Level Crossing Rate of the SSC Combiner Output Signal in the Presence of Log-normal Fading

Dragana Krstić[1], Petar Nikolić[2], Dušan Stefanović[3], Ilija Temelkovski[4]

Abstract - **Level crossing rate of the SSC combiner output signal are determined in this paper. The presence of the log-normal fading at the input is observed. The results are shown graphically for different variance values and decision threshold values.**

Keywords - **Level Crossing Rate, Log-normal Fading, SSC Combining**

I. INTRODUCTION

Many of the wireless communication systems use some form of diversity combining to reduce multupath fading appeared in the channel. Among the simpler diversity combining schemes, the two most popular are selection combining (SC) and switch and stay combining (SSC). SSC is an attempt at simplifying the complexity of the system but with loss in performance. In this case the receiver selects a particular antenna until its quality drops below a predetermined threshold. When this happens, the receiver switches to another antenna and stays with it for the next time slot, regardless of whether or not the channel quality of that antenna is above or below the predetermined threshold.

In the paper [1] Alouini and Simon develop, analyze and optimize a simple form of dual-branch switch and stay combining (SSC).

The consideration of SSC systems in the literature has been restricted to low-complexity mobile units where the number of diversity antennas is typically limited to two ([2], [3] and [4]). Furthermore, in all these publications, only predetection SSC has thus far been considered wherein the switching of the receiver between the two receiving antennas is based on a comparison of the instantaneous SNR of the connected antenna with a predetermined threshold. This results in a reduction in complexity relative to SC in that the simultaneous and continuous monitoring of both branches SNRs is no longer necessary.

In [5] the moment generating function (MGF) of the signal power at the output of dual-branch switch-and-stay selection diversity (SSC) combiners is derived.

In this paper level crossing rate of the SSC combiner output signal in the presence of log-normal fading will be determine. The results will be shown graphically for different variance values and decision threshold values.

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II. SYSTEM MODEL

The model of the SSC combiner with two inputs, considered in this paper, is shown in Fig. 1. The signals at the combiner input are r_1 and r_2, and r is the combiner output signal. The predetection combinig is observed.

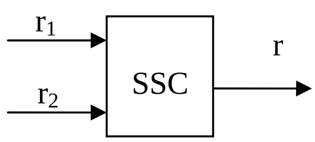

Fig. 1. Model of the SSC combiner with two inputs

The probability of the event that the combiner first examines the signal at the first input is P_1, and for the second input is P_2. If the combiner examines first the signal at the first input and if the value of the signal at the first input is above the treshold, r_T, SSC combiner forwards this signal to the circuit for the decision. If the value of the signal at the first input is below the treshold r_T, SSC combiner forwards the signal from the other input to the circuit for the decision.

We can select decision threshold value so that one of three parameters has to be minimal: the error probability, fade duration or average signal value.

If the SSC combiner first examines the signal from the second combiner input it works in the similar way.

The determination of the probability density of the combiner output signal is important for the receiver performances determination. The probability for the first input to be examined first is P_1 and for the second input to be examined first is P_2.

III. SYSTEM PERFORMANCES

The probability densitie functions (PDFs) of the combiner input signals, r_1 and r_2 , in the presence of log-normal fading, are:

$$p_{r_1}(r_1) = \frac{1}{\sqrt{2\pi}\,\sigma_1 r_1} e^{-\frac{(\ln r_1 - \mu_1)^2}{2\sigma_1^2}} \qquad r_1 \geq 0 \qquad (1)$$

$$p_{r_2}(r_2) = \frac{1}{\sqrt{2\pi}\sigma_2 r_2} e^{-\frac{(\ln r_2 - \mu_2)^2}{2\sigma_2^2}} \qquad r_2 \geq 0 \tag{2}$$

