Optimization of the Parameters of a Subwoofer with Phase-inverter

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Abstract – **One of the approaches for obtaining a flat frequency response of a loudspeaker system is to put a phase-inverter in the box, of the loudspeaker. Hence the expression for the frequency response becomes complicated and is not well studied in the literature. The authors have performed a optimization of the parameters of the subwoofer, that are important for obtaining a flat frequency response. The parameters are the coefficient of elasticity, tuning coefficient and the quality factor of the equivalent circuit. The most important achievement of this paper is the widening of the frequency range at low part, which is the most challenging task in designing loudspeaker boxes.**

Keywords - **Loudspeaker, phase-inverter, maximum flat frequency response.**

I. Introduction

In order to achieve high-quality of a sound rendering system, a loudspeaker box with low non-linear distortion should be designed. Among the many possibilities a loudspeaker with a so called sound inverter (or, as modern called bass reflex) offers the best sound at low cost.

The diaphragm of the loudspeaker on its movement inwards to the enclosed volume causes that the air becomes dense, and the pressure in the volume increases. At the input of the port a certain amount of momentum, caused by the pressure in the volume, appears. It makes the amount of in the vent to move. The output of the of the bass reflex makes the air in the close space to become dense, i.e. a sound wave is emitted. At the next moment the diaphragm of the driver is moving forward and also causes that the air in front of it becomes dense, i.e. also emits a sound wave. If both of the waves, caused by the diaphragm of the driver and from the output of the vent, and in the same phase, the sound pressure Pa in the space in front of the box will increase, it will be equal to the algebraic sum of the both pressures:

$$P = Pa + Pb \tag{1}$$

Where

Pb -is the moment value of the sound pressure, emitted by the driver, and

Pa - is the moment value of the sound pressure, emitted by the vent [1].

Going that way, the vent uses the energy of the back sound wave in the space of the front sound wave. The vent turns the negative property of the diaphragm producing a back wave into a positive one. It does so by inverting the phase and summing it with the front wave. That way there’s no need of putting sound isolating material in the box and for extra amplification for obtaining high sound pressure. That’s why the vent box loudspeaker system is the most energy efficient one. A typical figure of a bass-reflex system is given at fig.1

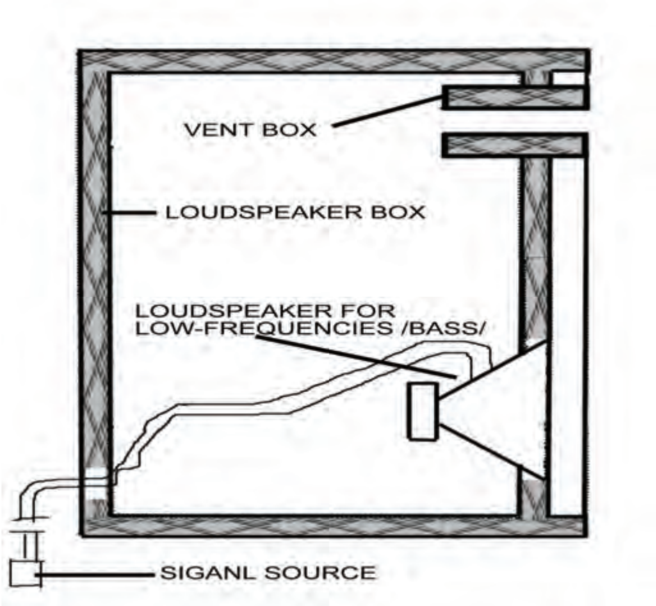

Fig.1 A typical figure of a bass-reflex system

II. Problems with obtaining maximum flat frequency response

The full and accurate analysis of the loudspeaker system with a bass reflex is made based on its equivalent circuit, taking into account its electrical parameters and deriving the analytical relation for the produced sound pressure level in the front space. The deriving of the equation is connected with a lot of complicated mathematical operations. In the literature [1] the ready derived relation can be found:

$$\mathrm{H} = k * \mathrm{H}_1(f) \tag{2}$$

where k is a constant.

The term $\mathrm{H}_1(f)$ is the frequency dependent one. It looks as follows

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$$\mathrm{H}_1(f)=\frac{1}{\sqrt{1+\frac{A_1}{x^2}+\frac{A_2}{x^4}+\frac{A_3}{x^6}+\frac{A_4}{x^8}}}, \qquad (3)$$

where the A'terms are defined as follows:

$$A_1=\frac{1}{Q_T^2}-2-2\alpha-2h^2; \qquad (4)$$

$$A_2=(1+\alpha)^2+h^2(4+2\alpha+h^2-\frac{2}{Q_T^2}); \qquad (5)$$

$$A_3=h^2(\frac{h^2}{Q_t^2}-2-2\alpha-2h^2); \qquad (6)$$

$$A_4=h^4 \qquad (7)$$

and the x is the normalized frequency about f_p, which is the resonance frequency of the loudspeaker itself.

