Sampling the Simplest Mathematical Functions

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Abstract: - **In some cases the equations describing the sampled mathematical functions are known. Also sometimes a synchronization is possible between the sampled function and the sampling process. In these cases the parameters of the equation(s) describing the functions could be calculated according to the paper and the sampled function could be restored. The difference between mathematical functions and real signals are summarized.**

Keywords: **sampling functions, known forms.**

I. INTRODUCTION

In some cases the mathematical representation (equation(s)) of a mathematical function is known and the parameters of the equation(s) should be calculated. The paper is dealing with that case and with mathematical functions only. The problem with sampling mathematical function is discussed in [1 - 7].

The most important differences between a “real signal” and a “mathematical function” are shown in [8, 9,10] and are summarized below:

* the real signal is uninterrupted function (smooth function).
* the real signal has “rounded angles” (not “ideal” or “broken angles”)
* the real signal has limited parameters in every moment (slew rate, power, spectrum, amplitude, etc.)

The task of sampling and reconstruction of the parameters of a mathematical function had deep theoretical roots in the past [1]. Now it has a practical meaning because often signals coming from the electronic test equipment are clear and could de replaced with very good accuracy (e.g. amplitude errors less than 0.5%) with idealized mathematical function. The task is discussed here but from different point of view and with the following assumptions.

1. The sampled mathematical function is representing oversimplified real signal. Not all of the basic properties of the real signal are respected. Especially the angles are considered “ideal” not “rounded” but the slew rate is considered always finite.
2. The mathematical equations of the sampled mathematical functions are known and the parameters into these equations should be calculated.
3. In order to calculate the parameters of the sampled function the principle “one sample per parameter” is applied.
4. In some cases the special disposition of the samples into the time is required, e.g a synchronization between the sampling and the sampled function is necessary. Some of the generators are providing such synchronization.

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II. DEFINITIONS AND THEOREMS FOR SAMPLING THE SIMPLEST FUNCTIONS

Definition 1: The direct current function (DCF) (a function representing a horizontal line) is given with the equation below

$$y=a \tag{1}$$

where a is a real number.

Theorem 1: In order to reconstruct a direct current function given with the equation above only one sample is needed.

Note: The accuracy of the reconstruction depends on the involved into sampling process equipment and will not be discussed here.

Definition 2: The linear function (LF) is given with the equation below

$$y = at + b \tag{2}$$

where a and b are real numbers.

Theorem 2: In order to reconstruct a linear function given with the equation above (e.g. to calculate the parameters a and b) only two samples are needed.

In practice often triangular functions (triangular pulses) are sampled and their parameters should be calculated. These functions are limited by three straight lines and often one of them is the abscise x of the coordinate system.

Theorem 3: In order to reconstruct a triangular function built from the two straight lines (two linear functions) given below

$$y_1 = at+b \tag{3}$$

$$y_2 = ct+d \tag{4}$$

and the abscise x of the coordinate system (e.g. to calculate the parameters a, b, c and d) four samples are needed under the condition that two samples are on the first edge of the triangular function (y_1) and two samples are on the second edge (y_2) of the function. The position of the abscise x (e.g. ground or zero level) should be known. Here a, b, c and d are real numbers and should be calculated.

Note: If synchronization is provided and the triangle is sampled in the common point of the two lines y_1 and y_2 only three samples are needed.

Theorem 4: In order to reconstruct a trapezoidal function built from three straight lines and the abscise x only six samples are needed under the condition that:

1. Two samples should be on the first edge of the trapezoidal function
2. One sample is on the first base of the trapezoidal function
3. Two samples are on the second edge of the same function.
4. One sample is on the second base of the trapezoidal function (could be the abscise x of the coordinate system).

Definition 3: SBLF (the simplest band limited function) is a function with two frequency (spectral) lines into its spectrum. The first is the direct current function (DCF) or zero

Hertz and the second is a sine or cosine function with frequency f. The following two formulas are applicable to SBLF:

$$A = A_m \sin(2\pi f + \theta) + B \quad (5)$$

or

$$A = A_m \mathrm{cosin}(2\pi f + \theta) + B \quad (6)$$

The SBLF is the simplest test function with two lines into spectrum and with four parameters to reconstruct (A_m, f, θ and B).

Definition 4: The function sampling factor (FSF) is defined only for the functions which could be represented as a finite sum of SBLF and is given with the formula below:

$$FSF = N_f = F_{sampling}/F_{max} \quad (7)$$

$F_{sampling}$ is the sampling frequency,

F_{max} is the maximal frequency in the presentation of the function (of the sine or cosine component with maximal frequency)

Theorem 5: In order to reconstruct an SBLF given with the equations above four sample are needed and FSF N>=4 is required.

