Temperature Profile of the Impulse Discharge

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Abstract **– New analytical model for determining the gas temperature profile of the impulse discharge in the cross-section of copper-bromide laser is proposed. The model consists of heat conductivity equation subject to special nonlinear boundary conditions of third and fourth kind, which allow obtaining the gas temperature distribution as a function of the laser parameters, such as geometrical design, constructive materials, input electric power, etc. Numerical results in the case of natural convection are presented.**

Keywords **- impulse discharge, Copper-Bromide laser, temperature profile.**

I. Introduction

The impulse discharge is very actively applied for exciting a range of impulse metal vapor lasers and its varieties with compounds, generated by self-restraint transitions from resonance to metastable levels (*r-m* lasers). These are copper vapor, gold vapor and manganese vapor lasers. Because of the physical nature of the laser level population processes, only this type of discharge induces an inversion polulation within the active laser levels and excites laser generation. Among the metal vapor lasers, the most investigated ones are lasers in which the active medium is modified by additing different halides of the working metal. More often hydrogen and hydrogen mixtures, such as HCl, HBr etc., are supplemented [1-2]. These modifications yield abasement of the operating medium temperature by 800-1000K and increase the frequency and energetic characteristics and laser radiation quality. Due to the above mentioned advantages this type of lasers are extensively used in modern of high-speed data recording systems, in material processing, in medicine, for athmosphere and world ocean investigation, in isotop spliting techniques, etc.

Among the metal vapor lasers the more perspective are the copper vapor lasers and copper compound vapor lasers. This type of lasers are the most powerful in the visible. Many papers are devoted to its investigation (see for instance [1-2] and quoted there literature). In order to enchance the laser caracterisrics and study the physical processes in the active laser medium different mathematical models and computer simulations of the kinetic prosesses have been developped [3-5]. The gas temperature is one of the most important thermodynamical characteriatcs of the active medium. This determines the laser application term, laser generation deteoretion, the distribution of the neutral atoms in the cross-section of the tube. The rise of the gas temperature produces a thermal population of the lower laser levels, which has influence on the laser power and laser beam mode composition. A higher temperature can provoke thermoionization gas discharge instability. Because of all this during the laser exploatation period and during the computer simulations, respectively, this parameter must be subject of detailed study. Two types of approach can be applied to determine the gas temperature – set the temperature on the tube walls [3-4] or choose some gas temperature distribution in the cross-section of the discharge tube, which do not change subsequently [5].

However, when multiple computer simulations are carried out, the geometrical design, tube constructive materials and thermal insulation the input electric power and working conditions often change. In this case the wall temperature is unknown. The stated initial temperature profile also changes. So without taking the correct gas temperature into account, the obtained results can be unreliable. Special attention to this problem is paid in [6].

In this paper a new analytical model is presented for more accurate determination of the gas temperature in the cross-section of the discharge in the copper-bromide vapor laser. The model is described by two-dimensional heat conductivity equation subject to nonlinear boundary conditions of the third and fourth kind, instead of the first kind boundary conditions, which were used till now. The model sets the air temperature (say T_0 = 300K) and takes into consideration the conduction and radiation processes in the heat exchange between the outside area of the laser tube and its surroundings. This approach allows obtaining the gas temperature and the wall temperature as a result of the solution of heat conductivity equation depending on the initial parameters, such as geometrical design, input electric power, etc.

II. Description of the Model

We study the impulse discharge in a copper-bromide vapor laser [7], operating at input electric power of 5KW and generating an output power of 120W. The geometrical design of the laser tube is shown in Fig. 1. The laser tube is manufactured of quartz, supplied with thermo-insulating glass wool, mineral wool or zirconium dioxide coating.

The following assumptions are imposed: 1) the temperature profile is seeked in a quasistationary regime; 2) the gas

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temperature between the pulse-repetition rates does not change considerably; 3) the total amount of the input electric power is transformed to heat.

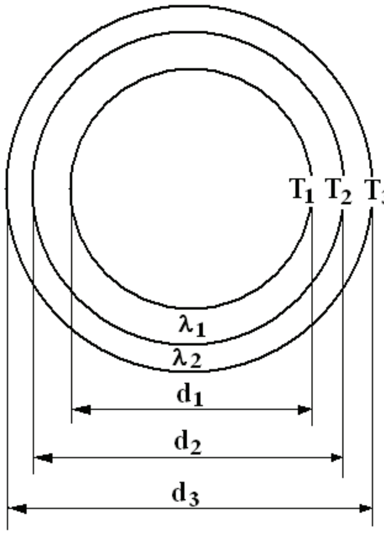

Fig. 1. Geometry of the cross-section of the laser tube: *d*1 = 56*mm*, *d*2 = 60*mm*, *d*3 = 70*mm*.

