A Quickened Genetic Nonlinear Reconstruction Algorithm for EIT

Vassil Guliashki[1]

Abstract – **This paper presents a quickened hybrid genetic algorithm for the inverse nonlinear problem of Electrical Impedance Tomography (EIT) in 2D domain. It belongs to the interior algorithms. Finite Element Method is used to solve the forward EIT problem regarding the nodal scalar potentials and current density values. The variational approach is applied to solve the inverse problem.**

Keywords – **Electrical Impedance Tomography, Genetic algorithms, Finite element method, Variational approach.**

I. Introduction

The Electrical Impedance Tomography (EIT) is a technique, proposed for non-destructive testing of materials, geophysical applications such as core sample analysis and investigations of the Earth contamination, as well as for biomedical purposes like making a diagnosis for breast cancer, investigation of chest organs and cerebral haemorrhaging (brain stroke). Algorithms for detection of flaws in materials are presented in [6,7,8,15,16]. The system, proposed in [8,16] permits geographically distributed research with remote measurement and data acquisition for eddy current test signals. In EIT technique low-frequency voltages, obtained as a result of injected currents in an inhomogeneous object, are measured by means of electrodes on its boundary. Then the interior electrical conductivity of the object is calculated. At the end EIT gets an image of the electric field inside the studied object, based on the conductivity distribution in it. Usually, the reconstruction of an EIT image consists of two parts: 1) The *forward problem* – where the scalar potentials (voltages), as well as the current density values inside the object are calculated, given an approximate conductivity distribution, boundary voltages and currents for a known geometry of the studied volume; 2) The *inverse problem* – where an adequate estimation of the conductivity distribution, based on the calculated (known) scalar potentials and current density values is received. The second is a nonlinear ill-posed problem (see [22]). Its feasible domain has valleys and/or plateaus (regions, where the objective function is almost flat).

The computational complexity of the exact methods for such a problem grows exponentially with the number of the unknown parameters, which – in EIT-problem – depends on the mesh chosen in Finite Element Method (FEM). The image quality is better when finer mesh is used, i.e. with more unknown parameters. To overcome the shortcomings of the exact methods many efficient approximate evolutionary algorithms, metaheuristic and hybrid methods have been created to find out quickly the global optimum of complex optimization problems.

A new hybrid genetic algorithm, solving the inverse EIT problem is proposed in this article.

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II. Formulation of the Problem

A. Experimental setup of the problem in 2D case

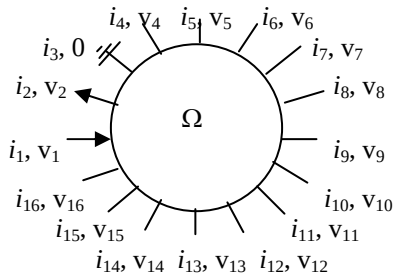

Fig. 1. Experimental setup for the EIT problem

The illustration of the experimental setup for the EIT problem is presented on Fig. 1. To perform measurements on the boundary of the studied 2D object 16 electrodes are used. The object is considered as an inhomogeneous, conducting body having a known overall shape Ω. For simplicity here is chosen the domain Ω to be a circle. It is divided by an uniform triangularization into 256 triangles (cells). This mesh is assumed to be fine enough, so that the FEM numerical calculations are sufficiently accurate. Direct currents $i1$ (input current) and $i2$ (output current) are applied to the body. The injected current between these two electrodes has value 10 mA. The potentials (voltages) are measured between pairs of the other electrodes, where one of the electrodes in each pair is the grounded electrode. Usually the voltage at the injection electrodes cannot be measured reliably and for this reason it is not included in the data set. The measured voltages have values about 1 V. Each electrode can be held to be equi-potential and the contact impedance is neglected. In this case the current field $J(x)$ and the electric field $E(x)$ are constrained by the Kirchhoff's laws:

$$\nabla \cdot J(x) = 0 \quad (1)$$

$$\nabla \times E(x) = 0 \quad (2)$$

and by the Ohm's law

$$J(x) = \sigma(x)E(x), \quad (3)$$

where σ(x) is the conductivity and J(x) is the current density. The body is assumed to be locally isotropic, so that σ(x) is a positive real number.

