Comparative Analysis of Integral Calculus Algorithms in Magnetic Signals Evaluation

Virginya Dimitrova[1]

Abstract - **The article presents three approaches for low-frequency magnetic signals calculation, based on direct integration of the Biot-Savart law using complete elliptic integrals; differences are into the chosen approach for complete elliptic integral calculation. The methods discussed are implemented in C# programming language and the results of their execution are compared both visually and numerically.**

Keywords – **integral calculus algorithms, complete elliptic integrals, low-frequency-magnetic field**

I. INTRODUCTION

The goal of the presented program component is to expose easy-to-use and easy-to-understand approach to evaluate the precision of low-frequency magnetic field calculation when different methods have been used for elliptic integrals solving.

Two ways for precision evaluation are developed:
- Numerical comparison;
- Visual comparison (2D graphs).

Both ways are based on the comparison of the results of law-frequency magnetic field calculation, performed using the *etalon* calculation and *matching* calculation. *Etalon* calculation is performed using the formula for on-axis points [1].

Matching calculation is performed using the formulas for off-axis points based on magnetic filed potential that involves complete elliptic integrals of the first and second kind [2]:

$$K = \int_{0}^{\pi/2} \frac{d\beta}{\sqrt{1 - k^2 \sin^2 \beta}} \qquad (1)$$

$$L = \int_{0}^{\pi/2} \sqrt{1 - k^2 \sin^2 \beta} \cdot d\beta \qquad (2)$$

In both cases calculations are performed for the same set of points of **z-**axis that is perpendicular to the plane of the current turn.

The differences into the precision of calculations are caused by the method chosen for elliptic integral calculation:
- Using diagram;
- Using elliptic integral Tables;
- Using Arithmetic-Geometric-Mean.

Magnetic field calculation for different cases of field sources – coils with great number of current turns requires a great deal of computation for large number of points into the area influenced by the field. We need a method that will guarantee a reduced complexity for a large amount of field points, namely with a matrix of points nearly 90000 or more.

The method must guarantee a good precision of calculations too. The main part of field computations is related to complete elliptic integrals evaluation. Consequently, improving the precision, decreasing the complexity of calculations and optimizing memory consumption can be achieved using the best suited method for complete elliptic integrals evaluation.

The first implemented method can be determined as a direct method and uses a diagram to compute the complete elliptic integrals of first and second kind K(k) and L(k) for the corresponding values of k. The main disadvantage of this method is the need to store floating point values into the memory as arrays elements (for example) thus limiting the step of discretization to avoid enlarging of arrays. Besides of errors of discretization this method does not ensure enough precision for the impossibility to obtain real values for K(k) and L(k) with more that two digits after the decimal point from the diagram. The method however has some advantages: relatively small size of the arrays and their static nature allows implementation with static arrays instead of linked lists and thus only two fast indexing operations per k value are needed to obtain K(k) and L(k) values.

Second implemented method is also a direct method as the first one, but is based on calculations (not on the predefined values stored into the memory). This method saves memory but instead of fast indexing operations uses so called modulus $m = 1-k^2$ and double precision constants into a lot of multiplication and addition operations. (The method is based on FORTRAN subroutine COMELP, rewritten in C# language).

The third implemented method can be determined as an iterative method (starts from a guess and finds successive approximations that converge to the solution) and is effective and numerically stable. This method computes Legendre elliptic integrals k(k) and L(k) by computing the equivalent Carlson elliptic integrals and corresponding RC, RF and RJ routines, originally written in FORTRAN and rewritten in C# language.

Elliptic integrals used in magnetic field calculation are expressed using Legendre's notation.

Numerical comparison is realized in two modes:
- Single values comparison;
- Multiple values comparison.

When the first mode is used the value of the **z**-coordinate of the point into which the magnetic induction **B** should be calculated is entered from the keyboard and the two compared numeric values as well as their difference are displayed.

Under the second mode the magnetic induction **B** is calculated for the predefined set of points and the corresponding compared values are displayed in rows-and-columns form (**DataGrid** Windows Forms control).

