Comments on the Method "Over Sampling and Averaging for Additional Bits of Resolution"

Petre Tzv. Petrov[1]

Abstract - **There is critical comments on the method "over sampling and averaging for additional bits of resolution". The method is applied for "adding bits" to the analog to digital converter (ADC) and for "improving the ADC resolution". It is shown that the results achieved by this method are unreliable, non-repetitive and the method "over sampling and averaging for additional bits" must be revised. The sampling theory is a victim of an oversimplification, overgeneralization and over sampling.**

Keywords - **over sampling and averaging for "additional bits"**

I. Introduction

It is claimed in [1, 2] that with method "over sampling and averaging" (OA) additional bits of information are added to the bits generated from analog to digital converter (ADC). In order to clarify the situation a research was done as follows: 1. The method OA was studied and applied. The results were analyzed. 2. The terms "accuracy" and "resolution" were compared. 3. The application of these terms from different manufacturer was compared [3, 4]. 4. The maximal amplitude error was evaluated when the simplest band wide signal (SBLS) is sampled [5]. 5. The method was applied for external and internal ADC and digital to analog converters (DAC). 6. The conclusions were made.

In fact the method OA consists of two parts. The first part is an increasing the sampling rate (This is useful up to certain limit. In general the errors are reduced). The second part. is averaging of the samples. (This is equivalent of low pass filtering and of decreasing of the sampling rate. The advantages of the first part are reduced or even disappear.)

II. "Over sampling" versus "signal sampling factor"

The term "over sampling" is used in term of "sampling at higher that "Nyquist rate" or " Shannon rate"[3, 4]. In fact that means "sampling with sampling factor greater than 2". Generally the term "over sampling" means that certain "redundancies" are introduced into signal processing according to the Nyquist-Shannon-Kotelnikov sampling theorem. In fact this is not exact because the theorem does not give sufficient information
for signal reconstruction [5].

Petre Tzv. Petrov is with Micro Engineering-Sofia, Bulgaria. Emails: ppetre@caramail.com and ptzvp@yahoo.fr

The parameter "sampling factor" (SF) or "signal sampling factor" (SSF) is given with Eq. 1:

$$N = F_d/F_s = F_d/F_{max} \qquad (1)$$

Where: F_d is the sampling frequency, F_s is the frequency of the sampled sinusoidal or co-sinusoidal signal, F_{max} is the maximal frequency of the sampled band limited signal (BLS). The SSF is more accurate that the term "over-sampling" because is giving the possibility to calculate the errors during the sampling process [5].

III. The method Over sampling and averaging

The method called "over sampling and averaging" (OA), described in [1,2] is based on the Eq.1:

$$F_{os} = 4^w F_{sn}, \qquad (1\ )$$

Where: F_{os} is the "over sampling frequency"; w is the number of the "added bits" ("additional bits of resolution"); F_{sn} is the initial sampling frequency ("original sampling frequency" or "Nyquist frequency").

Definition: The parameter "over sampling factor" (ratio) P is introduced with the equation below:

$$P = F_{os/}/F_{sn}, \qquad (2)$$

According to [1] the method OA is applicable if the noise added to the signal is "white noise", but that type of noise is not reproducible and does not exist. Also[1] with 12-bits ADC and the method OA are obtained 16 bits of or resolution or 4 more bits.. In total they have obtained "16-bits of useful data". The Table I is based on the Eqs. 1 and 2. The question is "If in [1] there are 4 bits beyond the bits guaranteed by the accuracy of the converter are they always reproducible?" The answer is "No". These additional bits are not always reproducible. They are filled with less or more unpredictable information. The comparison with higher accuracy measurement equipment shows that. In Table I a resume of the cases when 1 to 10 bits must be "added|" to the existing accurate bits of the converter.

It is claimed [1] that the signal to noise ratio (SNR) of 12-bits ADC is incremented from the theoretical maximum of 74 dB (for 12 bits) up to 90 dB which are equivalent of 16 bits. The "gain" of 26 dB is obtained because of these 4 additional bits. Following the method OA Table I was made and tested with different signals. (w ="additional bits" = "gained bits of resolution" and P is the over sampling factor (ratio)). P is showing how many times the sampling frequency is higher than the frequency of 100% amplitude modulation (the frequency of SSF N=2).