Since $\nabla \times E = 0$, E has the form

$$E(x) = -\nabla \Phi(x), \tag{4}$$

where $\Phi(x)$ is the scalar potential (the voltage). The equations (1)-(4) are equivalent to the single elliptic equation for $\Phi(x)$:

$$\nabla \cdot (\sigma(x) \nabla \Phi(x)) = 0 \text{ in } \Omega. \tag{5}$$

B. Boundary data and feasibility constraints

The experimental setup consists in injecting a measured current between two electrodes and measurement the voltage between pairs of other electrodes located on the boundary of the body. This procedure is repeated N times (N is the number of electrodes) clockwise, injecting current between all possible adjacent pairs of electrodes. For the setup on Fig. 1 we have N = 16. In case σ(x) is known, $\Phi(x)$ and J(x) are completely determined either by the boundary voltage $\Phi|_{\partial\Omega}$, or by the boundary current flux $J.n|_{\partial\Omega}$, where $n(x)$ is the unit outward normal to the boundary of the body $\partial\Omega$.

For the conductivity problem there are two distinct variational principles (see for example [1, 9]): the Dirichlet's principle:

$$\text{Min} \int_{\Omega} \sigma(x) |\nabla \Phi(x)|^2 \, dx \geq P, \tag{6}$$

where P is the power dissipated into heat (the measured power) in the true conductivity medium Ω, and its dual – the Thompson's variational principle, which takes the form:

$$\int_{\Omega} \sigma^{-1}(x) |J(x)|^2 \, dx \geq P. \tag{7}$$

These two constraints allow us to obtain upper and lower bounds on the feasible domain of the space that contains the solution to the inverse problem (for details see [1, 2]).

C. Formulation of the direct problem

The direct EIT problem is decomposed as two quadratic optimization problems: The first one has the form:

$$\text{Min} \int_{\Omega} \sigma(x) |\nabla \Phi(x)|^2 \, dx, \tag{8}$$

subject to:

$$\Phi(x) = V(\text{x}), \text{ for } x \in \partial\Omega, \tag{9}$$

where V(x), x ∈ ∂Ω, are the measured potentials on the boundary of the body.

The second optimization problem has the form:

$$\text{Min} \int_{\Omega} \frac{1}{\sigma(x)} |J(x)|^2 \, dx, \tag{10}$$

subject to:

$$-J(x).n(x) = I(\text{x}), \text{ for } x \in \partial\Omega, \tag{11}$$

$$\int_{\partial\Omega} I(x) dx = 0, \tag{12}$$

$$\nabla \cdot J(x) = 0, \text{ for } x \in \Omega, \tag{13}$$

where $I(x)$ are the currents on the boundary ∂Ω and $n(x)$ is the unit outward normal to the boundary ∂Ω.

The power dissipated into heat in Ω is:

$$\text{P} = \int_{\partial\Omega} I(x) V(x) dx. \tag{14}$$

The current density J(x) can be expressed by means of the electric vector potential $T(x)$ (see [22]):

$$J(\text{x}) = \nabla \times T(\text{x}). \tag{15}$$

Hence the second optimization problem can be written in the form:

$$\text{Min} \int_{\Omega} \frac{1}{\sigma(x)} |\nabla \times T(x)|^2 \, dx, \tag{16}$$

subject to:

$$-(\nabla \times T(x)).n(x) = I(\text{x}), \text{ for } x \in \partial\Omega, \tag{17}$$

$$\int_{\partial\Omega} I(x) dx = 0, \tag{18}$$

Starting with initial approximate values for σ(x), $x \in \Omega$, we solve the optimization problems (8)-(9) and (16)-(18) by means of the Finite Element Method (see for example [18, 21]) and calculate $\Phi(x)$, $T(x)$ and J(x), $x \in \Omega$.

D. Formulation of the inverse problem. A variational approach

The variational approach described in [12, 13] has been adopted here. Using data from N different measurements each time with different current injection pair of electrodes we solve N times the quadratic optimization problems (8)-(9) and (16)-(18). The essence of variational approach is to consider the linear equations (1) and (4) as constraints and to minimize the violation of nonlinear equation (3). So we solve the inverse EIT problem with unknowns σ(x), $x \in \Omega$, minimizing the error functional:

$$\boldsymbol{F} = \sum_{i=1}^{N} \frac{1}{2} \int_{\Omega} |\sigma(x)^{1/2} \nabla \Phi_i(x) + \sigma^{-1/2} J_i(x)|^2 \, dx, \tag{19}$$

subject to:

$$\Phi_i(x) = V_i(\text{x}), \; -J_i(x).n(x) = I_i(\text{x}), \; \nabla \cdot J_i(x) = 0, \; i=1,\ldots,N. \tag{20}$$

