Multiplication Products Quantization Noise Analysis for Orthogonal Complex IIR Digital Filters

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Abstract – **In this work a technique for analysis of multiplication product quantization errors for digital filters with complex coefficients is proposed. Then it is applied on complex orthogonal second-order digital filter sections. A very low sensitivity orthogonal narrow-band band-pass (BP) filter section is investigated due to the quantization of the multiplications. A comparative study with other popular complex orthogonal sections shows the advantages of the examined complex filter circuit. It is confirmed that the low sensitivity of the structure additionally decreases the multiplication quantization noises. The developed and investigated in this paper technique is universal enough to be applied on higher order orthogonal complex filters.**

Keywords – **complex orthogonal digital filters, sensitivity, quantization errors, noise analysis.**

I. Introduction

The quantization process is considered to be the general source of parasitic effects for digital filters. The finite word-length effects cause parasitic noises usually described as error signals.

The quantization effects can be separated into two categories in respect to the required analysis techniques:

Coefficient quantization – errors on account of representing filter coefficients as finite word-length fixed-point numbers. The filter coefficients are quantized once only and remain constant in the filter implementation. Coefficient quantization effects on filter characteristics perturb them from their ideal forms. If they no longer meet the specifications, the quantization design must be optimized by allocating more bits or choosing more proper filter realization. The structure of the digital filter has a significant effect on its sensitivity as a result of coefficient quantization.

Signal quantization – errors due to the finite-precision arithmetic operations of addition and multiplication. The quantization of the products within the filter (inner products) is normally accepted as a random process. It can be modeled as producing additive white noise sources in the filter, which simplifies the filter analysis.

The effect of signal quantization is to add an error or noise signal to the ideal output of the digital filter, which is compound of one or more of the following error sources: the quantization error of the analog-to-digital (A/D) converter at filter input; the errors resulting from the rounding or truncation of multiplication products within the filter; and requantization the output to the required word-length for a digital-to-analog (D/A) converter or another system.

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The paper considers the noise analysis due to multiplication products quantization, referred also by the general term arithmetic errors analysis. In case of real digital filters several good techniques for arithmetic noise estimation have been proposed earlier [1-3], whereas the signal quantization noise theory for the complex coefficients digital filters is still not well developed. Small amount of publications discuss some particular problems [4-5], but no general technique has been proposed so far.

This work suggests a new technique for multiplication products quantization noise analysis. It is applied to different orthogonal complex second-order sections. Experimentally is showed that the low coefficient sensitivity of the orthogonal complex circuit decreases the multiplication quantization output noises.

II. Noise Analysis of Complex Multiplication Products Quantization

As complex digital filter networks are considered in this work, all their building blocks and signals are complex too. Multiplication products quantization has to be done as finite-precision arithmetic is used. The quantization carried out by rounding generates parasitic noise signal $e_i(n)$. It is injected at a node after each multiplication quantization in the filter structure has been done and is termed as roundoff noise [1] [6].

On condition that the signal levels throughout the filter are much larger than the quantization step size, the following assumptions can be stipulated:

1. The error noise signal made at one sample time is uncorrelated with that made at any other sample, i.e. $e_i(n)$ and $e_i(k)$ are statistically independent for $n\neq k$. Hence, the noise signal is a stationary white noise with zero mean value and the variance σ_e^2, which in case of fixed-point representation with rounding is:

$$\sigma_e^2 = \frac{\delta^2}{12} = \frac{2^{-2B}}{12}. \qquad (1)$$

B is the word-length in bits and δ is the quantization step size.

2. The noise source introduced in one node is uncorrelated with those injected in all other nodes within the filter, i.e. $e_i(n)$ and $e_j(n+k)$ are statistically independent of each other for any value of n or k ($i \neq j$). This assumption leads to the conclusion that superposition can be employed.

Generally, two approaches for inner products quantization are possible – after and before summation.