The results from the experiments are as follows:

1. The additional calculated bits are not always predictable even when simple and stable sine wave or direct current (DC) signal is sampled. 2. The additional computational job for the

processor is making the method OA non applicable to most of the small and multi-channel systems. 3. The required data memory is rapidly increasing with over sampling ratio. 4. The method is practically equivalent of low pass filtering. 5. The method is practically equivalent of sampling with high sampling rate and re-sampling with much lower sampling rate. 6. The histograms from the experiments show that the output digital codes do not correspond to the Gaussian distribution. 7. The noise from the converter and the equipment is individual and cannot be approximated with white noise. This is especially important for production purposes.

TABLE I.
SUPPOSED ADDED BITS TO THE ADC OR DAC WITH OA

w, bits	P = $F_{os/}/F_{sn}$,]		w, bits	P = $F_{os/}/F_{sn}$
1	4		6	4096
2	16		7	16384
3	64		8	65536
4	256		9	262144
5	1024		10	1048576

IV. “ACCURACY” VERSUS “RESOLUTION”

The ”accuracy” (or “total reproducible error” or “total reproducible bits”) is the basic technical parameter of each conversion process. It is giving the absolute measurable error between the expected ideal code and the real code obtained from the converter. The accuracy is every time reproducible parameter and is always given by the manufacturer.. The accuracy is a price and application determining parameter.

The ”resolution” (sometimes called “precision”) is not an important technical parameter of the conversion process. In most of the cases it is a “commercial parameter” and cannot be used to compare reliably two products. The bits of ”resolution” are not an every time reproducible parameter. They are is not a basic price and application determining parameter. For example if a converter has an 6 bit of accuracy and 8 bits of resolution in this case only 6 bits are guaranteed and reproducible and the bits 7 and 8 are nor guaranteed and reproducible during the test process. In many cases they are neglected, because no decision could be based on them.

An example with the popular series of 8-bits ADC08XX with one channel is given on Table II and examples for multi channels ADC are given in Table III.

Although the ADC from each of the tables are ”pin to pin” compatible and with very similar characteristics they are not interchangeable. For example it is proven that ADC 0805 operating at 16 times higher frequency cannot replace ADC 0801, which has 4 times greater accuracy (+-0.25LSB against +- 1LSB). Both of them have the same number of bits and the same internal structure, but the LSB of ADC0805 is unstable. In fact if the clock frequency is increased from 100 kHz to 400 KHz or from 300KHz to 1200 kHz the conversion error is increased and not decreased. Moreover there is no way with averaging to reduce the error in a reproducible manner beyond the guaranteed accuracy of the converter. Consequently the method “over sampling and averaging for additional bits of resolution” is not a reliable method.

TABLE II
RESOLUTION (TOTAL BITS) AND ACCURACY (TOTAL ERROR) OF THE ADC080X SERIES.

ADC	Total bits	Resolu-tion, bits	Total error, LSB	RP, bits
ADC0800	8	8	+-2	6
ADC0801	8	8	+-0.25	8
ADC0802	8	8	+-0.5	8
ADC0803	8	8	+-0.5	8
ADC0804	8	8	+-1	7
ADC0805	8	8	+-1	7

RP – reproducible bits = bits of the accuracy

TABLE III.
RESOLUTION (TOTAL BITS) AND ACCURACY (TOTAL ERROR) OF THE ADC0808/9 MULTI CHANNEL ADC SERIES.

ADC	Total bits	Resolu-tion, bits	TUE	RP, bits
ADC0808	8	8	+-0.5LSB	8
ADC0809	8	8	+- 1LSB	7
ADC0816	8	8	+-0.5LSB	8
ADC0817	8	8	+- 1LSB	7

* TUE = Total unadjusted error = accuracy.

V. TESTING THE METHODS AND EXAMPLES

The methods “Over sampling and averaging” was tested with ADC with different word length connected to one microprocessor and sampling the same signal with same sampling rate and at the same moment. For example experiments with 6, 8, 10 and 12 bits ADC show clearly that there is no way with 6 bit ADC to replace 8, 10 or 12 bit ADC by simply increasing the sampling rate and averaging the samples. The accuracy of the converter depends from many design and production factors and cannot be overcome. Adding additional reproducible bits beyond the accuracy of the converter does not work.

Example 1: There is an “exception”. For example an 8-bit ADC with 8-bit accuracy is used to sample a noisy DC signal (or a very low frequency AC signal) and only 6-bits are stable. In this case it is possible by increasing the sampling rate (if the parameters of the noise are known) and averaging the samples to obtain 7- or even 8-bit of accuracy (the samples are accurate and predictable) but thee is no way by increasing more the sampling rate to obtain more than 8 accurate bits. The conclusion is easy expanded and tested for any number of bits and for different converters. In fact this is low pass filtering.

