Recovering Optical Image Transferred Through Atmospheric Turbulence. Wavelength analysis

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Abstract – **In this paper we study model-based restoration of long exposure space-to-ground images. The paper deals with the restoration of one dimensional image with least squares methods applied to our turbulence model. Methods for estimation of wavelength influence on restoration are proposed.**

Keywords – **Atmospheric turbulence, Least-squares methods, Restoration of images**

I. INTRODUCTION

Recently, practical algorithms have emerged that are capable of recovering spatial frequency detail that lies beyond the diffraction limit of an image sensor [1]. In this paper we will briefly review some application of our previous works [2,3]. We will use again a least-squares method based algorithm that achieves reconstruction of diffraction-limited one-dimensional images [3-5]. At the end of the analysis we will propose an algorithm for the practical estimation of wavelength influence on the restoration process.

II. ESTIMATION OF WAVELENGTH INFLUENCE ON RESTORATION

A. Turbulence model in terms of wavelength dependency

We use results from [2] for optical mean intensity

$$\langle I(x)\rangle = \langle I_0(x)\rangle \exp\left(-\gamma^2\right) + \frac{aL_m}{\sqrt{2}f} \times \tag{1}$$

$$\times \sqrt{\frac{\frac{2}{\eta}}{1+\frac{2}{\eta}\left(1+r^2\right)}} \left[1-\exp\left(-\gamma^2\right)\right] \times$$

$$\times \exp\left[-2\frac{\frac{2}{\eta}\left(1+r^2\right)}{1+\frac{2}{\eta}\left(1+r^2\right)}\frac{x^2}{x_0^2}\right] .$$

Where

$$\langle I_0(x)\rangle = \frac{aL_m}{\sqrt{2}f}\sqrt{\frac{1}{1+r^2}} \exp\left(-2\frac{x^2}{x_0^2}\right)$$

intensity without taking turbulence into account, parameters r and x_0 are defined by

$$r = \frac{\lambda}{\pi\ a\ \theta} = \frac{\lambda\ z}{\pi\ a\ b},\ x_0 = \sqrt{1+r^2}\, f\theta , \tag{2}$$

a and f are respectively the radius of input aperture and the focal length of registration optical system, L_m is the maximal Gaussian brightness of the cosmic object. Functions γ and η are defined by

$$\gamma^2 = \sqrt{\pi} C_n^2 k^2 \rho_{cV_0}^{5/3} Z_{eq} \tag{3}$$

and

$$\eta = \ln\left\{\left[\exp\left(\gamma^2\right)-1\right]\left[\exp\left(\frac{\gamma^2}{e^2}\right)-1\right]^{-1}\right\} , \tag{4}$$

where

$$k = 2\pi/\lambda \quad , \tag{5}$$

C_n^2, ρ_{cV_0}, Z_{eq} are the structure constant, the radius of spatial coherency and the equivalent thickness of the turbulence layer. On the other hand, we have [2]

$$\rho_{cV_0} = 2\lambda/\pi\theta . \tag{6}$$

In the previous research [2,3] we have presented results on a concrete wavelength ($\lambda = 0{,}5\ \mu m$). In actual fact, however, the influence of atmospheric turbulence over optical radiations of different wavelength is different (see (5) and (6)). It is necessary to point out that the effects from the impact of $k(\lambda)$ and $\rho_{cV_0}(\lambda)$ on γ^2, described by the substitution of (5) and (6) in (3), have contrary directions. As a result of their combined action, the multiplier $\lambda^{-1/3}$ is present in expression (3) which characterizes the atmospheric turbulence and originates in the method of smooth perturbations (MR-Method of Rytov).

We accept that we have information about atmospheric turbulence (for example from SODAR, SCIDAR [6] or LIDAR etc.). This means that functions (3) and (4) are known.

We present an illustration of the dependence of the optical intensity on the size x/x_0 with the wavelength being a parameter with the forward problem (Fig.1).

