A Method of Generating Non-Uniform Square QAM by Using Non-Linear Amplification

Dimitar Bojchev, Dobri Dobrev

Abstract - **The usual case of generating, transmitting and receiving a digital signal by using square QAM, provides equal protection of each symbol included in a super constellation of amplitude-phase positions of the signal vectors. In some cases it is necessary to protect one or a group of modulation symbols more than another. It can be done by using nonuniform modulation. In this article is proposed a method for generation and detection of nonuniform QAM signal.**

***Keywords* – nonuniform QAM, different protection of symbols**

I. INTRODUCTION

A special interest is the transmission of a square QAM signal through to the Gaussian channel. In this way, as it is well known, the probability density function of the signal vector modulation components position, can be expressed by normal distribution [1], [2], [3]:

$$P_i(N_0) = \frac{1}{\sigma_\xi \sqrt{2.\pi}}.e^{-\frac{(N_0 - I_i)^2}{2.\sigma_\xi^2}} \tag{1}$$

Respectively

$$P_q(N_0) = \frac{1}{\sigma_\xi \sqrt{2.\pi}}.e^{-\frac{(N_0 - Qq)^2}{2.\sigma_\xi^2}} \tag{2}$$

where σ_ζ^2 is the noise variance, I_i and Q_q are the means of the corresponding magnitude, e.g. the expecting positions of I and Q modulation component for *iq* signal vector position. Thus the area of right detection is circle and depends on noise variance and intensity – Fig. 1. It can be shown an influence of Additive White Gaussian Noise (AWGN) into a 256-QAM super constellation, depicted in Fig.2. Observing a rectangular constellation, the Euclidian distance between any pair of neighbor signal vector positions and respectively the symbols protection is equal in the frame of one quadrant. The Euclidian distance between two adjacent symbols can be expressed [3]:

$$d_{ij} = \sqrt{\int_0^T \left[S_i(t) - S_j(t)\right]^2 dt} \tag{3}$$

[1]Dimitar Bojchev is post graduated student in Faculty of Telecommunications, Technical University - Sofia, 8 Kliment Ohridski St. Darvenitsa, 1756, Sofia, Bulgaria, E-mail: dbojchev@tu-sofia.bg

[2]Dobri Dobrev is professor with the Faculty of Telecommunications,Technical University - Sofia, 8 Kliment Ohridski St.Darvenitsa, 1756, Sofia, Bulgaria, E-mail: dobrev@tu-sofia.bg

Where T is time length of the transmitted symbol.

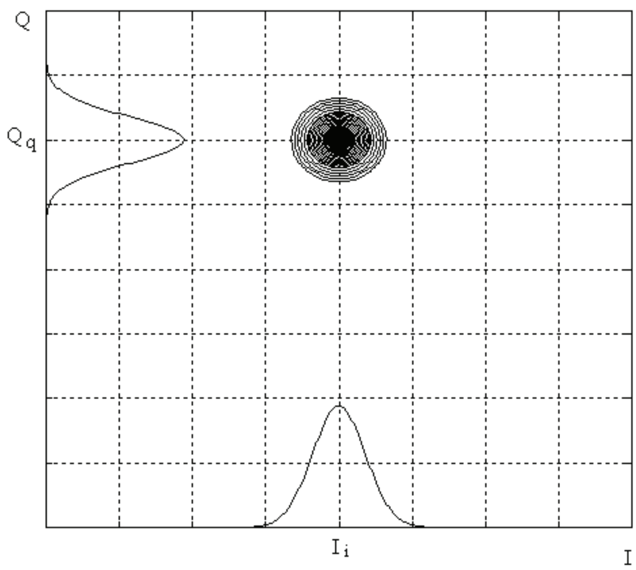

Fig.1. Probability density distribution of modulation components of one vector transmitted position true to the Gaussian channel

Considering detection process in its worst case, the symbol error probability function can be computed by cumulative density function of the Gaussian distribution [1]:

$$P_i(e) = \frac{1}{\sqrt{2.\pi}} \int_{\frac{d}{\sqrt{No/2}}}^{\infty} e^{-\frac{x^2}{2}} dx \tag{4}$$