After expanding the square in (19) we have:

$$\boldsymbol{F} = \sum_{i=1}^{N} [\frac{1}{2} \int_{\Omega} \sigma(x) |\nabla \Phi_i(x)|^2 \, dx + \frac{1}{2} \int_{\Omega} \frac{1}{\sigma(x)} |J_i(x)|^2 \, dx + \int_{\Omega} J_i(x).\nabla \Phi_i(x) dx] \tag{21}$$

The last term in (21) is irrelevant to the minimization of $\boldsymbol{F}$ seeking σ(x), because it is entirely determined by the boundary data. Minimization of the first term in (21) corresponds to the Dirichlet's variational principle and the minimization of the second term corresponds to the Thompson's variational principle. The expression for σ(x), which minimizes F in (21) has the form:

$$\sigma(x) = \left(\sum_{i=1}^{N} |J_i(x)|^2 \right)^{1/2} . \left(\sum_{i=1}^{N} |\nabla \Phi_i(x)|^2 \right)^{-1/2} \tag{22}$$

III. A Quickened Genetic Algorithm for the Inverse Nonlinear EIT Problem

From the point of view of mathematical programming (either linear or nonlinear) the optimization methods are divided into interior and exterior methods, depending on whether the iterative steps of the correspondent method are made inside or outside the feasible domain (see [3]). For example the least square method (see [23]) is an exterior method, whereas the Kohn and Vogelius method ([13]) is an interior method. Solving the EIT inverse problem both types of methods attempt to converge to a solution on the boundary of the feasible domain, but the exterior methods converge from outside the feasible domain, while the interior methods converge from inside the feasible domain. In [1] is pointed out that the exterior methods can achieve convergence quickly for data without errors. The interior methods have the advantage to be insensitive to data errors and perform stable, but they are often slowly converging.

The hybrid genetic algorithm proposed here belongs to the interior algorithms. To reconstruct the electrical field image we solve the problem:

$$\text{Min } \boldsymbol{G} = \sum_{i=1}^{N} \left[\int_{\Omega} \sigma(x) \, |\nabla \Phi_i(x)|^2 \, dx + \int_{\Omega} \frac{1}{\sigma(x)} \, |J_i(x)|^2 \, dx \right] \quad (23)$$

subject to the constraints (20).

ADI method (see [12]) performs iteratively the following procedure:

1) Using the last computed $\sigma(x)$ and the measured voltages, minimize (8) and (10) over $\Phi_i(x)$ and $J_i(x)$ for $i = 1,\ldots,N$.

2) Using the obtained $\Phi_i(x)$ and $J_i(x)$ minimize $\boldsymbol{G}$ from (23) over $\sigma(x)$, and update $\sigma(x)$ according to (22).

The authors pointed out that ADI method performs stable but very slowly. More rapid convergence is achieved by means of a modified Newton (MN) method (see [12]).

There are known several successful attempts applying a genetic or an evolution hybrid algorithm to solve this ill-posed problem (see for example [10, 11, 14, 17, 20]. In [10] a genetic algorithm is combined with the Davidon-Fletcher-Powell method (see [4]) and with Pareto-optimization. In [11] a genetic algorithm is coupled with Newton-Raphson method and mesh-grouping. In both these cases very encouraging results are obtained. Theory of simple genetic algorithms is given in [5].

The proposed hybrid genetic algorithm is designed to solve the inverse EIT problem (23), (20) overcoming the slowly converging of the interior methods and the instability of the exterior methods when the signal/noise ratio is greater than 1%. The main idea in the new genetic algorithm is to perform a given number k of ADI-steps ($k \le 20$), or less than k steps until a plateau of solutions is reached and then to continue the search by genetic procedure. After each genetic generation an acceleration phase is performed to increase the speed of the algorithm. Instead of mutation operator the algorithm performs a special kind of local search after the generations have finished.