Two-dimensional stationary heat conductivity equation in the cross-section of the discharge tube can be written in the form

$$\operatorname{div}\left(\lambda_g \operatorname{grad} T_g\right) + q_v = 0 \qquad (1)$$

where λ_g is the heat conduction coefficient of the gas, q_v is the volume density of the internal heat source, and T_g is the temperature in the tube.

In order to solve equation (1) more often the boundary condition of the first kind is used. It means that the temperature T_2 of the outside area of the quartzous tube is supposed to be given. The value of λ_g is approximated in the form $\lambda_g = \lambda_0 T_g^m$. In this case equation (1) possesses the following explicit solution ([8])

$$T_g(r) = \left[T_2^{m+1} + \frac{q_v(m+1)}{4\lambda_0}\left(R^2 - r^2\right)\right]^{1/(m+1)}, \qquad (2)$$

where R is the radius of the tube. Solution (2) is used for determination of the temperature profile of the copper-bromide vapor laser in [9], and for computer simulation of copper vapor laser in [4].

In our model we solve equation (1) under the following mixed boundary conditions of the third and fourth kind, which in cylindrical configuration take the form [10]

$$T_1 = T_2 + \frac{q_l \ln\left(d_2/d_1\right)}{2\pi\lambda_1}, \; T_2 = T_3 + \frac{q_l \ln\left(d_3/d_3\right)}{2\pi\lambda_2}, \qquad (3a)$$

$$Q = \alpha F_3\left(T_3 - T_0\right) + F_3 \varepsilon c\left[\left(T_3/100\right)^4 - \left(T_0/100\right)^4\right]., \qquad (3b)$$

The boundary condition (3a) describes the continuity of the heat flow through the boundary between two walls. Here q_l is the power per unit length, $q_l = Q/l_a$, $l_a = 2m$ is the *m*-active length [7], λ_1, λ_2 are the heat conductivity coefficients of the quartzous tube and the thermal insulation, respectively, d_j, $j = 1,2,3$ are the diameters of the inserted tubes (see Fig. 1). The boundary condition (3b) gives the way of heat exchange between the outside area of the laser tube and its surroundings. It consists of two terms. The first term at the right hand side of (3b) evolves from the Newton- Rihmann's law for heat exchange by convection. The second term gives the Stefan-Boltzmann law for heat exchange by radiation. The value of Q is equal to the overall heat flow and finds expression in the total consumed electric power, regarding the assumption 3, as it was stated above, α is the heat transfer coefficient, F_3 is the outside active area of the tube, ε is integral emisivity of the material, $c = 5.67 W/(m^2K^4)$ is the black body radiation coefficient, $T_0 = 300K$ is the temperature of the air in the surroundings. In equation (3b) the values of α and T_3 are unknown. To find T_3 we need preliminarily to calculate the heat transfer coefficient α.

For all types of convection the Nusselt criterion hold [10]:

$$Nu = \alpha H / \lambda, \qquad (4)$$

In the case of free convection the Grashoff criterion is given by the expression [10]:

$$Gr = g\beta H^3\left(T_3 - T_0\right)/\upsilon^2, \qquad (5)$$

For the horizontal tubes at natural convection the two upper criteria can be related by the equality [10]

$$Nu = 0.46 Gr^{0.25}, \qquad (6)$$

which is valid if $700 < Gr < 7.10^7$. In the previous expressions (4)-(6) H is a characteristic dimension of the body, here $H = d_3$, g is the gravitational acceleration, β is the coefficient of cubical heat expansion of the gas, which for the air is $\beta_{air} = 3,41.10^{-3}, K^{-1}$, υ is the kinematical viscosity, $\upsilon_{air} = 15,7.10^{-6} m^2/s$, λ is the heat conduction coefficient, $\lambda_{air} = 0,0251\ W/(mK)$. These data are valid for the air temperature 300*K* [10].

By using (4)-(6) the heat conduction coefficient α can be expressed as

$$\alpha = 0.46\lambda_{air}\left[g\beta_{air}d_3^3\left(T_3 - T_0\right)/\upsilon_{air}^2\right]^{0.25}/d_3. \quad (7)$$

The boundary condition (3b) represented with respect to power per unit length takes the form

$$q_l = 0.46\pi\lambda_{air}\left[g\beta_{air}d_3^3\left(T_3 - T_0\right)/\upsilon_{air}^2\right]^{0.25}\left(T_3 - T_0\right) + \pi d_3\varepsilon c\left[\left(T_3/100\right)^4 - \left(T_0/100\right)^4\right], \quad (8)$$

Now, the outside temperature T_3 can be calculated by solving nonlinear equation (8). It is not difficult to establish that this equation possesses an unique real solution. Then by means of (3a) the temperatures T_2 and T_1 can be determined.