Where $N_o / 2$ is double sided nose spectral density for each of quadrature components.
In another way, it can be expressed [2], [3]:

$$P(e) = \frac{M-1}{2} erfc\left(\frac{d_{ij}}{2.\sqrt{N_0}}\right) \tag{5}$$

It can be made a substitution of (3) in (5) and hence it becomes:

$$P(e) = \frac{M-1}{2} erfc\left(\frac{\sqrt{\int_0^T \left[S_i(t) - S_j(t)\right]^2 dt}}{2.\sqrt{N_0}}\right), \tag{6}$$

where S_i and S_j are any pair of adjacent symbols.

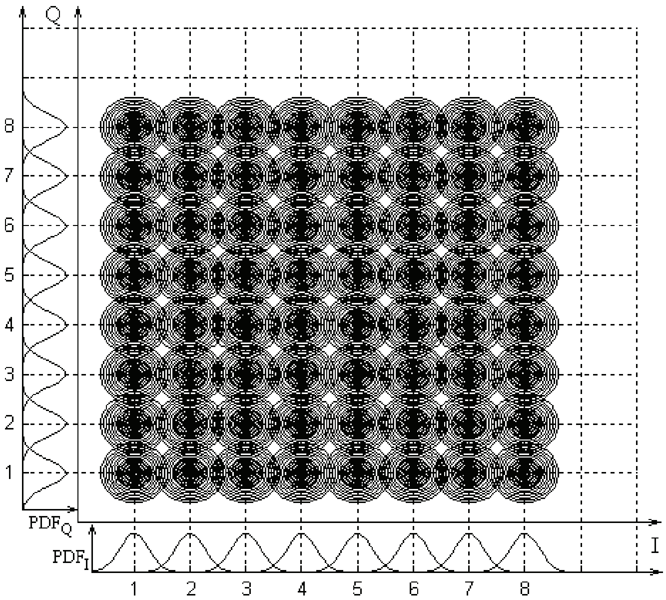

Fig.2. 256-QAM super constellation, passed through a Gaussian channel

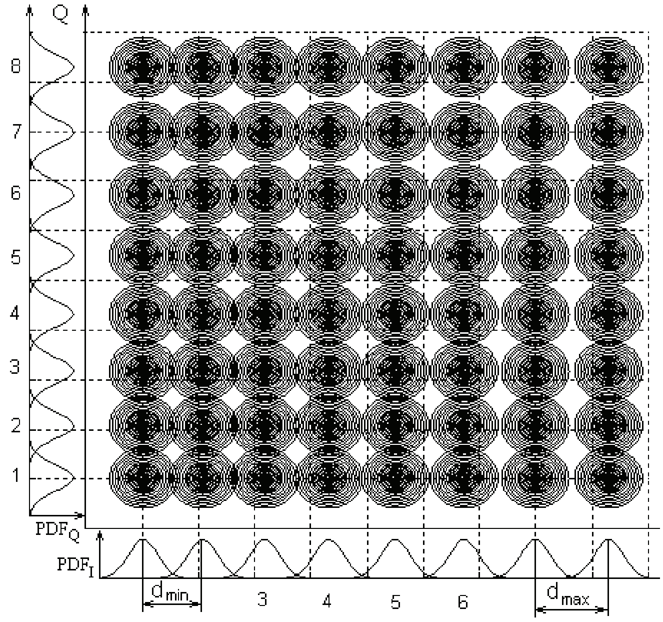

Fig.3. Deformation of 256QAM super constellation, passed through to a third order non-linearity with coefficients a_2=19,9.10^{-3}; a_3 = 2,98.10^{-5}

II. GENERATING NON-UNIFORM CONSTELLATION

A non-uniform QAM can be obtained by using stage with non-linear transfer function. It can be expressed by third order series expansion [6],[7],[8],[9],[10]:

$$U_{out} = a_0.[Us(t)]^0 + a_1[Us(t)]^1 + a_2[Us(t)]^2 + a_3[Us(t)]^3, \quad (7)$$

where $Us(t)$ is the input signal of the considering stage, a_0,a_1,a_2,a_3 are the polynomial coefficients

