\min \tag{5}
\end{equation*}
$$

where $y_{i}$ correspond to experimental data of $\langle I(x)\rangle_{N}$ (derived from $n$-sized CCD matrix for example); $\varphi\left(x_{i} ; a\right)$ correspond to theoretical $\langle I(x)\rangle_{N}$ (see formula (4)); parameter $a$ equivalent to $x_{0}$.
Because we have only one parameter in minimization we use the first derivative case

$$
\begin{equation*}
\left.\sum_{i}^{n}\left[y_{i}-\varphi\left(x_{i} ; a\right)\right] \frac{\partial \varphi}{\partial a}\right|_{x=x_{i}}=0 \tag{6}
\end{equation*}
$$

Now we derive the first derivative of (4)

$$
\frac{\partial \varphi}{\partial a}=4 \frac{c_{1} x^{2} \exp \left(-2 \frac{x^{2}}{a^{2}}\right)}{a^{3}}+\frac{4 c_{2} c_{3} x^{2} \exp \left(-2 \frac{c_{3} x^{2}}{a^{2}}\right)}{a^{3}}
$$

where $c_{1}, c_{2}, c_{3}$ are known constants (see (4)).
We put (7) and (4) in (6) and we derive the final equation to solve. The root of this equation is $a$ (see (5) and (6)) respectively $x_{0}$. We calculate this equation with numerical methods (not discussed here).

## D. Numerical example

In our simulations we found that error is less than $1 \%$ in 100 points example in lack of noise.

## III. Additional Remarks

This is first paper of solution of so called inverse restoration problem using previously developed turbulence model [1,2].

## IV. Conclusion

In this paper we solve numerically the problem of restoration of turbulence degraded images using previously developed turbulence model. Next step is to investigate sum of signal with noise (for example Gaussian).

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[^0]:    ${ }^{1}$ Kalin L. Dimitrov is with the Technical University of Sofia, Faculty of Communication Technics and Technologies, Kl.Ohridski Blvd, 1756 Sofia, Bulgaria, E-mail: kld@tu-sofia.bg

