Evaluation of the Industry Standard Sampling Rates

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Abstract – **Video, audio, measurement and control industries are using a lot of standard sampling rates based on the classical signal conversion theory. In the paper most of them are evaluated from the point of view of amplitude errors. New parameters for errors evaluation are proposed. The results are showing that new and deeper research is needed in order to evaluate fully the effect of the signal conversion process.**

Keywords - **standard sampling rates, evaluation, errors**

I. INTRODUCTION

The industry is using a large wide variety of sampling rates [1, 2, 3, 4] with not well defined errors during the conversion of the real world signals (usually called analog signals). The basic parameters of the real world signals are discussed in [5]. This variety of sampling rates applied to the same type of signals (audio, video) with destination to the human beings without clear definition of the testing signals and introduced errors is a proof that the theory and practice of the field are not well developed. The present situation should be understood clearly and corrected according to the properties of the real signals and the end user.

The basic and most useful test signals for the signal conversion theory and practice are:

1. Direct current (DC).
2. Sine and cosine wave without direct current. This is the simplest band wide signal (SBLS) with zero phase and DC component. The definition of SBLS is given below.
3. The SBLS with phase and DC components which are not zero .
4. The sum of SBLSs. In practice every real world signal could be presented as a finite sum of SBLSs.
5. Linearly changing signal (triangular signal).
6. Saw tooth signal.

Definition: The simplest band limited signal (SBLS) is a signal with two lines into its spectrum. The first line is the direct current (DC) and the second is a sine or cosine wave.

The following two formulas are applicable to the two basics SBLS:

$$A = A_m \sin(2\pi f + \theta) + B \quad (1)$$

$$A = A_m \mathrm{cosin}(2\pi f + \theta) + B \quad (2)$$

The SBLS is the simplest test signal with two lines into spectrum and with four parameters to reconstruct (A_m,f, θ and B).

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The terms SBLS and “algebraic sum of SBLSs” are intended to replace the term “band limited signal” used widely in the technical publications. The goal is to introduce a way for better errors evaluation during the sampling and reconstruction process.

It is important to mention that once converted into digital form the analog signal (AS) cannot be reconstructed “exactly” or with “arbitrary” accuracy. Irreversible errors are introduced during the signal conversion. This is in contradiction with the statement of the classical sampling theorem given in [6, 7, 8].

In fact the theorems of Kotelnikov, Nyquest and Shannon are all about “mathematical functions”, not about the process of sampling and reconstruction of real signals. Every real signal could be presented as a mathematical function, but not every function could be realized as real signal. Consequently the stated theorem dos not describe the sampling process. The history of that “theorem” is given in [1].

Moreover there is no two analog to digital converter (ADC) with exactly the same characteristics and there is no couples of ADC and digital to analog converter (DAC) with exactly matched characteristics. Consequently during every signal conversion unknown errors are introduced and the original analog signal cannot be reconstructed without errors in any signal parameter.

The parameters which should be evaluated during the signal conversion process are defined for SBLS and sum of SBLSs below:

1. Maximal amplitude errors.
2. Minimal amplitude errors.
3. Average amplitude errors.
4. Phase error.
5. DC errors.
6. Introduced in band and out band frequencies into the reconstructed signals as separate values.
7. Total introduced frequencies components by the sampling process.

In that paper the parameters defined in the next section are evaluated.

II. BASIC DEFINITIONS AND TYPE OF ERRORS

As we mention above there are a lot of sampling rates (F_d) combined with different number of accurate bits (n) of the converters used widely in the industry of transmission and recorder of sounds and video

This “diversity” and lack of objective criteria is partially created with the inaccurate basis of the sampling theory and particularly by Classical sampling theorem (CST) stated in [6, 7, 8, 9]

The terms used in that papers are defined below.

Definition: Sampling factor (SF) N or “signal sampling factor” (SSF) is given with the formula below

$$N = F_d/F_s = F_d/F_{max} \quad (3)$$

where

F_d is the sampling frequency. (Also it is called "digitalizing frequency", "sampling rate", "discretizing frequency" or "frequency of the digital samples").

F_s is the frequency of the sampled sinusoidal or co sinusoidal signal (SS or CS). The DC offset and the phase are accepted to be zero in order to simplify the analysis.

F_{max} is the maximal frequency of the sampled band limited signal (BLS)

Although N could be any number in most of the cases N greater than or equal to 4 is required.

N is different for each signal component and consequently each signal component is converted with different set of errors.

