Computation of Capacitance of Circular Conductors

Sarhan M. Musa[1] , Matthew N. O. Sadiku[2]

Abstract – **Circular conductors have manufacturing advantage compared to rectangular conductors in building microwave components such as filters and couplers which make it very popular for microwave designers. The accurate and efficient evaluation results of the self and coupling capacitance of circular conductors can help the designers to optimize the layout of the integrated circuits. In this paper we present computing of the capacitance per unit length of circular conductors. Mainly, we focus our computation on three types of systems: single circular wire above ground plane in a dielectric layer, single circular conductor above a perfectly conducting ground plane in a lossy dielectric layer, and three circular rod arrays in a rectangular trough with homogenous dielectric layer. Comparisons with published results demonstrate excellent agreement with the modeling and simulation using finite element method (FEM).**

Keywords – **Circular conductor, capacitance, finite element method.**

I. Introduction

In recent years, investigators, designers, and researchers become more interested in circular conductors due to its application in high speed and high density digital electronics such as chip carriers and printed circuit technologies. Therefore, analysis and computation of electrical characterization such as capacitance matrices for circular conductors are essential to be accurate and efficient.

Previous attempts at the problem include using boundary element method (BEM) [1-2], method of moment (MOM) [3-4], wavelet expansion method [1], analytical method [1-2], method of images [5], conformal transformation method (CMT) [5-6], integral equation method [7], multipole theory method (MTM) [8]. We illustrate that our method using FEM is suitable and effective as other methods for modeling of circular conductors.

In this work, we use finite element method (FEM) with COMSOL multiphysics package to calculate the capacitance per unit length of open and shielded circular conductors. We use finite-element method (FEM) in modeling the transmission lines structure, because FEM is especially suitable for the computation of electric and electromagnetic fields in strongly inhomogeneous media. Also, it has high computation accuracy and fast computation speed. We consider three circular conductor systems: (1) single circular wire above ground plane in a dielectric layer with zero conductivity, (2) single circular conductor above a perfectly conducting ground plane in a lossy dielectric layer, and (3) three circular rod arrays in a rectangular trough with homogenous dielectric layer. We compare our results with previous investigators and find them to be close.

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II. Results and Discussion

The models are designed in 2D using electrostatic and magnetostatics environment.

In any electromagnetic field analysis the placement of far-field boundary is an important concern, especially when dealing with open solution regions. It is necessary to take into account that the natural boundary of a line at infinity and the presence of remote objects and their potential influence on the field shape [9]. In all our simulations, the circular conductor is surrounded by a w X h shield, where w is the width and h is the thickness of the shield. In the boundary condition of the model's design, we use ground boundary which is zero potential ($V = 0$) for the shield. We use port condition for the circular conductor to force the potential or current to one or zero depending on the setting.

A. Single Circular Wire above Ground Plane in a Dielectric Layer with Zero Conductivity

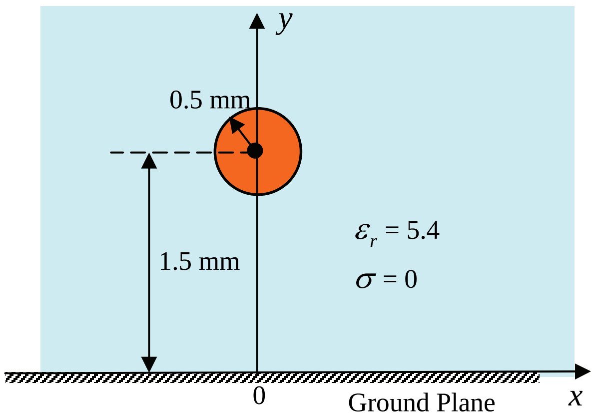

Fig. 1. Cross-section of single circular wire above ground plane with zero conductivity.

The geometry is enclosed by a 10 X 10 mm shield. The shielded is not included in Fig. 1 but it is included in Fig. 2. Figure 2 shows potential distribution in streamline plot for of single circular wire above ground plane with zero conductivity.

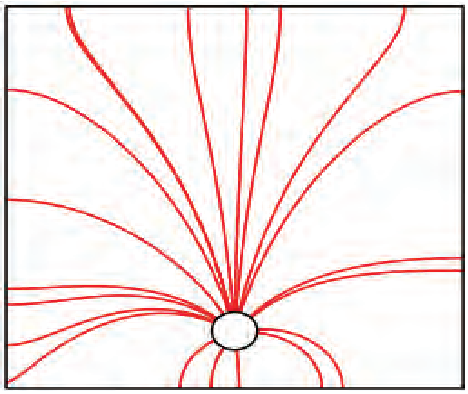

Fig. 2. Potential distribution in streamline plot for single circular wire above ground plane in a dielectric layer with zero conductivity.