Example 2: Many microcontrollers as PIC, MCS-51 derivatives, 68HC11 are with build in ADC with accuracy from 8 to 12 bits. Experiments show that there is no way (even after intensive computations) to obtain more accurate (predictable) bits that the bits guaranteed by the manufacturer. The bits added by computations are not always reproducible.

Example 3: With voltmeter with precision +-1% are done manually a lot of fast AC measurements and the samples are averaged but accuracy better than +-1% was not achieved. The “manual sampling rate” was varied but the additional reproducible bits ware not obtained. The results were compared against +-0.2% millimeter.

Example 4: A fast 8-bit (+-0.2%) video ADC cannot replace a slow 12-bit (+-0,0122%) industrial ADC. Four more bits cannot be gained according to the method "over sampling and averaging for adding bits"

Example 5: A comparator (e.g. LM311) could replace 1-bit ADC with two levels and with error Eadc=+-25% (+-0,5*LSB, LSB=50%*Vfs. (Vfs is the full scale voltage of the converter). In order to apply the method OA a fast RISC PIC microcontroller was added and the output of the comparator was averaged. According to the method OA a converter with higher than one bit accuracy could be produced but this was not observable, because the accuracy (the predictable number of the produced bits) depends on many factors and not only from the averaged samples. The answer of the question "Is it possible with a simple comparator and processor to replace a multi bits ADC? " is "No".

As a conclusion the incoming samples from n bit ADC can not be interpolated in a reproducible manner to samples from n+1, n+2, etc ADC. In order to achieve increased number of predictable and repetitive bits a lot of hardware conditions should be met (e.g. a better reference source and comparator) not only increasing the sampling rate and averaging the samples.

VI. The method OA and the DACs

If the method OA is really applicable to the ADC it should be applicable in clear and reproducible manner with the DAC. But there is no way to increase the accuracy or the resolution of the DAC by simply increasing of the frequency of the converted samples. In order to do that special hardware techniques should be used or simply DAC with more accurate bits should be used. But they are not part of the method AO.

VII. Testing the method OA with the simplest band wide test signal

Definition: The simplest band wide signal (SBLS) has two lines in its frequency spectrum. The first line is the direct current component (DC) and the second line is the alternative current (AC) component which is a sine or co-sine wave. Also is the simplest alternative current (AC) test signal (ACTS) which could be produced and very often is used as a basic test signal. SBLS was defined with the Eq. (3):

$$A = A_m \sin(\omega t+ \theta) + A_{dc} = A_m \sin(2\pi ft+ \theta) + A_{dc} \quad (3)$$

Where A is amplitude of the SBLS, ω is the angular frequency, f is the linear frequency, A_m amplitude of sine wave component and A_{dc} amplitude of the direct current component of BLS.

Sometimes graphically (or mechanically) SBLS is produced by circular movement of the vector with five components as follows: x and y are the coordinates of the center of the circle, ω is the frequency of rotation, r is the radius of the circle and θ is the starting position of the rotation of the vector.

Assumption θ=0 and A_{dc}=0 (or x=0 and y =0) does not decrease the number of the parameters of the SS to reconstruct. They are always at least four (and at least four samples for one period of the sine wave are needed for the reconstruction). We can simplify the Eq. 3 for analyzing it. We are obtaining the simplified formula (which is the formula of the simplest alternative current testing periodical signal) with zeroed phase and DC component:

$$A = A_m \sin(\omega t) = A_m \sin(2\pi ft) \quad (4)$$

We are introducing the principle "One sample per parameter to reproduce". According to it at least four samples are required to reproduce all parameters of that signal (not two as the classical sampling theorem is stating).

VIII. Basic amplitude errors

When this signal is sampled with Analog to digital converter (ADC) with infinite (or sufficiently big) number of bits the deviation from the maximum of the sine wave component is called amplitude error. Here three amplitude errors are defied: maximal error (E_{max}), possible minimal error (E_{min}) and average (typical) error (E_{av}). The following equations are applicable and easy verifiable

$$E_{max} = 1-\sin(90-180/N) = 1-\cos(180/N) \quad (5)$$

$$E_{min} = 0 \quad (6)$$

$$E_{av} = 1- \sin(90-180/(2N)) = 1-\cos(90/N) \quad (7)$$

It is seen that if the sampling factor N is increased from zero to infinity neither of these errors is not following the rule " the error is decreasing twice if the sampling factor is increasing four times". The dependency of the maximal amplitude error from the SSF from 2 to 37 is shown in Table IV.