Definition: Band limited signal (BLS) is every signal which contains at every moment limited number of sinusoidal and co sinusoidal components between one minimal frequency F_{min} and one maximal F_{max}. In fact every BLS is a sum of limited number of SBLSs.

Definition: Fixed band limited signal (FBLS) is every signal which contains limited and constant number of the same sinusoidal and co sinusoidal components between one minimal frequency F_{min} and one maximal F_{max}. Every BLS is a sum of limited number of SBLSs.

Definition: Sampling phase to amplitude modulation is an amplitude modulation of the samples during the sampling process as consequence of changing the phase of the sampled signal when the instability of the sampling rate F_d is neglected.

Definition: The basic parameters of the sampling process with constant sampling rate F_d are:

$N = F_d/F_s$ - signal sampling factor;

φ0 - angle of the first sample (time between the beginning of the coordinate system or of the period of the signal and the moment of the first sample) ;

n - number of the accurate (reproducible) bits of the converter. (The "resolution" of the converter given with. the total number of the bits (m) is usually greater than the bits of the "accuracy" (n).)

There is a relation between the SSF N and the number of bits n as follows:

1. From one side for each N there is minimum value for the number of bits n_{min} which is not increasing the amplitude errors above defined limits.
2. From the other side for each N there is no use to increase the number of bits n_{max} above one value because this is not decreasing the amplitude errors in significant way.

Definition: $F_{d100\%}$ is the first sampling frequency of 100% modulation. The term is intended to replace "the Nyquest frequency" or the"frequency of exact reconstruction". The corresponding equation is

$$F_{d100\%} = 2\ F_{max.} \quad (4)$$

$F_{d100\%}$ is also called "the main (first) frequency of 100% modulation". It is defined with the following conditions:

1. Signal sampling factor N=2.
2. With maximal amplitude error between 0 and 100% included.

If we change the phase of the sampled with N=2 SS or CS from 0 to 90 degrees the amplitude of the samples will change from 0 to the maximal value of the SS or CS. Consequently the output samples are modulated by the phase of the sampled signal.

Definition: $F_{s100\%}$ is the signal frequency of 100% amplitude modulation when the phase of the SS or CS is changing from 0 do 90 degrees for the given sampling frequency $F_{d.}$ The corresponding equation is

$$F_{s100\%} = 0.5 * F_d \quad (5)$$

Definition: F_{s3db} is the sampling frequency guarantied maximal error E_{max} less than or equal to 3db (approximately 30%) at given signal frequency F_s. The corresponding equation is

$$F_{d3db} = 4*F_{max} = 4*F_s \quad (6)$$

This frequency is also called the "frequency of 3dB amplitude modulation".

Definition: F_{d1dB} is the sampling frequency guarantying maximal amplitude error of 1 dB (approximately 10%) at given signal frequency F_s. It is defined with the equation:

$$F_{d1bB} = F_{d10\%} = 7*F_s \quad (7)$$

Definition: $F_{0.1dB}$ is the sampling frequency guaranteeing maximal amplitude error of 0.1 dB (approximately 1%) at given signal frequency F_s.

$$F_{d0.1B} = F_{d1\%} = 22*F_s \quad (8)$$

Definition: F_{s3dB}, F_{s1bB}, $F_{s0.1B}$ are respectively the signal frequencies guaranteeing maximal amplitude error of 3dB, 1dB or 0.1 dB at the given sampling rate F_d.

Definition: $\theta_{max1000}$ is the angle of the maximal deviation from the amplitude value of the SS at selected sampling rate when a SS with 1000Hz is sampled. The phase and DC components are zeros.

Definition: φ0 = 0 angle of the first sample. (φ0 is the difference in time between the starting point of the signal (t1) and the moment when the first sample is taken (t2).)

Definition: Maximal amplitude error E_{ssmax} during the conversion of SS (DC and phase components are zeros) is given with the equation below:

$$E_{ssmax}=(1-\sin(90-180/N)=(1-\cos(180/N)) \quad (9)$$

Definition: Maximal amplitude error E_{csmax} during the conversion of CS (DC and phase components are zeros) is given with the equation below:

$$E_{csmax}=(1-\cos(90-180/N)=(1-\sin(180/N)) \quad (10)$$

III. Evaluation of the Standard Sampling Rates

Most of the industry standard sampling rates and the principal application are listed in Wikipedia in the article "Sampling rates" [2]. They are evaluated from the point of view of the maximal amplitude errors when sinusoidal and co-sinusoidal signal are sampled and the signal is directly reconstructed and the results are given in the Table I.