Table I shows the finite element results for the capacitance per unit length of single circular wire above ground plane in a dielectric layer with zero conductivity. The results in Table I are compared with the work of previous investigations. They are in good agreement.

B. Single Circular Conductor above a Perfectly Conducting Ground Plane in a Lossy Dielectric Layer

Figure 3 shows the cross section for single circular conductor above a perfectly conducting ground plane in a dielectric layer with loss tangent.

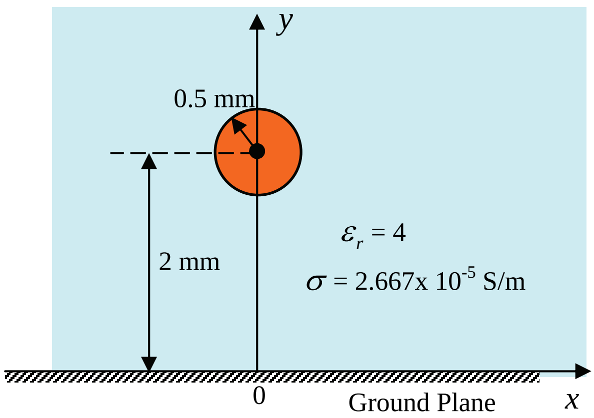

Fig. 3. Cross-section of single circular conductor above a perfectly conducting ground plane in a lossy dielectric layer.

The geometry is enclosed by a 10 X 10 mm shield. The shielded is not included in Fig. 3 but it is included in Figs. 4 and 5. Figure 4 shows potential distribution in streamline plot. Figure 5 shows potential distribution in contour plot.

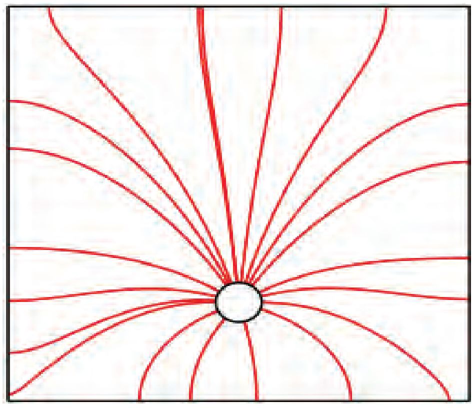

Fig. 4. Potential distribution in streamline plot for single circular conductor above a perfectly conducting ground plane in a dielectric layer with loss tangent.

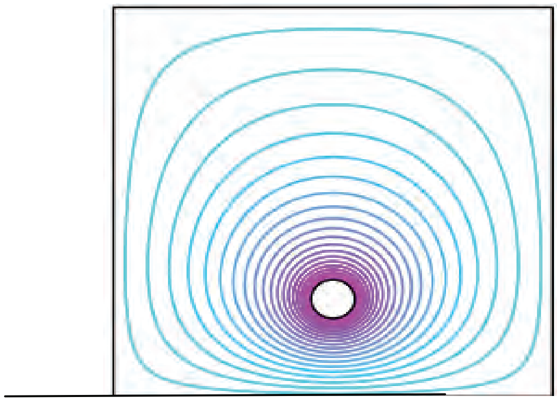

Fig. 5. Contour plot of single circular conductor above a perfectly conducting ground plane in a lossy dielectric layer.

Table II shows the finite element results for the capacitance per unit length of single circular conductor above a perfectly conducting ground plane in a lossy dielectric layer. The results in Table II are compared with the work of previous investigations. They are in good agreement.

C. Three CircularRrod Arrays in a Rectangular Trough with Homogenous Dielectric Layer with Zero Conductivity

Figure 6 shows a multiple circular conductors geometry of three circular rod arrays in a rectangular trough with homogenous dielectric layer and zero conductivity with the following parameters:

R_1 = radius circular conductor 1 = 1.5 mm

R_2 = radius circular conductor 2 = 1.5 mm

R_3 = radius circular conductor 3 = 2 mm

The circular conductors are in a rectangular trough with dimension 22 X 8 mm. Figure 7 shows the potential distribution in streamline plot. Figure 8 shows potential distribution in contour plot.