The cumulative probability densities (CDFs) are given by:

$$F_{r_1}(r_T) = \int_0^{r_T} p_{r_1}(x)\,dx \tag{3}$$

$$F_{r_2}(r_T) = \int_0^{r_T} p_{r_2}(x)\,dx \tag{4}$$

r_T is the treshold of the decision. In the presence of log-normal fading CDFs are:

$$F_{r_1}(r_T) = \int_0^{r_T} \frac{1}{\sqrt{2\pi}\sigma_1 x} e^{-\frac{(\ln x-\mu_1)^2}{2\sigma_1^2}} dx = \frac{1}{2} + \frac{1}{2} erf\left(\frac{\ln r_t - \mu_1}{\sigma_1\sqrt{2}}\right) \tag{5}$$

$$F_{r_2}(r_T) = \int_0^{r_T} \frac{1}{\sqrt{2\pi}\sigma_2 x} e^{-\frac{(\ln x-\mu_2)^2}{2\sigma_2^2}} dx = \frac{1}{2} + \frac{1}{2} erf\left(\frac{\ln r_t - \mu_2}{\sigma_2\sqrt{2}}\right) \tag{6}$$

where *erfc(x)* is the error function and it is defined as [7]:

$$erf\,(x) = \frac{2}{\sqrt{\pi}} \int_0^{x} e^{-t^2}\,dt$$

The joint probability densities of the combiner input signals, r_1 and r_2, and their derivatives $\dot{r}_1$ and $\dot{r}_2$, in the presence of log-normal fading, are:

$$p_{r_1\dot{r}_1}(r_1, \dot{r}_1) = \frac{1}{\sqrt{2\pi}\sigma_1 r_1} e^{-\frac{(\ln r_1-\mu_1)^2}{2\sigma_1^2}} \cdot \frac{1}{\sqrt{2\pi}\,\beta_1 r_1} e^{-\frac{\dot{r}_1^2}{2\beta_1^2 r_1^2}} \qquad r_1 \geq 0 \tag{7}$$

$$p_{r_2\dot{r}_2}(r_2, \dot{r}_2) = \frac{1}{\sqrt{2\pi}\sigma_2 r_2} e^{-\frac{(\ln r_2-\mu_2)^2}{2\sigma_2^2}} \cdot \frac{1}{\sqrt{2\pi}\,\beta_2 r_2} e^{-\frac{\dot{r}_2^2}{2\beta_2^2 r_2^2}} \qquad r_2 \geq 0 \tag{8}$$

The probabilities P_1 and P_2 are:

$$P_1 = \frac{F_{r_2}(r_T)}{F_{r_1}(r_T) + F_{r_2}(r_T)} = \frac{1 + erf\left(\frac{\ln r_t - \mu_2}{\sigma_2\sqrt{2}}\right)}{2 + erf\left(\frac{\ln r_t - \mu_1}{\sigma_1\sqrt{2}}\right) + erf\left(\frac{\ln r_t - \mu_2}{\sigma_2\sqrt{2}}\right)} \tag{9}$$

$$P_2 = \frac{F_{r_1}(r_T)}{F_{r_1}(r_T) + F_{r_2}(r_T)} = \frac{1 + erf\left(\frac{\ln r_t - \mu_1}{\sigma_1\sqrt{2}}\right)}{2 + erf\left(\frac{\ln r_t - \mu_1}{\sigma_1\sqrt{2}}\right) + erf\left(\frac{\ln r_t - \mu_2}{\sigma_2\sqrt{2}}\right)} \tag{10}$$

The expression for the joint probability density function of the SSC combiner output signal and its derivative will be determined first for the case: $r < r_T$:

$$p_{r\dot{r}}(r\dot{r}) = P_1 \cdot F_{r_1}(r_T) \cdot p_{r_2\dot{r}_2}(r\dot{r}) + P_2 \cdot F_{r_2}(r_T) \cdot p_{r_1\dot{r}_1}(r\dot{r}) \tag{11}$$

and then for $r \geq r_T$:

$$p_{r\dot{r}}(r\dot{r}) = P_1 \cdot p_{r_1\dot{r}_1}(r\dot{r}) + P_1 \cdot F_{r_1}(r_T) \cdot p_{r_2\dot{r}_2}(r\dot{r}) + P_2 \cdot p_{r_2\dot{r}_2}(r\dot{r}) + P_2 \cdot F_{r_2}(r_T) \cdot p_{r_1\dot{r}_1}(r\dot{r}) \tag{12}$$

