Outage Probability of SIR-Based Dual EGC Diversity Over Correlated Rayleigh Fading Channel

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Abstract - **In this paper the performance analysis of SIR-based dual EGC receiver with two correlated branches over Rayleigh fading channel is considered. Analytical and numerical results for the outage probability, assuming correlated Rayleigh fading for both the desired signals and co-channel interferers, are derived. This is the real scenario in practical dual equal-gain diversity systems with insufficient antenna spacing. Numerical results demonstrate the effect of balanced and unbalanced SIRs and various values of the correlation coefficient on the EGC receiver performance.**

Keywords – **Co-channel interference, Equal-gain combining, Outage probability, Rayleigh fading channels**

I. INTRODUCTION

Spatial diversity or "signals from multiple antennas" are often used to reduce the effects of fading in wireless communications. There are a lot of types of combining techniques such as selection combining (SC), equal-gain combining (EGC), maximal-ratio combining (MRC), or a combination of MRC and SC called generalized selection combining (GSC) [1], [2]. Among them, EGC provides performance better than SC and comparable to MRC but with simpler implementation complexity. It is a great practical solution. In EGC the received signals are co-phased, equally weighted, and then summed to form the resultant signal.

Particularly, independent fading assumes antenna elements to be placed sufficiently apart, which is not always realized in practice due to insufficient antenna spacing when diversity is applied in compact terminals [2]. In this kind of terminal, the fading among the channels is correlated, resulting in a degradation of the diversity gain obtained. Therefore, it is important to understand how the correlation between received signals affects the system performance. Also, in cellular communications systems where the level of the co-channel interference is sufficiently high as compared to the thermal noise, the most effective performance criterion is to select the highest signal-to-interference ratio (SIR; SIR-based selection diversity) [3].

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In two early works [5], [6], useful analytical expressions for the outage performance of EGC receivers operating in an interference-limited Rayleigh fading environment have been obtained. The results were derived in environment with multiple co-channel interferers but no correlated branches. In another related work [4], the outage performance of EGC receivers under Nakagami-*m* fading channels and CCI (co-channel interference) has been studied. The case of correlated desired signals and uncorrelated interferers was observed. A method for the evaluation of the outage probability in dual SIR-based SC with correlated Rayleigh fading, for both desired signals and interferers, has been published in [3]. An approach to the performance analysis of dual SIR-based SC over correlated Nakagami-*m* fading has been also reported in literature [7]. Moreover, analytical study investigating the diversity system in the presence of co-channel interference has been studied [8]-[15].

In this paper outage probability of SIR-based dual EGC diversity has been analyzed. The main contribution of the paper is that proposed analysis is carried out assuming correlative Rayleigh fading for both the desired signals and co-channel interferers, which is the real scenario in practice. The channel and system model are presented. Based on joint probability densities for correlated Rayleigh fading, formulas for outage probability in terms of double infinite sums are derived. Numerical results show the effect of various systems' parameters and discussion illustrates the proposed mathematical analysis. All provided numerical results are presented graphically.

II. CHANNEL AND SYSTEM MODEL

Channel model with Rayleigh fading has been in interest to model various propagation channels, which describes multipath scattering with different clusters of reflected waves. This channel model is used in many wireless communications applications. In this paper, a wireless communication system with dual SIR-based EGC diversity is considered. The desired signal received by the *i*-th antenna, $D_i(t)$, can be written as [15]:

$$D_i(t) = R_i(t) e^{j\phi_i(t)} e^{j[2\pi f_c t + \Phi(t)]}, i = 1,2 \tag{1}$$

with f_c as the carrier frequency, $\Phi(t)$ the desired information signal, $R_i(t)$ a Rayleigh distributed random amplitude process, and $\phi_i(t)$ the random phase uniformly distributed in $[0,2\pi)$. The resultant interfering signal received by the *i*-th antenna is:

$$C_i(t) = r_i(t) e^{j\theta_i(t)} e^{j[2\pi f_c t + \psi(t)]}, i = 1,2 \tag{2}$$

where $r_i(t)$ is also a Rayleigh distributed random amplitude process, $\theta_i(t)$ is the random phase, and $\psi_i(t)$ is the information signal. This model refers to the case of a single co-channel interferer.

Moreover, the performance of the EGC can be carried out by considering the effect of only the strongest interferer, assuming that the remaining interferers are combined and considered as lumped interference that is uncorrelated between antennas. Furthermore, $R_i(t)$, $r_i(t)$, $\phi_i(t)$ and $\theta_i(t)$ are assumed to be mutually independent and the level of the interference is sufficiently high for the effect of thermal noise on system performance to be negligible (interference-limited environment). Now, due to insufficient antennae spacing, both desired and interfering signal envelopes experience correlative Rayleigh fading with joint PDFs [3]:

$$f_{R_1R_2}(R_1,R_2)=\frac{4R_1R_2}{\Omega_{d1}\Omega_{d2}(1-\rho)}\times\exp\left[-\frac{1}{1-\rho}\left(\frac{R_1^2}{\Omega_{d1}}+\frac{R_2^2}{\Omega_{d2}}\right)\right]\times\mathrm{I}_0\left(\frac{2\sqrt{\rho}R_1R_2}{(1-\rho)\sqrt{\Omega_{d1}\Omega_{d2}}}\right)\quad(3)$$

and

$$f_{r_1r_2}(r_1,r_2)=\frac{4r_1r_2}{\Omega_{c1}\Omega_{c2}(1-\rho)}\times\exp\left[-\frac{1}{1-\rho}\left(\frac{r_1^2}{\Omega_{c1}}+\frac{r_2^2}{\Omega_{c2}}\right)\right]\times\mathrm{I}_0\left(\frac{2\sqrt{\rho}r_1r_2}{(1-\rho)\sqrt{\Omega_{c1}\Omega_{c2}}}\right)\quad(4)$$

where $\Gamma(.)$ is the Gamma function, $\mathrm{I}_\nu(.)$ being the first kind and νth order modified Bessel function, ρ the correlation coefficient, $\Omega_{di}=\overline{R_i^2}$ and $\Omega_{ci}=\overline{r_i^2}$ the average signal desired and interference powers at the i-th branch. Let $\zeta_1=R_1^2/r_1^2$ and $\zeta_2=R_2^2/r_2^2$ be the instantaneous SIRs at the input diversity branches. The output SIR of equal-gain combiner is defined as

$$\zeta=\zeta_{OUT}=\zeta_1+\zeta_2\quad(5)$$

III. Outage Probability

Outage probability is a measure of the system's performance. It can be defined and related to the different criteria of reception. The outage probability, can be defined as probability which SIR falls below a given threshold and be expressed as [7]:

$$P_{out}=P_{robability}(\zeta<\beta)=\int_0^\beta f_\zeta(t)dt=F_\zeta(\beta).\quad(6)$$

The joint PDF of the instantaneous SIRs at the two input branches of EGC, is given as [3]:

$$f_{\zeta_1,\zeta_2}(t_1,t_2)=\frac{1}{4\sqrt{t_1t_2}}\int_0^\infty\int_0^\infty f_{R_1,R_2}\left(x_1\sqrt{t_1},x_2\sqrt{t_2}\right)\times f_{r1,r2}(x_1,x_2)x_1x_2dx_1dx_2.\quad(7)$$

By employing the infinite series representation of the modified Bessel function:

$$\mathrm{I}_\nu(z)=\sum_{k=0}^\infty\frac{z^{\nu+2k}}{2^{\nu+2k}k!\Gamma(\nu+k+1)}\quad(8)$$

and by replacing Eqs. (3) and (4) into Eq. (7), the joint PDF can be defined:

$$f_{\zeta_1,\zeta_2}(t_1,t_2)=\sum_{i_1=0}^\infty\sum_{i_2=0}^\infty\frac{\rho^{i_1+i_2}(\Gamma(i_1+i_2+2))^2(1-\rho)^2(ab)^{i_2+1}}{(i_1!i_2!)^2}\times\frac{t_1^{i_1}t_2^{i_1}}{(a+t_1)^{i_1+i_2+2}(b+t_2)^{i_1+i_2+2}}\quad(9)$$

where $a=\Omega_{d1}/\Omega_{c1}$, $b=\Omega_{d2}/\Omega_{c2}$.

For evaluating the PDF at the output of EGC combiner we used Eq. (10)

$$f_\zeta(t)=\int_0^t f_{\zeta_1\zeta_2}(t-t_2,t_2)dt_2.\quad(10)$$

Eq. (11) follows straightforward from Eqs. (7) and (10), the form of $f_\zeta(t)$ can finally be written as:

$$f_\zeta(t)=\sum_{i_1=0}^\infty\sum_{i_2=0}^\infty\frac{\rho^{i_1+i_2}(\Gamma(i_1+i_2+2))^2(1-\rho)^2(ab)^{i_2+1}}{(i_1!i_2!)^2}\times\int_0^t\frac{(t-t_2)^{i_1}t_2^{i_1}}{(a+(t-t_2))^{i_1+i_2+2}(b+t_2)^{i_1+i_2+2}}dt_2.\quad(11)$$

The bivariate (joint) CDF of ζ_1,ζ_2 can be expressed as

$$F_\zeta(\beta)=\int_0^\beta f_\zeta(t)dt\quad(12)$$

where β is the protection ratio, defined as required ratio of the desired signal power to the interference power at the output of the combiner.

By substituting Eq. (11) into Eq. (12), the outage probability for SIR-based dual diversity system in channel with Rayleigh fading and EGC receiver can be evaluated.

IV. Numerical Results

Numerical results, according to analytical expressions of the system with EGC receiver operating over Rayleigh fading in the presence of CCI are presented. All achieved results for probability density function and outage probability are shown on the graphics below.