Acceleration phase

During this phase a step is calculated like in the Nelder and Mead simplex method (see [19]). The members (solutions) in the current population are ordered in increasing order of their G-values. The weight center σ_c of the group of first five members is calculated. Let the last five members with the worst G-values be denoted by σ_{wi} , i=1,2,…5; Each σ_{wi} is reflected towards σ_c making a step $y = \sigma_c - \sigma_{wi}$ to create a new solution. The constraints $\sigma(x) \ge 0$, (6) and (7) are used to reduce the length of this step if it is necessary. In case someone of the so generated solutions is better than one of the current population, the better solution replaces the worse. Each successful step leading to better solution is repeated again and again, taking into account (6), (7) and $\sigma(x) \ge 0$, until the G-value cannot be improved anymore.

Special kind of local search

As mentioned in [14] the conductivity distribution is piecewise constant for lots of biomedical subjects like in head or in chest of the human body. For this reason we can use a model having D parts with constant conductivity. In the best solution some conductivities are greater and some smaller than the mean conductivity. The local search proposed consists of following operations:

1. Choose the cell with the greatest conductivity σ_g and try to increase it by δ to $\sigma_g + \delta$, where δ is a small positive number. In case the G-value is decreased repeat this operation.
2. Performed analog operation with the smallest conductivity σ_s, which should be decreased by δ to $\sigma_s - \delta$ until the G-value decreases.
3. For the cell with σ_g try to replace the conductivity of each its neighboring cell by σ_g and evaluate the G-value. The conductivity values are replaced in this way until the G-value improves.
4. Perform the operation of 3. by σ_s in the neighborhood of cell with σ_s.

"Pseudo-code" form of the quickened genetic algorithm

Generate an initial population P_0 with size $s = 5D$ members, where D is the supposed number of regions in Ω having constant conductivity; Set $i := 0$; (iteration counter);

Evaluate the members in P_i;

Starting from the best member solution (with minimal value of G) perform k, or less than k, ADI steps until no more improvement is reached.

While no stopping criteria are met **do**

Set $i := i+1$;

For j=1, s; **do**

Select two members I_1 and I_2 by means of „Roulette wheel selection" in P_{i-1};

Apply *one-point crossover* operator to I_1 and I_2 for creating offspring O_1 and O_2;

Decide whether or not O_1 and O_2 should enter P_i to replace older (worse) members;

EndFor

Create the population P_i from P_{i-1}, replacing the worst members in P_{i-1} with the best generated children

solutions**;** Perform the acceleration phase.

EndWhile

Perform a special kind of local search around the best found solution σ (*x*)*.

The algorithm stops when the error functional value *G* becomes smaller than the prescribed value or when the iteration limit is reached.

IV. CONCLUSION

The proposed hybrid genetic algorithm makes an attempt to combine the good features of genetic algorithms and of interior methods in order to perform stable and robust search when the data are contaminated with great noise. The new algorithm combines the simple genetic procedure with steps of the ADI method, accelerating steps and a special kind of local search around the best obtained solution. We expect that the new algorithm may need only a few genetic generations to find the global optimal solution. A program on MATLAB has been created and will be tested on a number of test examples.

REFERENCES

[1] Berryman, J. G., „Some Nonlinear Reconstruction Algorithms for Electrical Impedance Tomography“, LLNL, UCRL-JC-143031, *Geometric methods in inverse problems and PDE control*, Minneapolis, Minnesota, July 16 - 27, 2001, *Conference Proceedings*, http://www.osti.gov/bridge/product.biblio.jsp?osti_id=15013262

[2] Berryman, J. G. and R. V. Kohn, „Variational Constraints for Electrical Impedance Tomography“, Physical Review Letters 65, 1990, pp. 325-328.

[3] Fiacco, A. V. and McCormick, G..P., *Nonlinear Programming: Sequential Unconstrained Minimization Technique*, 1990 (SIAM, Philadelphia)

[4] Gill P., Murray W. and Wright M., *Practical Optimization*, 1981 (New York: Academic)

[5] Goldberg D. E., *Genetic Algorithms in Search, Optimization and Machine Learning,*, Addison Wesley, Reading, Mass, 1989.

[6] Gouljashki V. and G. Marinova, “Fast 3-D ECT Numerical Analysis”, *Proceedings of the 1st Japanese-Bulgarian-Macedonian Joint Seminar on Applied Electromagnetics, Sofia*, BULGARIA, September, 14.-15., 1998, JBMSAEM’98, pp. 139-144.