In many hardware implementation schemes the multiplication operation is carried out as a multiply-add operation with the result $m(n)$ stored in a double-precision register. In such cases the quantization operation can be realized after all the multiply-add operations have been completed, reducing the number of quantization error sources to one for each sum of product operations, as it is depicted in Fig. 1a. The resulting noise $\gamma(n)$ at the digital filter output will be considerably lower on account of using internal storage devices of double-precision.

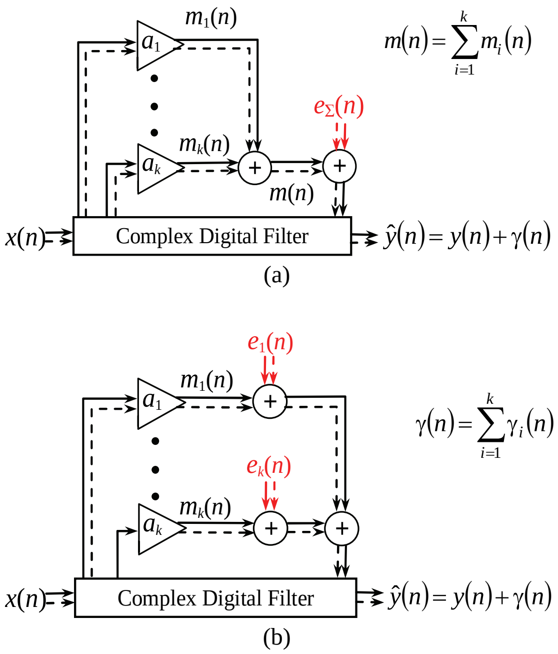

Fig.1: Noise models of multiplication products quantization (a) after summation; (b) before summation

More common for the fixed-point filter implementations is quantization operation to be performed separately for each multiplication product $m_i(n), i=1\div k$ (Fig.1b). Recall the assumption that error sources $e_i(n)$ are statistically independent of each other, then each error source develops a white noise $\gamma_i(n)$ at the output of the filter. Therefore, if the variance of the noise reached to the filter output as a result of each one multiplication quantization is calculated by itself, superposition holds for these noise variances, and they can be simply added.

For a filter structure with k multipliers the total output noise variance due to the quantization of all k multiplication products will be as follows:

$$\sigma_\gamma^2=\sum_{i=1}^{k}\sigma_{\gamma_i}^2=\frac{2^{-2B}}{12}\sum_{i=1}^{k}\sigma_{\gamma_i,n}^2=\sigma_e^2\sigma_{\gamma,n}^2 . \qquad (2)$$

$\sigma_\gamma^2=\sigma_{\gamma_{\mathrm{Re}}}^2+j\sigma_{\gamma_{\mathrm{Im}}}^2$ implies the nominal value of the total complex output variance and $\sigma_{\gamma,n}^2=\sigma_{\gamma_{\mathrm{Re}},n}^2+j\sigma_{\gamma_{\mathrm{Im}},n}^2$ is named the noise gain or normalized output noise variance.

The variance of the noise sequence at the output of the filter results from the error signal $e_i(n)$ only, as given by [6]:

$$\sigma_{\gamma_i}^2=\sigma_e^2\frac{1}{2\pi j}\oint G_i(z)\,G_i\left(z^{-1}\right)z^{-1}dz=\sigma_e^2\,\sigma_{\gamma_i,n}^2 , \qquad (3)$$

where $G_i(z)$ is defined as *noise transfer function* (NTF), i.e. the transfer function from the noise source $e_i(n)$ to the filter output $y(n)$. NTF is a complex transfer function and depends on the structure of the digital filter.

The total complex output noise variance can be calculated by Eq.(1) to Eq.(3) providing all quantities in these formulas are complex.

III. Output Noise Variance Estimation for Orthogonal Complex Second-Order Digital Filter Sections

The LS2-based orthogonal complex filter section [8] is derived after the orthogonal form of the circuit transformation [7] is applied on the low-pass (LP) second-order real prototype. It is a narrow-band BP filter with canonical number of elements, realized by fixed-point arithmetic and demonstrates very low coefficient sensitivity [8] even in case of a very short word-length.