Definition 5: The simplest dual tone (dual frequency) function (DTF) A_{dt} could be regarded as a sum of two separate SBLS A_1 and A_2, e.g.

$$A_{dt} = A_1 + A_2 \quad (8)$$

Or

$$A_{dt} = A_{m1} \sin(2\pi f_1 + \theta_1) + B_1 + {} + A_{m2} \sin(2\pi f_2 + \theta_2) + B_2 \quad (9)$$

Theorem 6: In order to reconstruct the simplest dual tone function A_{dt} given with equation above at least eight samples are needed. Moreover the knowledge of one of the two components B_1 or B_2 may be necessary and the function sampling factor N_f for the highest frequency component should be at least 4.

Definition 6: The simplest dual tone (frequency) function with zero offset (DTZF) A_{dtzf} is given with the equation below:

$$A_{dt} = A_{m1} \sin(2\pi f_1 + \theta_1) + A_{m2} \sin(2\pi f_2 + \theta_2) \quad (10)$$

Theorem 7: In order to reconstruct the simplest dual tone function (DTF) A_{dt} given with equation above at least six samples are needed. The knowledge that the two DCF components are zeros is essential and the function sampling factor N_f for the highest frequency component should be at least 4.

Theorem 8: Every mathematical function which could be represented with sum of k SBLFs could be reconstructed with:

1. 4*k samples where k in the number of the sine and/or cosine functions in the sum and
2. The function sampling factor N_f for each sine or cosine component should be at least 4.

III. Engineering Considerations – Sources Generating Functions with Known Form

Some modern electronic sources are generating functions with known and almost ideal form (e.g. with amplitude accuracy better than +-0.1%). These functions cloud be sampled, the parameters could be evaluated (calculated) and the function could be physically reconstructed. These sources are:

* functions generators (sine wave, triangular, saw tooth, etc. generators);
* arbitrary function generators;
* single phase (active) sine wave rectifiers;
* multiphase (active) sine wave rectifiers;
*some sensors, etc.

The following applications are examples of the theory discussed in the paper:

* measuring the slew rate of the electronic stage (amplifier);
* measuring the rise and fall times of the electronic circuit;
* testing current and voltage generators;
* testing electronic integrators;
* testing integrating analog to digital converters,
* testing switching power amplifiers,
* testing oscillators.

The real signals generated from modern equipment could be used to evaluate the parameters of the following sampled and reconstructed mathematical functions:

* sine function,
* cosine function,
* triangular function,
* saw toots function,
* trapezoidal function,
* cut triangular function (pseudo trapezoidal function),
* pseudo rectangular function (the ideal rectangular function is not a real function),
* half wave positive or negative sine or cosine wave
* two phase non filtered function,
* triple phase non filtered function,
* rising exponential function,
* falling exponential function,
* declining sine wave function,
* rising sine wave function, etc.

These functions could be represented with mathematical equations or as a finite sum of the SBLFs and the number of the samples in order to calculate the parameters could be evaluated with the methods shown above.

In all cases there are two main parameters of the sampled functions which should be taken into considerations and which have to have finite values::

* the maximal slew rate;
* the peak to peak amplitude.

The DCF and phase components of these functions could have zero or non zero values and should be taken into consideration.

Very often the synchronization of the sampling process with the sampled function is possible, eg.:

* with the start of the function,
* with zero crossings of the function,
* with the maximal value,
* with the minimal value.

Moreover the testing equipment could generate triggering pulses in order to synchronize the analog to digital converter with the converted function.

Applying the classical sampling theorem and the classical sampling theory is not the best solution with these cases.

The main tasks are listed below:

* sampling the function,
* calculating the parameters of the function from the samples
* recalculated the samples of the function with the frequency of the reconstruction
* reconstruction of the functions from the recalculated samples with digital to analog converter, on the electronic display, etc.

The function sampling factor (the frequency) of the reconstruction could be different from the frequency of the initial sampling necessarily for the parameters calculations

IV. Filters before ADC and after DAC

The filters before the analog to digital converter (ADC) is not necessarily if the function source is internally band limited (e.g. sine wave function generator).

The filter after digital to analog converter (DAC) is considered obligatory in order to convert the generated trapezoidal function with rounded angles into function as close as possible to the converted analog function applied to input of the ADC.

The sampling rate of the reconstruction could be equals or different to the sampled rated of the ADC. In all cases the function sampling factor should as greater as possible, e.g greater that ten, but sampling factor greater than 1000 is not unusual if possible. Also a very good quality electronic oscilloscope is needed in order to observe the parameters of the sampled and reconstructed functions.

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