In fact non-linearity depends on square and cubic polynomials coefficients. In this case formula (7) can be expressed as:

$$U_{out} = Us(t) + a_2[Us(t)]^2 + a_3[Us(t)]^3 \quad (8)$$

For example, by using a Matlab simulation, the deformation of a one quadrant 256-QAM super constellation can be shown, obtained by coefficients a_2=19,9.10^{-3}; a_3 = 2,98.10^{-5} – Fig.3. The Euclidian distance between adjacent vector signal positions and respectively between adjacent thresholds is different in comparison with rectangular uniform QAM. The distance can be expressed [3]:

$$d^e = d\sqrt{2.E_g} \quad (9)$$

Where E_g is a symbol energy.

The absolute distance between minimal and maximal value in percents is:

$$\Delta d^e = (d_{\max} - d_{\min})\sqrt{2.E_g}.100[\%] \quad (10)$$

It is important to be noted that in the receiver has to be included a stage with inverse non-linearity.

To avoid mixing of signal spectral components, one must not use an analog non-linearity stage. The transmitter's non-linearity has to be realized by addition to I and Q modulating signals of a sum of square and cubic means of them e.g. corresponding modulation component with a value, defined by the polynomial coefficients a_2 and a_3. In this case the most suitable decision is realized by digital signal processing architecture. In the receiver detector equipment has to be included inverse non-linearity processing. It is necessary to reconstruct the exact uniform QAM constellation amplitude-phase positions of the signal vector.

III. ESTIMATING OF NOISE PROTECTION EFFICIENCY

The main question is to define the influence of the variance of Euclidian distance into the symbols nose protection. It can be done by analyzing the symbol error probability of maximal and minimal Euclidian distance. Thus, it can be written:

$$P(e)_\Delta = \frac{M-1}{2} erfc\left(0,7.(d_{\max} - d_{\min}).\sqrt{\frac{E_g}{N_0}}\right) \quad (11)$$

A Matlab simulation is made of 256-QAM and 10% distance between maximal and minimal Euclidian value in different signal-to-noise ratio – Fig.4. The same simulation is also made for 128-QAM –fig.5. The figures show that the effect of decreasing a symbol error ratio in dependence of introduced nonlinearity into the communication channel is proportional of the dynamic range. Furthermore it depends on a modulation order. Since the dynamic range is expressed in dB, the dependence is approximately linear. Hence it can be composed a simple approximates equation:

$$SER(\Delta d) = -k.\Delta d + SER_0 \quad (12)$$

The coefficient k and the constant SER_0 depend on signal-to-noise ratio. It is shown in fig.6, fig.7 for 128-QAM and 256-QAM super constellation.

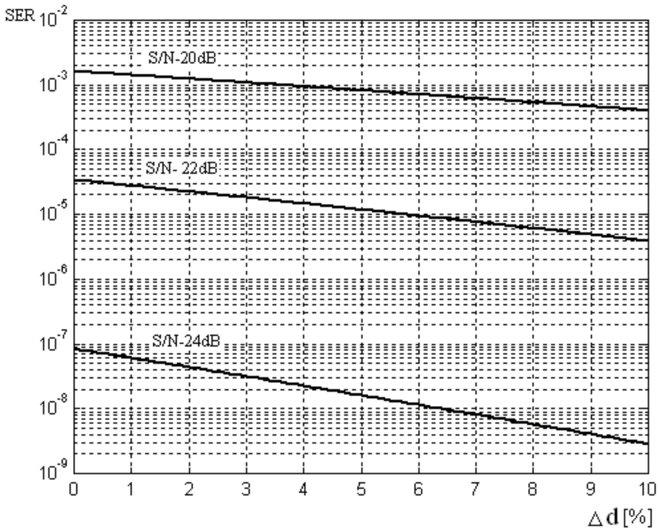

Fig.4. An increasing of symbol error ratio of 256-QAM super constellation in dependence of variance of the Euclidian value distance