According to [1] "If the sampling factor N is multiplied by four one more bit is added to the converter, or the resolution in multiplied by 2". It is seen from the Table IV that if we increase the sampling factor four times we will not obtain one more bit (the maximal amplitude error will not decrease two times). The maximal amplitude error does not follow exponential law.

IX. Final comments on the method "Over sampling and averaging"

The method OA was tested with direct current signal, sine wave and SBLS.

The "additional bits" obtain by the method OA sometimes could be added from generator of numbers with almost the same success.

The errors internal to the ADC cannot be overcome by the method OA because they are more or less individual and more that the guaranteed by the manufacturer bits cannot be achieved.

The increasing of the resolution (adding bits with OA) does not increase the accuracy of the measurement.

"Adding bits" with OA means that if we sample a DC with very high sampling rate and with very low accuracy ADC we will having the possibility to obtain a lots of bits of resolution but in practice a lot of not repeatable bits are obtained.

OA is useful as a kind of low pass filtering and is giving the possibility to use all accurate bits of the converter but cannot be used to go beyond the guaranteed accurate bits.

It seems that in general the method “over sampling and averaging for additional bits of resolution” has no theoretical and practical value. The “added bits” are not reproducible during the tests and are not useful.

If the frequency of the converter is increasing the conversion errors are increased too. Consequently there is a lost of data bits and there is no gain of additional bits.

The “delta sigma modulation” and the “over sampling and averaging” has nothing in common as a theory and practice. These are two different methods.

The theory of the “white noise” cannot be taken seriously because such a noise does not exist, cannot be generate and cannot be regulate effectively.

Charging a microcontroller with large among of data from the ADC in order to calculate non-reproducible bits is not effective technical task.

The errors during the conversion is closely related with the shape of the signal, the used hardware and software, the power supply and many application parameters and cannot be considered as “white noise”.

The “noise” is not a good thing. Adding noise (disturbing signal in that case) to the useful signal is not desirable from the point of view of the accuracy.

TABLE IV
RELATION BETWEEN THE SAMPLING FACTOR N AND EMAX FOR N FROM 2 TO 37.

$N = F_d/F_s$	E_{max} [%]	$N = F_d/F_s$	E_{max} [%]
2	100	20	1.23
3	50	21	1.11
4	29.3	22	1.02
5	19.1	23	0.9314
6	13.4	24	0.856
7	9.9	25	0.789
8	7.61	26	0.729
9	6.09	27	0.676
10	4.89	28	0.629
11	4.05	29	0.586
12	3.40	30	0.548
13	2.91	31	0.513
14	2.51	32	0.482
15	2.18	33	0.453
16	1.92	34	0.427
17	1.70	35	0.403
18	1.51	36	0.381
19	1.36	37	0.360

The method OA is demanding exponential increase of the sampling rate and is producing non reproducible bits from the ADC. It is not applicable to DAC. The reverse over sampling and de-averaging for DAC does not work.

As mention before OA applied to the ADC means two things: 1/ Choosing the frequency of sampling much higher than SSF N=2. e.g.. N=32 is giving over sampling factor $P=32/2=16=4^2$. 2/ Adding and averaging each P samples (in this case sixteen consecutive samples) and producing only one sample with supposed higher resolution or even accuracy (in this case the “gain” sould be 2 bits), e.g. for N=2, E_{max} = 100% and with N=32, E_{max}=0.482 (Table IV). But this is not observed.

X. CONCLUSIONS ABOUT THE SIGNAL SAMPLING FACTOR

WithSSF N=2 the amplitude error of the digital copy of the sampled SS/CS could vary from 0 to 100%.

With SSF N=4 the amplitude error of the digital copy of the sampled SS/CS could vary from 0 to 29.3% (0 to 3 dB).

The CST based on Fourier series (oversimplified series) is uncompleted and cannot be used as a reference point during the signal sampling and reconstruction.

The SSF N divisible by 4 (N = 4*k, where k = 1, 2, 3…) is guarantying the possibility of zero DC error during the direct signal reconstruction and is giving the possibility to obtain zero amplitude errors because the signal could be sampled in its maximums, minimums and zero crossings.

Law of the limiting SSF was formulated: There is no sense to increase the SSF N=Fd/Fs>0 higher the value limited by the accuracy and rapidity of the converter, because the obtained additional bits by calculations are nor reproducible.

Oversimplification, overgeneralization and over sampling did a lot of harm in the sampling theory.

The sampling rate of 20Hz to 20 kHz audio signal should be at least 80 kHz in order to guarantee always less than 3db maximal amplitude error.

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