In Fig. 2 a signal flow graph of orthogonal complex LS2 filter due to complex multiplication product quantization is shown. As long as the quantized multiplication signals are complex, the originated noise sources $e_a(n)$ and $e_b(n)$ will be complex too. Hence, the complex output signal $y(n)=y_{\mathrm{Re}}(n)+jy_{\mathrm{Im}}(n)$ will be mixed with complex output noise $\gamma(n)=\gamma_{\mathrm{Re}}(n)+j\gamma_{\mathrm{Im}}(n)$.

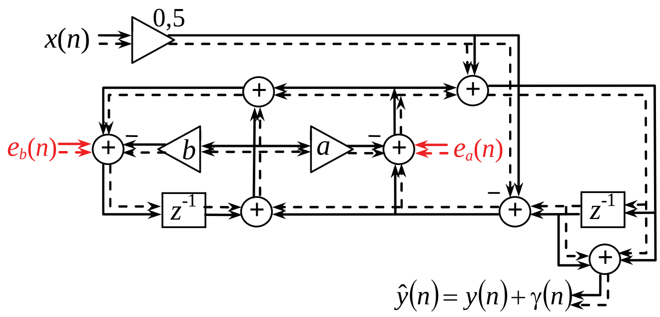

Fig.2: Noise model of BP orthogonal complex LS2 filter section due to complex multiplication product quantization

The noise model consists two noise sources thus realizing two complex NTF:

$$G_{e_a}(z)=\frac{Y_{\mathrm{Re}}(z)+jY_{\mathrm{Im}}(z)}{E_a^{\mathrm{Re}}(z)+jE_a^{\mathrm{Im}}(z)}=G_{e_a}^{\mathrm{Re}}(z)+jG_{e_a}^{\mathrm{Im}}(z)=$$
$$=a\frac{(2a+b-3)z^{-2}+(1-b)z^{-4}}{D(z)}+j\frac{-az^{-1}+a(3-2a-2b)z^{-3}}{D(z)}; \qquad (4)$$

$$G_{e_b}(z)=\frac{Y_{\mathrm{Re}}(z)+jY_{\mathrm{Im}}(z)}{E_b^{\mathrm{Re}}(z)+jE_b^{\mathrm{Im}}(z)}=G_{e_b}^{\mathrm{Re}}(z)+jG_{e_b}^{\mathrm{Im}}(z)=$$
$$=\frac{-1+(2-2a-b)z^{-2}+(1-b)z^{-4}}{D(z)}+j\frac{-2z^{-1}\left[(1-a)+(b-1)(b-1+a)z^{-2}\right]}{D(z)}, \qquad (5)$$

where $D(z)=1+\left[(2a+b-2)^2-2(1-b)\right]z^{-2}+(1-b)^2z^{-4}$.

The nomalized complex output noise variance can be evaluated after the superposition is employed:

$$\sigma^2_{\gamma,n} = \sigma^2_{\gamma_{\mathrm{Re}},n} + j\sigma^2_{\gamma_{\mathrm{Im}},n} = \sigma^2_{\gamma_a,n} + \sigma^2_{\gamma_b,n} \, ,$$

where

$$\sigma^2_{\gamma_a,n} = \frac{1}{2\pi j}\oint G_{e_a}(z)\, G_{e_a}\left(z^{-1}\right) z^{-1} dz \, ;$$

$$\sigma^2_{\gamma_b,n} = \frac{1}{2\pi j}\oint G_{e_b}(z)\, G_{e_b}\left(z^{-1}\right) z^{-1} dz \, .$$

In order to be compared, the output noise variance for the DF-based orthogonal complex second-order digital filter section [8] is derived by analogy.

IV. Experiments and Discussions

In [9] arithmetic noise analysis is applied for orthogonal complex first-order sections. It is shown experimentally that the low coefficient sensitivity of the orthogonal complex circuit accompany low output noise variance due to the multiplication products quantization.

In this work second-order real and orthogonal narrow-band structures are investigated with respect to the roundoff errors. Some experimental results for the output noise variance due to multiplication products quantization from 3 to 8 bits referring to the real LP narrow-band prototype sections are shown in Fig. 3.