We have now for $r < r_T$:

$$\begin{aligned} p_{r\dot{r}}(r,\dot{r}) = {} & \frac{1+erf\left(\frac{\ln r_t - \mu_2}{\sigma_2\sqrt{2}}\right)}{2+erf\left(\frac{\ln r_t - \mu_1}{\sigma_1\sqrt{2}}\right)+erf\left(\frac{\ln r_t - \mu_2}{\sigma_2\sqrt{2}}\right)} \cdot \\ & \cdot \left(\frac{1}{2}+\frac{1}{2}erf\left(\frac{\ln r_t - \mu_1}{\sigma_1\sqrt{2}}\right)\right) \frac{1}{\sqrt{2\pi}\sigma_2 r} e^{-\frac{(\ln r-\mu_2)^2}{2\sigma_2^2}} \frac{1}{\sqrt{2\pi}\,\beta_2 r} e^{-\frac{\dot{r}^2}{2\beta_2^2 r^2}} + \\ & + \frac{1+erf\left(\frac{\ln r_t - \mu_1}{\sigma_1\sqrt{2}}\right)}{2+erf\left(\frac{\ln r_t - \mu_1}{\sigma_1\sqrt{2}}\right)+erf\left(\frac{\ln r_t - \mu_2}{\sigma_2\sqrt{2}}\right)} \cdot \\ & \cdot \left(\frac{1}{2}+\frac{1}{2}erf\left(\frac{\ln r_t - \mu_2}{\sigma_2\sqrt{2}}\right)\right) \frac{1}{\sqrt{2\pi}\sigma_1 r} e^{-\frac{(\ln r-\mu_1)^2}{2\sigma_1^2}} \frac{1}{\sqrt{2\pi}\,\beta_1 r} e^{-\frac{\dot{r}^2}{2\beta_1^2 r^2}} \end{aligned} \tag{13}$$

and for $r \geq r_T$:

$$\begin{aligned} p_{r\dot{r}}(r,\dot{r}) = {} & \frac{1+erf\left(\frac{\ln r_t - \mu_2}{\sigma_2\sqrt{2}}\right)}{2+erf\left(\frac{\ln r_t - \mu_1}{\sigma_1\sqrt{2}}\right)+erf\left(\frac{\ln r_t - \mu_2}{\sigma_2\sqrt{2}}\right)} \frac{1}{\sqrt{2\pi}\sigma_1 r} e^{-\frac{(\ln r-\mu_1)^2}{2\sigma_1^2}} \cdot \\ & \cdot \frac{1}{\sqrt{2\pi}\,\beta_1 r} e^{-\frac{\dot{r}^2}{2\beta_1^2 r^2}} + \frac{1+erf\left(\frac{\ln r_t - \mu_2}{\sigma_2\sqrt{2}}\right)}{2+erf\left(\frac{\ln r_t - \mu_1}{\sigma_1\sqrt{2}}\right)+erf\left(\frac{\ln r_t - \mu_2}{\sigma_2\sqrt{2}}\right)} \cdot \\ & \cdot \left(\frac{1}{2}+\frac{1}{2}erf\left(\frac{\ln r_t - \mu_1}{\sigma_1\sqrt{2}}\right)\right) \frac{1}{\sqrt{2\pi}\sigma_2 r} e^{-\frac{(\ln r-\mu_2)^2}{2\sigma_2^2}} \frac{1}{\sqrt{2\pi}\,\beta_2 r} e^{-\frac{\dot{r}^2}{2\beta_2^2 r^2}} + \\ & + \frac{1+erf\left(\frac{\ln r_t - \mu_1}{\sigma_1\sqrt{2}}\right)}{2+erf\left(\frac{\ln r_t - \mu_1}{\sigma_1\sqrt{2}}\right)+erf\left(\frac{\ln r_t - \mu_2}{\sigma_2\sqrt{2}}\right)} \cdot \\ & \cdot \frac{1}{\sqrt{2\pi}\sigma_2 r} e^{-\frac{(\ln r-\mu_2)^2}{2\sigma_2^2}} \frac{1}{\sqrt{2\pi}\,\beta_2 r} e^{-\frac{\dot{r}^2}{2\beta_2^2 r^2}} + \\ & + \frac{1+erf\left(\frac{\ln r_t - \mu_1}{\sigma_1\sqrt{2}}\right)}{2+erf\left(\frac{\ln r_t - \mu_1}{\sigma_1\sqrt{2}}\right)+erf\left(\frac{\ln r_t - \mu_2}{\sigma_2\sqrt{2}}\right)} \left(\frac{1}{2}+\frac{1}{2}erf\left(\frac{\ln r_t - \mu_2}{\sigma_2\sqrt{2}}\right)\right) \cdot \end{aligned}$$

$$\cdot\frac{1}{\sqrt{2\pi}\sigma_1 r}e^{-\frac{(\ln r-\mu_1)^2}{2\sigma_1{}^2}}\frac{1}{\sqrt{2\pi}\,\beta_1 r}e^{-\frac{\dot{r}^2}{2\beta_1^2 r^2}} \tag{14}$$