TABLE I

VALUES OF THE CAPACITANCE (in pF/m) COEFFICIENTS FOR SINGLE CIRCULAR WIRE ABOVE GOUND PLANE IN A DIELECTRIC LAYER WITH ZERO CONDUCTIVITY

C	Reference [1-2] Wavelet expansion method	Reference [1-2] BEM	Reference [1-2] MOM	Reference [1-2] Analytical	Our Work
C_{11}	169.9379	169.9840	153.1754	170.1889	177.1165

TABLE II

VALUES OF THE CAPACITANCE (in F/m) COEFFICIENTS FOR SINGLE CIRCULAR WIRE ABOVE GOUND PLANE IN A LOSSY DIELECTRIC LAYER

C	Reference [10] Numerical solution	Reference [10] Analytical solution	Our Work
C_{11}	1.073 x10^{-10}	1.078 x10^{-10}	1.056 x10^{-10}

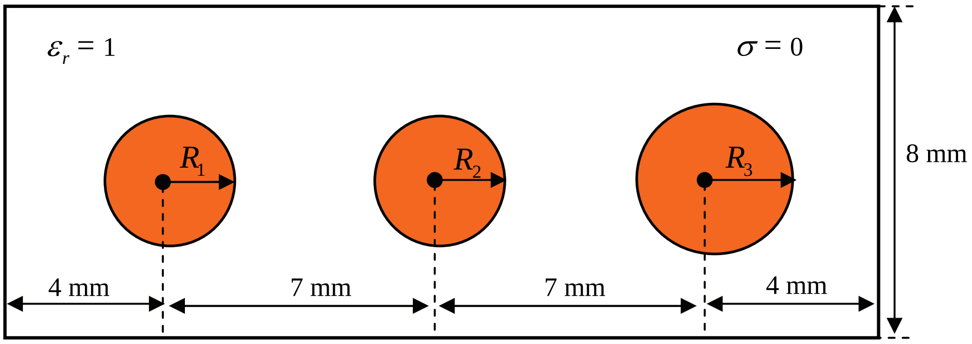

Fig. 6. Cross-section of three circular rod arrays in a rectangular trough with homogenous dielectric layer.

TABLE III

VALUES OF THE CAPACITANCE (in pF/m) COEFFICIENTS FOR THREE CIRCULAR ROD ARRAYS IN A RECTANGULAR TROUGH WITH HOMOGENOUS DIELECTRIC LAYER AND ZERO CONDUCTIVITY

C	Reference [8] MTM	Reference [8] BEM	Our Work
C_{11}	49.85	49.80	49.88
C_{12}	-4.802	-4.793	-4.805
C_{13}	-0.0063	-0.0064	-0.0064
C_{22}	47.51	47.46	47.54
C_{23}	-6.259	-6.249	-6.262
C_{33}	67.36	67.30	67.39

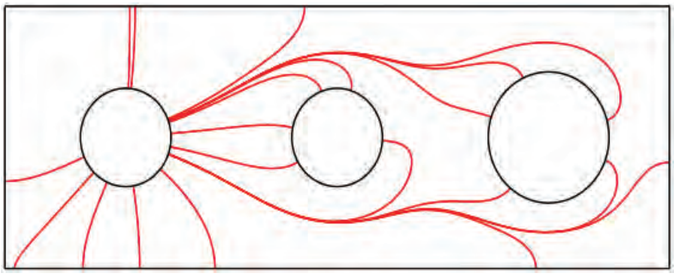

Fig. 7. Potential distribution in streamline plot for three circular rod arrays in a rectangular trough with homogenous dielectric layer and zero conductivity with node 1 as input.

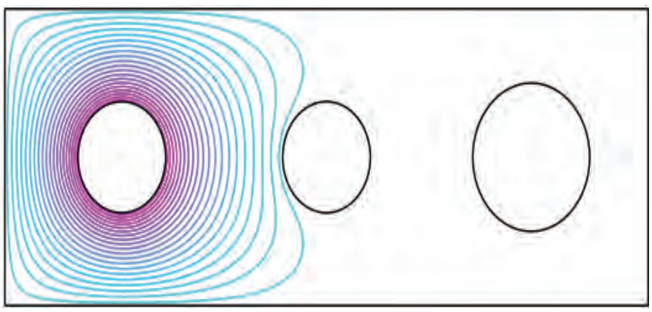

Fig. 8. Contour plot of three circular rod arrays in a rectangular trough with homogenous dielectric layer and zero conductivity.

Table III shows the finite element results for the capacitance per unit length and inductance per unit length of three circular rod arrays in a rectangular trough with homogenous dielectric layer and zero conductivity. The results in Table III are compared with the work of previous investigations. They are in good agreement.

III. Conclusion

In this paper, we have presented the computing and modeling of three types of circular conductor transmission line systems: single circular wire above ground plane in a dielectric layer with zero conductivity, single circular conductor above a perfectly conducting ground plane in a lossy dielectric layer, and three circular rod arrays in a rectangular trough with homogenous dielectric layer. The results obtained using COMSOL for the capacitance per unit length agree well with those found in the literature.

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