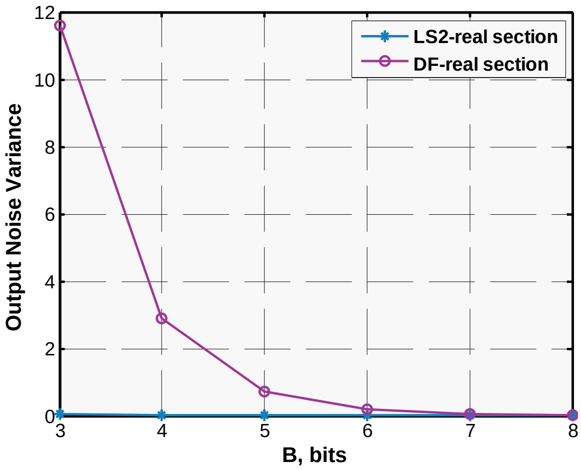

Fig.3: Output noise variances due to multiplication product quantization for LS2 and DF real sections

TABLE I

Multiplication products quantization in bits	Output noise variances of the real LP sections	
	DF	**LS2**
3	11.59193139858002	0.03982671126229
4	2.89054662868667	0.00995321478997
5	0.72249792141146	0.00248821008630
6	0.18060730243049	0.00062203181678
7	0.04515164102438	0.00015550792210
8	0.01128790972702	0.00003887698017

It is evident that the low sensitivity LS2 section output noise variance is significantly lower than that of the DF-section when the multiplication product is rounded to 3 bits only. The difference decreases exponentially with increasing word-length as numerical results in Table I show.

Complex output noise variances of the orthogonal sections after rounding of the multiplications to different word-length are determined in addition. They are achieved for the same very narrow-band BP complex orthogonal filters. The exact numerical results are shown in Table II.

TABLE II

Multiplication products quantization in bits	Complex modules of the complex output noise variances of the orthogonal complex sections	
	DF-based	LS2-based
3	1.08873247770671	0.00394689452292
4	0.27218311942668	0.00098672363073
5	0.06804577985667	0.00024668090768
6	0.01701144496417	0.00006167022692
7	0.00425286124104	0.00001541755673
8	0.00106321531026	0.00000385438918

In order to compare the obtained complex output noise variances, their complex modules have been graphically presented in Fig.4. It can be concluded that the low sensitivity LS2-based orthogonal complex section shows few times lower output noise after 3 bits multiplication signal quantization. The shorter word-length quantization of the inner products signifies lower power consumption and faster computation process. For low sensitivity circuits the resistance to quantization effects provides better signal to noise ratio, i.e. higher quality digital signal processing.

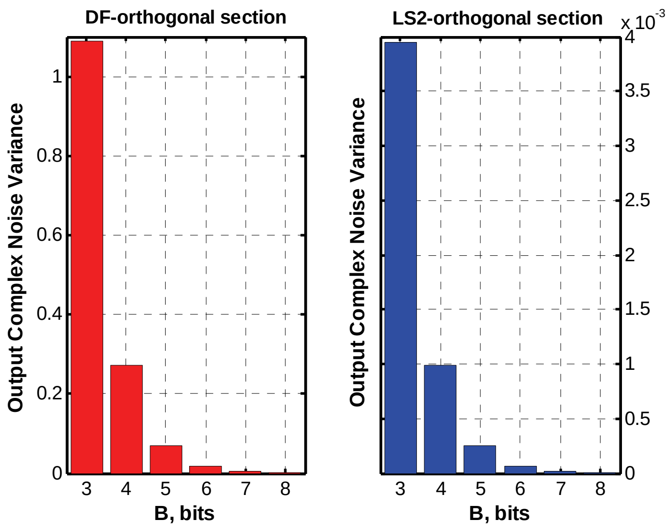

Fig.4: Output noise variances due to multiplication product quantization for LS2- and DF-based orthogonal sections

Analytic sinusoidal signals were applied at the input of DF- and LS2-based orthogonal complex sections and the multiplication signals were quantized to different word-length. The noise signals generated due to this process has been filtered and the noise reaching the outputs is graphically depicted in Fig 5.