For the channels with identical parameters it is, for $r < r_T$:

$$p_{r\dot{r}}(r,\dot{r}) = \left(\frac{1}{2}+\frac{1}{2}erf\left(\frac{\ln r_t-\mu}{\sigma\sqrt{2}}\right)\right)\frac{1}{\sqrt{2\pi}\sigma r}e^{-\frac{(\ln r-\mu)^2}{2\sigma^2}}\frac{1}{\sqrt{2\pi}\,\beta r}e^{-\frac{\dot{r}^2}{2\beta^2 r^2}} \tag{15}$$

and for $r \ge r_T$:

$$p_{r\dot{r}}(r,\dot{r}) = \left(\frac{3}{2}+\frac{1}{2}erf\left(\frac{\ln r_t-\mu}{\sigma\sqrt{2}}\right)\right)\frac{1}{\sqrt{2\pi}\sigma r}e^{-\frac{(\ln r-\mu)^2}{2\sigma^2}}\frac{1}{\sqrt{2\pi}\,\beta r}e^{-\frac{\dot{r}^2}{2\beta^2 r^2}} \tag{16}$$

The level crossing rate is:

$$N(r_{th}) = \int_0^{\infty} \dot{r}\, p_{r\dot{r}}(r_{th},\dot{r})\, d\dot{r} \tag{17}$$

for $r_{th} < r_T$:

$$N(r_{th}) = \frac{1+erf\left(\frac{\ln r_t-\mu_2}{\sigma_2\sqrt{2}}\right)}{2+erf\left(\frac{\ln r_t-\mu_1}{\sigma_1\sqrt{2}}\right)+erf\left(\frac{\ln r_t-\mu_2}{\sigma_2\sqrt{2}}\right)}\cdot \left(\frac{1}{2}+\frac{1}{2}erf\left(\frac{\ln r_t-\mu_1}{\sigma_1\sqrt{2}}\right)\right)\frac{1}{\sqrt{2\pi}\sigma_2 r_{th}}e^{-\frac{(\ln r_{th}-\mu_2)^2}{2\sigma_2{}^2}}\frac{\beta_2 r_{th}}{\sqrt{2\pi}} +$$
$$+\frac{1+erf\left(\frac{\ln r_t-\mu_1}{\sigma_1\sqrt{2}}\right)}{2+erf\left(\frac{\ln r_t-\mu_1}{\sigma_1\sqrt{2}}\right)+erf\left(\frac{\ln r_t-\mu_2}{\sigma_2\sqrt{2}}\right)}\cdot \left(\frac{1}{2}+\frac{1}{2}erf\left(\frac{\ln r_t-\mu_2}{\sigma_2\sqrt{2}}\right)\right)\frac{1}{\sqrt{2\pi}\sigma_1 r_{th}}e^{-\frac{(\ln r_{th}-\mu_1)^2}{2\sigma_1{}^2}}\frac{\beta_1 r_{th}}{\sqrt{2\pi}} \tag{18}$$

and for $r_{th} \ge r_T$:

$$N(r_{th}) = \frac{1+erf\left(\frac{\ln r_t-\mu_2}{\sigma_2\sqrt{2}}\right)}{2+erf\left(\frac{\ln r_t-\mu_1}{\sigma_1\sqrt{2}}\right)+erf\left(\frac{\ln r_t-\mu_2}{\sigma_2\sqrt{2}}\right)}\cdot\frac{1}{\sqrt{2\pi}\sigma_1 r_{th}}e^{-\frac{(\ln r_{th}-\mu_1)^2}{2\sigma_1{}^2}}\cdot\frac{\beta_1 r_{th}}{\sqrt{2\pi}}+\frac{1+erf\left(\frac{\ln r_t-\mu_2}{\sigma_2\sqrt{2}}\right)}{2+erf\left(\frac{\ln r_t-\mu_1}{\sigma_1\sqrt{2}}\right)+erf\left(\frac{\ln r_t-\mu_2}{\sigma_2\sqrt{2}}\right)}\cdot$$
$$\cdot\left(\frac{1}{2}+\frac{1}{2}erf\left(\frac{\ln r_t-\mu_1}{\sigma_1\sqrt{2}}\right)\right)\frac{1}{\sqrt{2\pi}\sigma_2 r_{th}}e^{-\frac{(\ln r_{th}-\mu_2)^2}{2\sigma_2{}^2}}\frac{\beta_2 r_{th}}{\sqrt{2\pi}}+$$
$$+\frac{1+erf\left(\frac{\ln r_t-\mu_1}{\sigma_1\sqrt{2}}\right)}{2+erf\left(\frac{\ln r_t-\mu_1}{\sigma_1\sqrt{2}}\right)+erf\left(\frac{\ln r_t-\mu_2}{\sigma_2\sqrt{2}}\right)}\frac{1}{\sqrt{2\pi}\sigma_2 r_{th}}e^{-\frac{(\ln r_{th}-\mu_2)^2}{2\sigma_2{}^2}}\frac{\beta_2 r_{th}}{\sqrt{2\pi}}+$$
$$+\frac{1+erf\left(\frac{\ln r_t-\mu_1}{\sigma_1\sqrt{2}}\right)}{2+erf\left(\frac{\ln r_t-\mu_1}{\sigma_1\sqrt{2}}\right)+erf\left(\frac{\ln r_t-\mu_2}{\sigma_2\sqrt{2}}\right)}\cdot$$
$$\cdot\left(\frac{1}{2}+\frac{1}{2}erf\left(\frac{\ln r_t-\mu_2}{\sigma_2\sqrt{2}}\right)\right)\frac{1}{\sqrt{2\pi}\sigma r_{th}}e^{-\frac{(\ln r_{th}-\mu_1)^2}{2\sigma_1{}^2}}\frac{\beta_1 r_{th}}{\sqrt{2\pi}} \tag{19}$$

For the channels with identical parameters it is valid for $r_{th} < r_T$:

$$N(r_{th}) = \left(\frac{1}{2}+\frac{1}{2}erf\left(\frac{\ln r_t-\mu}{\sigma\sqrt{2}}\right)\right)\frac{1}{\sqrt{2\pi}\sigma r_{th}}e^{-\frac{(\ln r_{th}-\mu)^2}{2\sigma^2}}\frac{\beta r_{th}}{\sqrt{2\pi}} \tag{20}$$

and for $r_{th} \ge r_T$:

$$N(r_{th}) = \left(\frac{3}{2}+\frac{1}{2}erf\left(\frac{\ln r_t-\mu}{\sigma\sqrt{2}}\right)\right)\frac{1}{\sqrt{2\pi}\sigma r_{th}}e^{-\frac{(\ln r_{th}-\mu)^2}{2\sigma^2}}\frac{\beta r_{th}}{\sqrt{2\pi}} \tag{21}$$

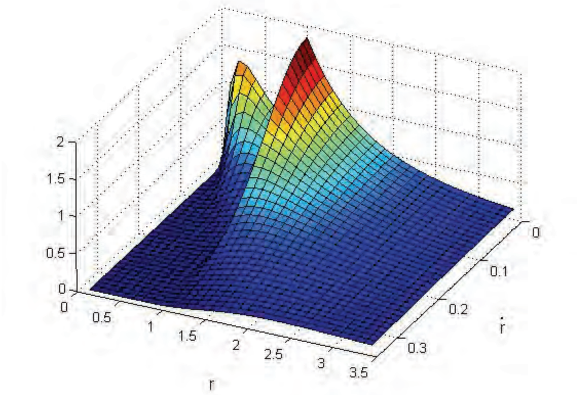

Fig. 2. The PDF of the SSC combiner output signal and its derivative $p_{r\dot{r}}(r,\dot{r})$ for r_T =1, σ=1, μ=0.5 and β=0. 1

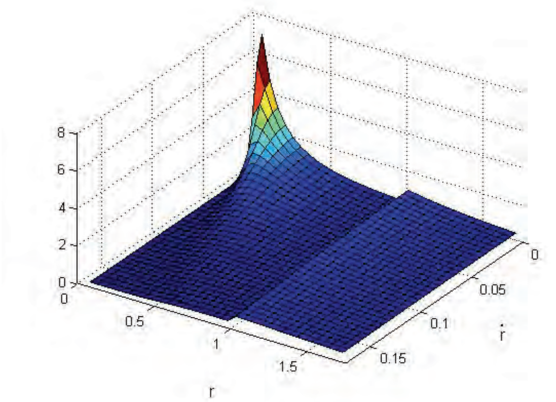

Fig. 3. The joint PDF of the SSC combiner output signal and its derivative $p_{r\dot{r}}(r,\dot{r})$ for r_T =1, σ=1.5, μ=0.5 and β=0.15