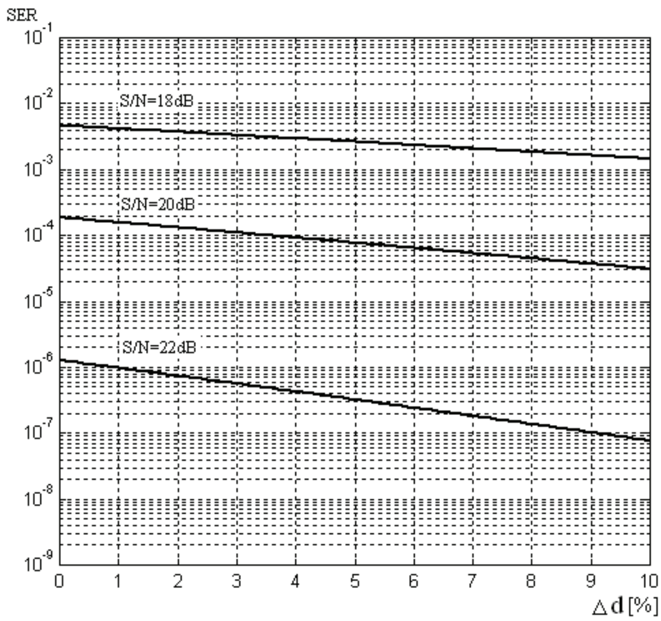

Fig.5. An increasing of symbol error ratio of 128-QAM super constellation in dependence of varying of the Euclidian distance value

Bearing in mind the above, the major question is a polynomial coefficients adjustment. Practically it is more convenient to work with relative estimation of Euclidian distance varying:

$$\delta_{d^e} = \frac{d_{\max}}{d_{\min}} \quad (13)$$

The number of amplitude levels is:

$$m = \frac{\sqrt{M}}{2} \quad (14)$$

The exact value m-th position, after non-linear processing can be calculated by substitution in (8) :

$$I_m = m + a_2.m^2 + a_3.m^3 \quad (15)$$

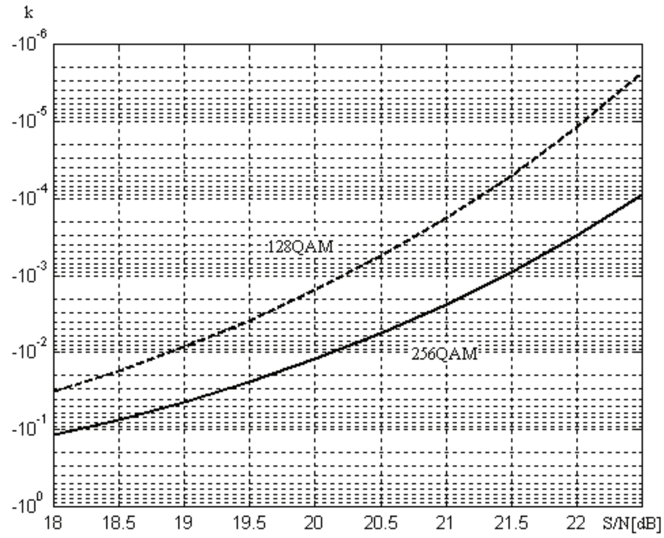

Fig.6. The coefficient k in dependence of signal to noise ratio SNR for 128-QAM and 256 QAM super constellation

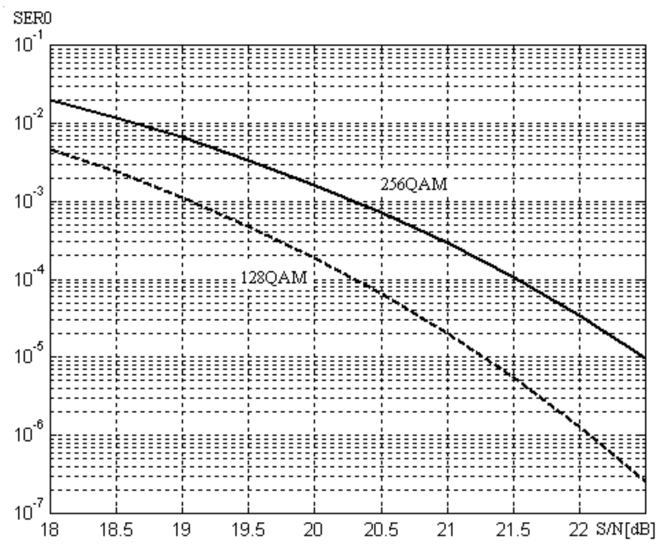

Fig.7. The constant SER_0 in dependence of signal to noise ratio SNR for 128-QAM and 256 QAM super constellation