Since 1000 Hz is considered suitable testing frequency when comparing different sampling rates and evaluating the number of the bits the signal sampling factor N_{1kHz} is given at that frequency. Another suitable frequency for testing audio signal is 440Hz.

TABLE I
EVALUATION OF THE BANDWIDTH AT SEVERAL LEVELS OF SOME OF THE INDUSTRY STANDARD SAMPLING RATES.

F_d , [Hz]	$F_{s100\%}$ (F_d/2)	F_{s3dB} (F_d/4)	F_{s1bB} (F_d/7)	N_{1kHz}
5500	2750	1375	785.7	5.5
7333	3666.5	1833.25	1048	7.33
8000	4000	2000	1143	8
11025	5512.5	2756.25	1575	11.025
16000	8000	4000	2286	16
18900	9450	4725	2700	18
22050	11025	5512.5	3150	22.050
22254	11127	5563.5	3179	22.254
32000	16000	8000	4571	32
37800	18900	9450	5400	37.8
44056	22028	11014	6294	44.056
44100	22050	11025	6300	44.1
47250	23625	11812.5	6750	47.2
48000	24000	12000	6857	48
50000	25000	12500	7142	50
50400	25200	12600	7200	50.4
88200	44100	22050	12600	88.2
96000	48000	24000	13714	96
176400	88200	44100	25200	176.4
192000	96000	48000	27429	192
13.4 MHz	7.7 MHz	3.35 MHz	1.94 MHz	13400

In the table the amplitude error from the finite number of bits (n) of the converter is neglected. In most of the cases this could be done when the following formula is applied:

$$n >= \log_2 (1/ E_{max}) +3, \text{[bits]} \qquad (11)$$

where E_{max} is the maximal amplitude error for the corresponding signal sampling factor N.

IV. CONCLUSIONS AND SUGGESTIONS

The signal conversion theory is in need of new and better formulated theoretical basis in order to evaluate errors during the signal conversions. Wide range of theoretical models could be developed but they should be not oversimplified [10, 11].

An example of oversimplified theorem is the classical sampling theorem and the Fourier series.

When experimenting with SS and CS with SSF N=2 the following theorem could be used for signal reconstruction.

Theorem: If we sample a sine or co-sine wave signal exactly into its maximum and minimum and convert it in a square wave signal with the same frequency and amplitude we can reconstruct the original SS or CS with appropriate low pass or band pass filter plus an amplifier with appropriate gain.

This synchronization is giving the possibility to reconstruct also the direct current component (DCC) of the SS/CS.

Some of the terms in the signal conversion theory should be reevaluated and possibly replaced with more representative terms. An example is given below

1. The term “over sampling converter” is incorrect and difficult to understand because we can sample at any frequency. At that case what will be a “over sampled converter”? Connecting that term with “Nyquist frequency” is incorrect because at that frequency and near to it there is no full guaranty that the signal will be reconstructed entirely.

2. The co-called “CD quality” (Sampling rate of 44.1 kHz) is guaranteed only 11.025 kHz bandwidth at – 3dB which obviously not enough to code the full audio band width for the human being which sometimes is extended to more than 20 kHz. It is much better to call that quality a “middle quality audio sampling”. In Table II are given several suggestions how to select the sampling frequency and number of the accurate bits of the converter in order to guarantee to respective quality of the directly reconstructed signal.

TABLE II
SUGGESTED VALUES FOR CONVERSION OF THE AUDIO SIGNALS WITH LESS THAN 3 DB MAXIMAL AMPLITUDE ERROR AT SELECTED QUALITY

Signal	F_{smax} [Hz]	F_d [Hz]	n [bits]
MQ speech	4000	16000	>=14
MQ audio	10000	40000	>=16
HQ audio	20000	80000	>=18
FQ audio	25000	100000	>=22

Abbreviations in the table: LQ – low quality, MQ – middle quality, HQ – high quality, FQ – full quality for human beings

Most of the industry standard frequencies are based of the old and inaccurate theory. They are guaranteeing 3dB signal band width lower than expected. Clear definition of the introduced errors is required.

The paper is giving good presentation of the amplitude and phase performances of the industry standard sampling rates when a sine wave signal is sampled.

When a sum of sine and or cosine waves or sum of SBLSs is sampled the results are more complicated and the generated errors cannot always be predicted and reproduced in a simple manner.

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