IV. Numerical Results

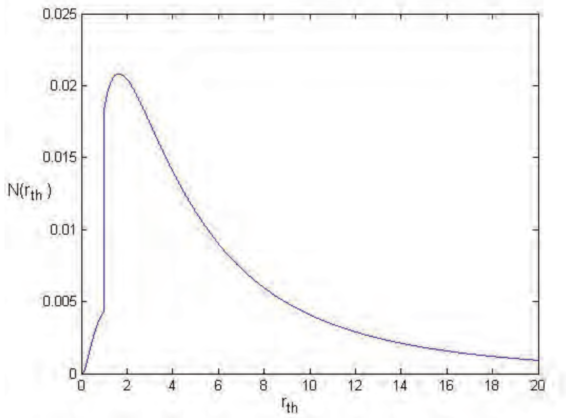

Fig. 4. Level crossing rate $N(r_{th})$ for r_T =1, σ=1, μ=0.5 and β=0.1

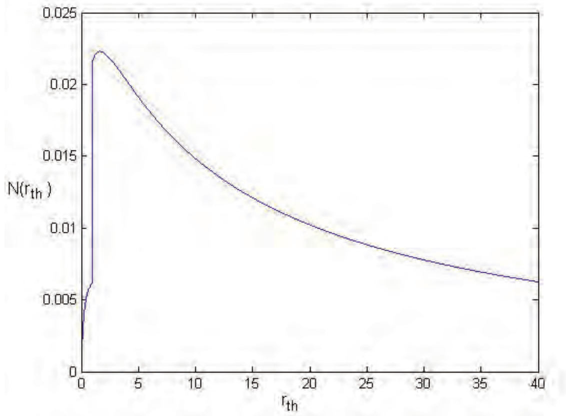

Fig. 5. Level crossing rate $N(r_{th})$ for r_T =1, σ=2, μ=0.5 and β=0.2

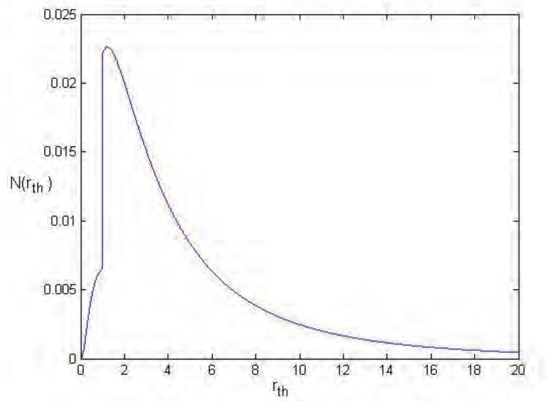

Fig. 6. Level crossing rate $N(r_{th})$ for r_T =1, σ=1, μ=0.2 and β=0.1

The joint probability density functions (PDFs) of the SSC combiner output signal are shown in Figs. 2 and 3. for diferent values of r_T, σ, μ and β. The level crossing rate curves $N(r_{th})$, for diferent parameters are given in Figs. 4 to 7.

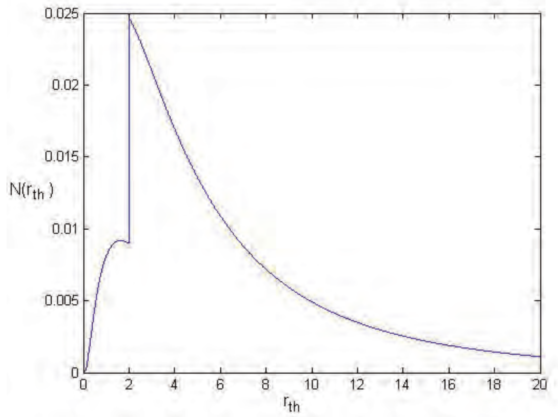

Fig. 7. Level crossing rate $N(r_{th})$ for r_T =2, σ=1, μ=0.5 and β=0.1

V. Conclusion

In this paper the level crossing rate (LCR) of the SSC combiner output signal is determined in the presence of log-normal fading. The results are shown graphically for different variance values and decision threshold values.

We determine LCR in order to obtain Fade Duration of SSC Combiner [7]. In our future work these system performances can be derived for correlated log-normal fading and for some other fading distributions.

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