A formula (15) is correct if a_1 =1, e.g. there is no amplification in non-linearity processing. In this way can be defined $m-1$-th position.

$$I_{m-1} = m - 1 + a_2.(m-1)^2 + a_3.(m-1)^3 \quad (15)$$

Thus d_{max} will be:

$$d_{\max}^{e} = I_m - I_{m-1} = a_2.(m-1) - a_3.(m^2 - 3.m + 1) - 1 \quad (16)$$

And

$$d_{\min}^{e} = I_2 - I_1 = 3.a_2 + 7.a_3 + 1 \quad (17)$$

After subtracting (16) of (17) and substituting (13) it can be written:

$$\Delta d^{e} = d_{\min}^{e}(\delta - 1) = a_2.(m-4) - a_3.(m^2 - 3.m - 6) \quad (18)$$

If $d_{\min}$=1 and then:

$$\delta = a_2.(m-4) - a_3.(m^2 - 3.m - 6) + 1 \quad (19)$$

Finally by substituting (14) in (19) it will be obtained:

$$\delta = a_2.\left(\frac{\sqrt{M}}{2} - 4\right) - a_3.\left(\frac{M}{4} - \frac{3}{2}.\sqrt{M} - 6\right) + 1 \quad (20)$$

By using formula (20), a choice can be made of cubic and square coefficients a_2 and a_3. The computing of exact values is an interactive process.

Another important question is the gain in equivalent dynamic range at positions with maximal distance in comparison with those with minimal, especially the influence of symbol error ratio (SER). It can be estimated by using (11). In fig. 8 are composed two graphics of SER in dependence of signal to noise ratio (SNR) for 128-QAM and 256-QAM.

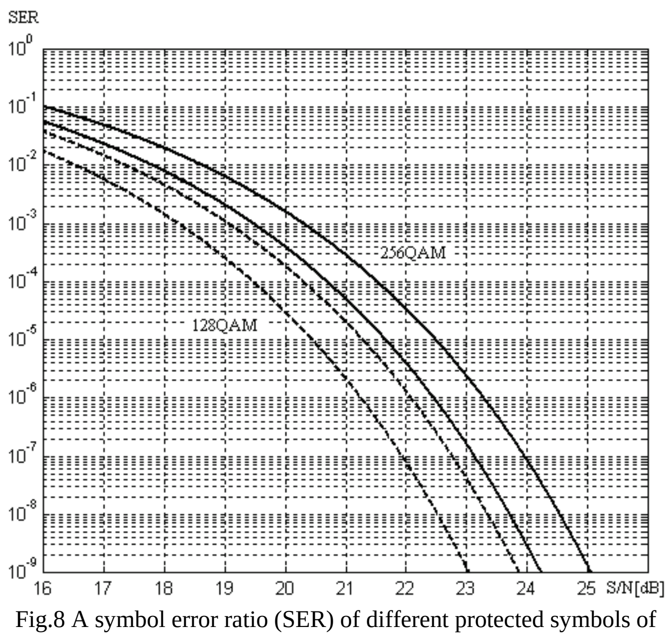

Fig.8 A symbol error ratio (SER) of different protected symbols of 128-QAM(dash line) and 256-QAM constellation in dependence of signal to noise ratio SNR

IV. Conclusion

In the current paper is proposed a method for different protection of QAM super constellation symbols when the signal passed trough to a Gaussian Channel. This goal is achieved by using non-linear processing and is realized literal, determinate shifting for each of the amplitude-phase signal vector positions. Hence is obtained a non-uniform QAM constellation. In the receiver side the constellation is reconstructing by inverse non linear processing. Furthermore is made an analysis of noise protection efficiency and an expression is given for numerical estimation.

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