In Fig. 1, the PDF of EGC output is plotted for balanced ($a=b$) as well as for unbalanced ($a=4b$) SIRs at the input branches. The dependence output PDF, for various values of correlation coefficient ρ is also presented.

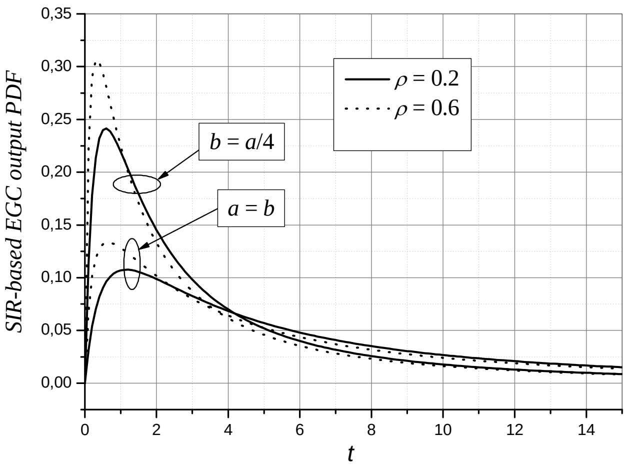

Fig. 1. SIR-based dual EGC output PDFs

In Fig. 2, the outage probability is plotted versus normalized the protection ratio β and for the fixed correlation coefficient ρ. The protection ratio β is normalized by value of desired signal power to interference power ratio of the first input branch. Balanced ($a=b$) and unbalanced ($a=4b$, $a=b/4$) SIRs are assumed at the two input branches.

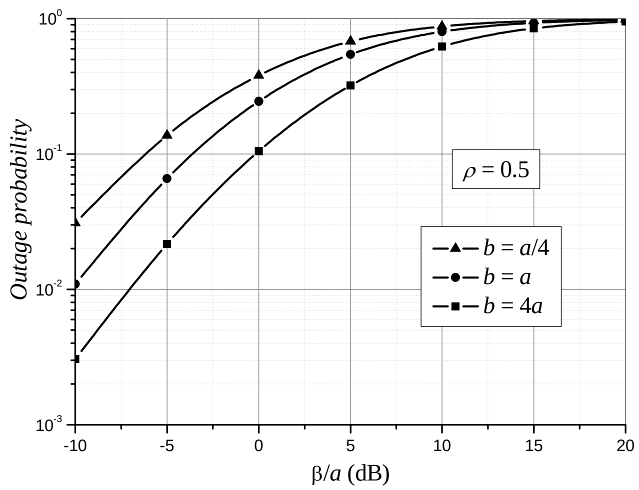

Fig. 2. Outage probability for balanced and unbalanced SIRs and correlation coefficient $\rho = 0.5$

It is very interesting to observe that if the value of desired signal power to interference power ratio of the second input branch increases, the outage probability decreases. But, for low values of desired signal power to interference power ratio of the second input branch, outage performance of the system deteriorates in comparation of balanced SIRs case.

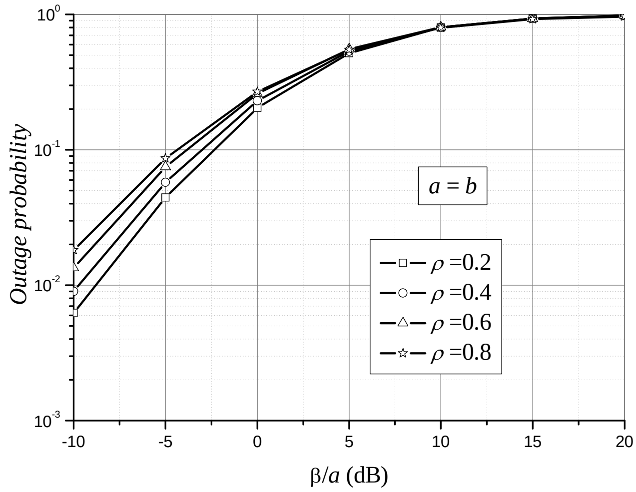

Fig. 3. Outage probability for balanced SIRs and various values of correlation coefficient

In Fig. 3, the outage probability is plotted versus the β/a, for several values of correlation coefficient ρ and balanced SIRs. It is evident that for strong interference (β/a increases) the outage probability increases slowly as the correlation coefficient ρ increases. For lower values of β/a, ρ has a significant effect on the outage performance. Moreover it is obvious that system's performance deteriorates when correlation coefficient ρ increases.

V. Conclusion

In this paper, the performance of a dual SIR-based equal gain-combining system, operating over Rayleigh channel, was studied. The case of correlated both the desired signals and interferences because of insufficient antenna spacing which is real scenario in practice, was observed. Equal gain combining has performance close to that of maximal ratio combining but with lower implementation complexity**.** For this type of diversity, useful analytical expressions for the probabilty density function and outage probability at the output of EGC, were presented. Using this analytical approach, the outage probability, was also efficiently presented graphically. The effects of various parameters, such as the input SIR unbalance, and the level of correlation to the system's performance, were interpreted.

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