[1]Virginya T. Dimitrova is with the Faculty of Computer Systems and Control, Technical University, 1000 Sofia, Bulgaria, E-mail: vergy@tu-sofia.bg

In regard to the visual comparison two different graphical representations of the results of precision analysis are supplied:

▪ 2D graph with **z**-coordinates of points on **X-**axis and calculated **B**-values (using *etalon* method and *matching* method) in different colors on **Y**-axis;

▪ 2D graph with **z**-coordinates of points on **X-**axis and the difference of the calculated **B**-values (*etalon* method and *matching* method) on **Y** axis.

II. DESIGN OF THE PROGRAM

The core of the program is the algorithm shown in fig 1. It supplies all the possibilities needed to satisfy the goal of the program.
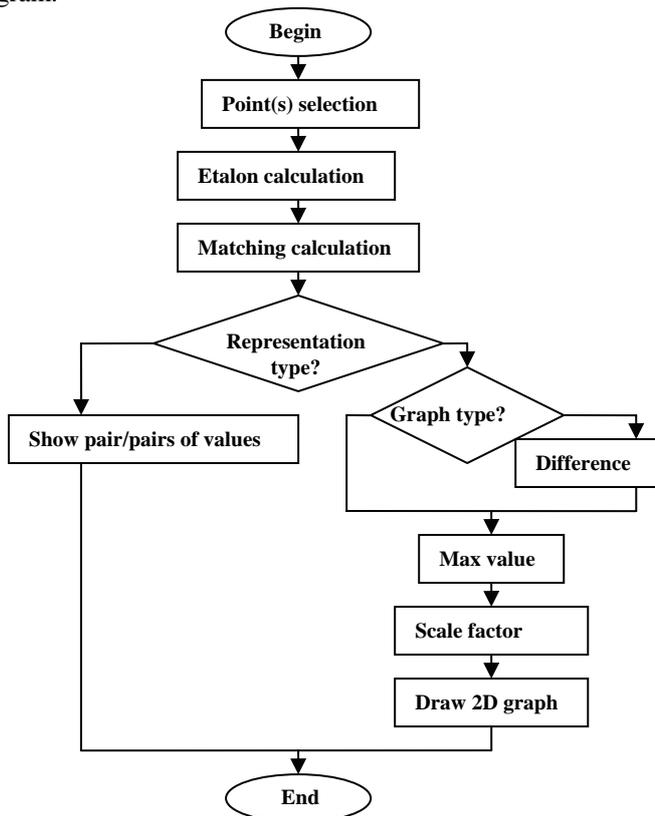

Figure 1. Flow-chart of the program

III. IMPLEMENTATION

Program decision is created using Visual Studio .NET program environment, C# programming language and Windows Forms application template.

Methods for magnetic induction calculation are placed into a separate **FieldCalcs** class. To achieve the universality (ability to calculate magnetic induction for every point into the space) the constructor has two parameters – the values of ρ and **z**. For the current implementation the value of the first parameter is always 0.0 and only the second parameter changes its value.

Separate instance methods for calculation are supplied:

• **BonZ()** *etalon* method for on-axis points and

• **BEveryWhere()** *matching* method for off-axis points.

The **BEveryWhere()** method has a parameter passed using a .NET delegate mechanism thus allowing to dynamically change the method used for complete elliptic integrals calculation (fig. 2).
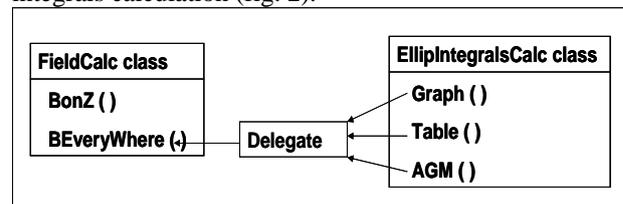

Figure 2. Program components

IV. CONCLUSION

The presented program component has been successfully applied for the evaluation of the effectiveness, numerical stability and precision of the calculations in regard to low-frequency magnetic field generated by different configurations of field sources. The structure and implementation of the program component allows methods for complete elliptic integral calculation, exposed by the **FieldCalc** class, to be replaced, improved or extended with additional methods without changes in the design of the component or applications that reference it.

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