[7] Gouljashki V., Marinova G., “Algorithm FCD, Determining the Spatial Parameters of Flaws in Materials”, *JSAEM Studies in Applied Electromagnetics and Mechanics 1999*, ISBN 4-931455-11-5, ISSN 1343-2689, (Joint Seminar Sapporo 1999), Hokkaido University, Sapporo, JAPAN, November 1-3, 1999, pp. 78-81.

[8] Guliashki V. and G. Marinova, “Distributed System for Parallel Data Processing of ECT Signals for Electromagnetic Flaw Detection in Materials”, 3-rd Japanese-Bulgarian-Macedonian Joint Seminar on Applied Electromagnetics JBMSAEM’2K, September 15-17. 2000, Ohrid, Macedonia, *Conference Digest*, pp. 71-73.

[9] Gray, G. A., “A Variational Study of the Electrical Impedance Tomography Problem”, Ph.D. Thesis, Rice University, Houston, Texas, April 2002.

[10] Hsiao C., Chahine G. and Gumerov N., „Application of a Hybrid Genetic / Powell Algorithm and a Boundary Element Method to Electrical Impedance Tomography”, *Journal of Computational Physics* 173, 2001, pp. 433-454.

[11] Kim S., Kim M., Kim K., Cho H. and Lee B., „Mesh Grouping and Boundary Estimate Algorithms for Potential Applications of Electrical Resistance Tomography”, *Journal of the Korean Physical Society*, vol. 47. No. 4, October 2005, pp. 597-602.

[12] Kohn, R. V. and McKenney A., „Numerical Implementation of a Variational Method for Electrical Impedance Tomography”, *Inverse Problems*, vol. 6, 1990, pp. 389-414.

[13] Kohn, R. V. and Vogelius M., „Relaxation of a Variational Method for Impedance Computed Tomography“, *Communications on Pure and Applied Mathematics* 40, 1987, pp. 745-777.

[14] Li Y, Xu G., Guo L., Wang L., Yang S., Rao L., He R. and Yan W., “Resistivity Parameters Estimation Based on 2D Real Head Model Using Improved Differential Evolution Algorithm”, 2006, http://embc2006.njit.edu/pdf/329_Li.pdf

[15] Marinova G. and V. Gouljashki, “2D Eddy Current Imaging for Steam Generator Tubing Based on Spline Interpolation Mapping and Improved Statistical Filtering”, *Proceedings of the 1st Japanese-Bulgarian-Macedonian Joint Seminar on Applied Electromagnetics*, Sofia, BULGARIA, September, 14.-15., 1998, JBMSAEM’98, pp. 145-150.

[16] Marinova G., Gouljashki V., “Distributed System for Research in Electromagnetic Flaw Detection in Power Plant Steam Generator Tubes”, *JSAEM Studies in Applied Electromagnetics and Mechanics 1999*, ISBN 4-931455-11-5, ISSN 1343-2689, (Joint Seminar Sapporo 1999), Hokkaido University, Sapporo, JAPAN, November 1-3, 1999, pp. 82-85.

[17] Olmi, R., Bini, M. and Priori, S., “A genetic algorithm approach to image reconstruction in electrical impedance tomography”, *IEEE Transactions on Evolutionary Computation*, Volume 4, Issue 1, April 2000, pp.:83 – 88.

[18] Papasov, S. P. and Farhy S. L., *Theoretical electro technique. Part* 3, 1988, (Sofia, Technika)

[19] Reklaitis G. V., Ravindran A., Ragsdell K. M., *Engineering Optimization. Methods and Applications*, John Wiley and Sons, 1983.

[20] Rolnik V.P, Seleghim P.Jr., „A Specialized Genetic Algorithm for Electrical Impedance Tomography of Two-Phase Flows”, *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, vol. XVIII, № 4, October – December 2006, pp. 378-389.

[21] Strang G. and Fix G., *An Analysis of the Finite Element Method*, 1973 (Englewood Cliffs, NJ: Prentice-Hall)

[22] Tamburino A., S. Ventre and G. Rubinacci, “Electrical Resistance Tomography: Complementarity and Quadratic Models”, *Inverse Problems*, vol. 16, 2000, pp. 1585-1618.

[23] Yorkey T., Webster J. and Tompkins W., “Comparing Reconstruction Algorithms for Electrical Impedance Tomography”, *IEEE Transactions on Biomedical Engineering*, BME-34, 1987, pp. 834-852.