In Fig.5a the real part of the complex noise reaching to the real output is shown for both second-order orthogonal complex structures. The imaginary output noise signals are presented in Fig. 5b. Obviously the complex noise arrived at the output of the orthogonal circuits due to the quantization of the complex inner signals is considerably more for the DF-based section than for the LS2-based section. It is obvious that the level of the output noise after quantization of multiplications and the sensitivity of the system are directly proportional to each other. Therefore, very low sensitivity complex filter derivation is important to achieve a better noise resistance and to improve complex signal filtering and quality of the digital signal processing in general.

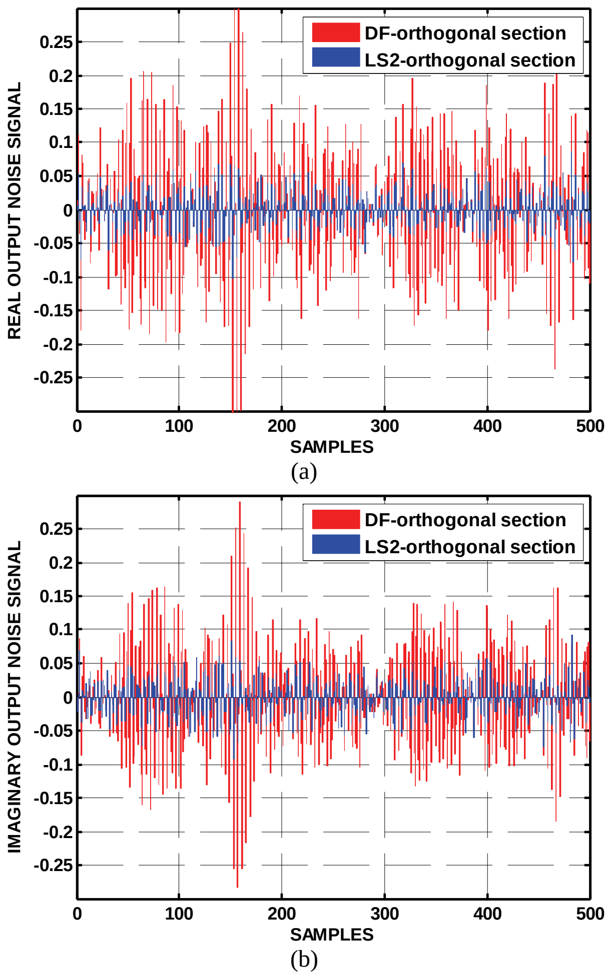

Fig.5: The output noise signals after quantization of the multiplications to 3 bits for LS2 and DF - based orthogonal complex sections (a) real outputs; (b) imaginary outputs

V. Conclusions

In this paper a new technique for complex arithmetic noise analysis is suggested. The resulting error signals at the output of orthogonal complex second-order digital filter sections after multiplication products quantization are examined. The technique has been developed under the assumption that there is no correlation between the noise and the input and output signals to the multipliers.

The real prototype properties are inherited by its complex filter counterpart with respect to the noise analysis after inner signal quantization. It was shown that both real and orthogonal complex LS2 filter sections, having very low coefficient sensitivity, demonstrate low output noise variance on account of multiplication signal quantization. The DF-based orthogonal complex circuit shows many times higher output roundoff errors than LS2 section. Hence, as for coefficient quantization, the filter structure affects considerably signal quantization noise levels. The low sensitivity of the complex filters in a very limited word-length environment makes available low computational complexity and provides better quality of the filtering process.

The suggested technique is general enough to be applied for orthogonal complex filters of higher order. The narrow-band complex coefficients realizations are preferred in many telecommunication applications and are normally implemented with fixed-point arithmetic which usually provides lower power consumption and minimum computing time. After relevant alterations it can be effectively applied for all types of signal quantization errors estimation of complex coefficient systems in general.

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