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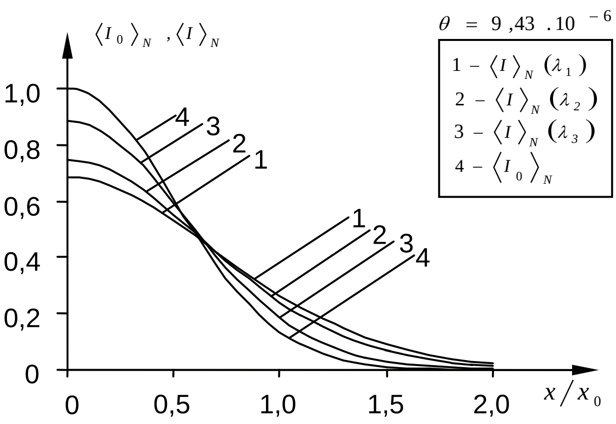

$\lambda[\mu\mathrm{m}] = 0,4;\ 1;\ 3,5;$

Fig.1. Wavelength dependency

B. Formulation of analysis

The formulation is shown on Fig.1.

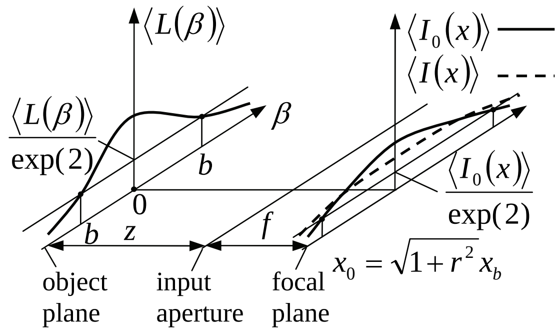

Fig. 2. Formulation of analysis

Our main goal is to find scale x_0 (see (4)). If we know x_0, there is a possibility simply to calculate $\langle I_0 \rangle$ from (1) i.e. the initial image distribution.

C. Least square solution (LSS)

The method of least squares provides a means of estimating the values of coefficients in an equation. Typically, the estimates are based upon some sample of data. The idea of least squares is to minimize the total amount of error due to these estimates. Note that the technique of least squares is different from linear regression, though they have similar objectives [6].

We implement least squares method in the following way

$$\sum_i^n [y_i - \varphi(x_i; a)]^2 = min \qquad (7)$$

where y_i corresponds to the experimental data of $\langle I(x) \rangle_N$ (derived from *n*-sized CCD matrix for example); $\varphi(x_i; a)$ corresponds to the theoretical $\langle I(x) \rangle_N$ (see formula (1)); parameter *a* equivalent to x_0.

Because we have only one parameter in the minimization, we use the first derivative case

$$\sum_i^n [y_i - \varphi(x_i; a)] \frac{\partial \varphi}{\partial a}\bigg|_{x = x_i} = 0 \qquad (8).$$

Now we derive the first derivative of (4)

$$\frac{\partial \varphi}{\partial a} = 4 \frac{c_1 x^2 exp\left(-2\frac{x^2}{a^2}\right)}{a^3} + \frac{4 c_2 c_3 x^2 exp\left(-2\frac{c_3 x^2}{a^2}\right)}{a^3} \qquad (9)$$

where c_1, c_2, c_3 are known constants (see (2)).

We put (9) and (1) in (8) and we derive the final equation to solve. The root of this equation is a (see (7) and (8)) respectively x_0. We calculate this equation with numerical methods (not discussed here).

D. Numerical example

We suggest that, after the application of LSS and derivation of the coefficient a for the different wavelengths, an assessment be made that is based on the calculation of the derived error.

This would, to a great extent, support the application of larger wavelengths. Due to the necessity for experimental data, no numerical example has been given at this stage. We hope this will take place in our further papers.

III. Conclusion

In this paper we propose methods for estimation of the solution of the problem of restoration of turbulence-degraded images with respect to the wavelength, using a previously developed turbulence model.

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