Finally, from (2) we obtain the following expression to calculate the gas temperatures in all internal points in the active discharge medium:

$$T_g\left(r\right) = \left[T_1^{m+1} + \frac{q_v\left(m+1\right)}{4\lambda_0}\left(R_1^2 - r^2\right)\right]^{1/(m+1)}, \quad (9)$$

where R_1 is the radius of the internal tube.

III. Results and Discussion

In this section some numerical results for the temperature profile in the case of natural convection, obtained by our analytical model are presented. The values of the used physical constants are shown in Table 1.

Table 1. Data used in the calculation of the temperature profile. The lowest row shows the corresponding references.

Q, W	l_a, m	q_v, W/cm^3	q_l, W/m	$\lambda_g = \lambda_0 T^m$, $W/(mK)$	λ_1, $W/(mK)$	λ_2, $W/(mK)$	ε
5000	2	1.015	2500	$\lambda_0 = 5.8935.10^{-5}$ $m = 1.091$ 15*Torr* Ne and 0.3*Torr* H_2	1.96 (T = 800÷ 1100K)	0.12 (T = 800÷ 1100K) (mineral wool)	0.72
[7]	[7]			[9]	[11]	[10]	[11]

The equations (8), (3a) and (9) are solved consecutively. Curve 1 in Fig. 2 shows the obtained temperature distribution in the cross-section of the discharge tube. The corresponding values of the intermediate temperatures are: $T_3 = 662K$, $T_2 = 1174K$, and $T_1 = 1188K$, respectively, the temperature in the centre of the tube is $T(0) = T_{\max} = 2200K$.

As it was mentioned above, the usually used technique in the literature required to give the outside surface temperature T_2 of the quartzous tube, which would be measured (for instance by a thermocouple). After that, assuming $T_2 = T_1$ in (2) one can calculate the internal temperature profile in the discharge tube [3, 7]. We found the absolute error in this case to be around $\Delta T = T_1 - T_2 = 14K$. As the temperature difference is small, this approach is practically admissible.

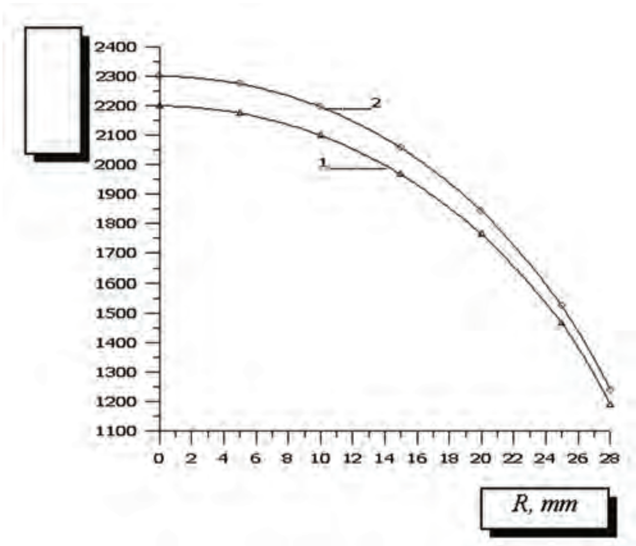

Fig. 2. Gas temperature distribution in the cross-section of the active discharge medium at natural convection: curve 1- at power Q = 5000W and curve 2 - at power Q = 5500W.

By our model an "apriori" estimation of the possible temperature variation depending on the input electrical power can be obtained. As an example, an increase of 10% in the input electric power causes the increase of maximum temperature $T_{\max}$ up to 100%, which is illustrated in Fig. 2. This type of results cannot be found using the above mentioned methods, while the temperature T_2 remains unknown.

Fig. 3 shows the relative change of the temperature $T(0)$ in the centre of the discharge with the change of the insulation coating thickness. The otained results indicate a 8.7 % increase in the maximum temperature with an 11% increase in the thickness of the thermal insulation.

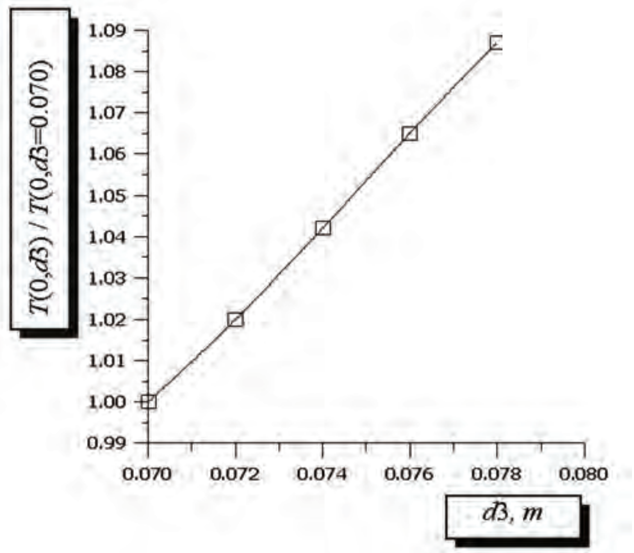

Fig. 3. Relative variation of the calculated temperature $T(0)$ in the centre of the tube, while increasing the thickness of the thermal insulation.

Another advantage of the presented model is that, at a given temperature T_3, equation (8) allows analyzing the heat exchange behavior through the outside laser surface to the surroundings. For our data we obtain that the dominant heat exchange mechanism is heat radiation (66%) with the remainder being heat conduction (34%).

IV. CONCLUSION

A new approach in solving the heat conductivity equation for copper-bromide vapor laser is presented. Mixed boundary conditions of the third and fourth kind are introduced to describe more precisely the heat exchange between the laser tube and its surroundings.

The temperature profile of the discharge is calculated in the cross-section of the laser tube in the case of natural convection. It is established that the dominant heat exchange mechanism in this case is heat radiation. The proposed model allows carry out further computer simulations in order to optimize the working laser characteristics by changing geometric design, constructive materials, input electric power and other basic parameters.

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REFERENCES

[1] D. V. Shiyanov, V. B. Sukhanov, G. S. Evtushenko and O. S. Andrienko, "Experimental Investigation of the Effect of HBr Additions *on the Lasing Characteristics of a CuBr Laser", Quant. Electronics*, vol. 34, no 7, pp. 625-629, 2004.

[2] D. V. Shiyanov, G. S. Evtushenko, V. B. Sukhanov and V. F. Fedorov, "Effect of Gas Mixture Composition and Pump Conditions on the Parameters of the CuBr—Ne—H2(HBr) Laser", *Quant. Electronics*, vol. 37, no 1, pp. 49-52, 2007.

[3] R. G. Carman, D. J. W. Brown and A. Piper, "A Self-Consistent Model for the Discharge Kinetics in a High-Repetition-Rate Copper-Vapor Laser", *IEEE J. of Quant. Electr.*, vol. 30, no 8, pp. 1876-1895, 1994.

[4] S. I. Yakovlenko, "Critical Electron Density in a Self-contained Copper Vapour Laser in the Restricted Pulse Repetition Rate", *Quant. Electronics*, vol. 30, no 6, pp. 501-505, 2000.

[5] A. M. Boichenko, G. S. Evtushenko, O. V. Zhdaneev and S. I. Yakovlenko, "Theoretical Analysis of the Mechanisms of Influence of Hydrogen Additions on the Emission Parameters of a Copper Vapour Laser", *Quant. Electronics*, vol. 33, no 12, pp. 1047-1058, 2003.

[6] G. G. Petrash, "On the Simulation of a Copper Vapour Laser with Hydrogen Admixtures", *Quant. Electronics*, vol. 35, no 6, pp. 576-577, 2005.

[7] D. N. Astadjov, K. D. Dimitrov, D. R. Jones, V. K. Kirkov, C. E. Little, N. V. Sabotinov and N. K. Vuchkov, "Copper Bromide Laser of 120-W Average Output Power", *IEEE J. of Quant. Electr*., vol. 33, no 5, pp. 705-709, 1997.

[8] M. J. Kushner and B. E. Warner, "Large-bore Copper-vapor Lasers: Kinetics and Scaling Issues", *J. Appl. Phys.*, vol. 54, no 6, pp. 2970-2982, 1983.

[9] D. N. Astadjov, N.K.Vuchkov and N.V. Sabotinov, "Parametric Study of the CuBr Laser with Hidrogen Additives", *IEEE J. of Quant. Electr*., vol. 24, no 9, pp. 1926-1935, 1988.

[10] M. Oprev, S. G. Batov and D. Z. Uzunov, *Heat Technology*, Sofia, Technika, 1978.

[11] *Physical Quantities*, Handbook, Moscow: Energotomizdat, 1991.