$$x=\frac{f}{f_p} \qquad (8)$$

Q_T is the quality factor of the equivalent circuit α
Is the so called coefficient of elasticity, which shows how many times the volume of the box is bigger than the equivalent volume of the driver

$$\alpha=\frac{V_{box}}{V_{driver}} \qquad (9)$$

where V_{box} is the volume of the loudspeaker box, and V_{driver} is the equivalent volume of the driver. The last coefficient h is the so called tuning coefficient, that shows how many times the resonance frequency of the vent is bigger than that of the loudspeaker box:

$$h=\frac{f_v}{f_p}, \qquad (10)$$

where f_v is the resonance frequency of the vent, and the f_p is the one of the loudspeaker box.

When optimizing the coefficients, most of the authors propose that the first three coefficients should be equaled to zero, because it's impossible to have all of the A'terms zero [1][2] . This approach gives a good approximation of the parameters for home and semi-professional applications. The loudspeaker system is always the component with the worse quality of the whole sound immersing system. That's why it's necessary to derive more accurate optimization of the coefficients of the frequency response, in order to become as flat one as possible [2].

III. Proposed Optimizing of the Coefficients of the Vent Box

The problem, which was solved in this paper, is defined as follows – Maximize the energy of the eta-funtcion, i.e. minimize the energy of the function

$$\mathrm{H}_2(f)=\sqrt{1+\frac{A_1}{x^2}+\frac{A_2}{x^4}+\frac{A_3}{x^6}+\frac{A_4}{x^8}} \qquad (11)$$

by optimizing the coefficients α , h and Q_T .

The resonance frequency of the driver is set to be 40 Hz, as this value is very common in practice [1]. The equivalent Volume of the loudspeaker is not more than $40\,V^3$. The driver can be put alone in the loudspeaker box, or with combination with speakers for the mid- and high-frequency range.
For implementation sake the values of the three parameters are constrained as follows:

- one can not use as big α as wanted, because the volume of the box will become enormous big and not suitable for transportation and very hard to integrate in rooms with low or middle volume [3].
- as for the coefficient h , the size of the vent, which is in linear connection with the resonance frequency of the vent, also cannot be so small or big, as some of the non-constrained optimizations propose, that's why it should be limited [3].
- the quality factor is one of the more difficult to change in the design of the loudspeaker system, especially it's very hard to get it more than 0.6, that's why it's limited to that value [4].

The Optimization toolbox of Matlab was used. The function for optimization is fminimax, i.e. the desired values of the coefficients are derived by solving the minimax problem [5]. The maximum function evaluations for every iteration were 1000, and the maximum iterations themselves were also 1000. The error tolerance of the optimization was set to be 1e-003.

The frequency range of the optimization was from x=[0.4 8], this means for liner frequency f=10:320 Hz.

TABLE I
RANGE OF THE PARAMETERS TO BE OPIMIZED

α	1-12,
h	0.04-1.8
Q_T	0.01-0.6

The reasons for the constrains are:
Each of these ranges was divided in 288 steps, and each of the whole 23887872 sets of parameters was given to fminimax as initial set of parameters. The output range of parameters was constrained as the input one. The most important linear constraint that was set is that the eta-function should be in the

range from 1 to 0.707, i.e. the maximum allowable frequency distortion is 3 dB.

The optimization was performed on 4 computers with the following configuration:

TABLE II
COMPUTER CONFIGURATION

CPU	Intel Pentium 4 2.8 GHz
DDR RAM	512 MB
OS	WINDOWS XP SP 2

The optimization was performed parallel on the four computers for 48 hours.

The algorithm of the optimization is defined as follows:

1. Set the ranges for α , h , Q_T .
2. Set three nested in each other loops, one for α , h , Q_T respectively.
3. For each combination of the three parameters to be optimized call the function fminimax.
4. The goal function if fminimax is does the following:
    a. Evaluates the frequency response of the loudspeaker with the current set of coefficients.
    b. Evaluates the inverse of the energy of the frequency response and returns the result.
5. Fminimax then minimizes the inverse of the energy of the frequency response with respect to the given constraints, which are as follows:
    a. The evaluated normalized frequency response should be not less that 0.707 in the frequency range from 40 to 320 Hz.
    b. The evaluated normalized frequency response should be not more than 1 in the frequency range from 40 to 320 Hz /taking into account the loudspeaker is a passive system/.
6. After the performed optimization of all of the sets of coefficients, take these ones, which have no violation of the constraints.
7. From the latter sets of coefficients take this one, which provides the widest range of the frequency response at level -3 dB.

IV. ANALYSIS OF THE RESULTS

The optimized coefficients, as well as the two sets of coefficients, that are proposed in the literature, are given in table three [1] [4] :

TABLE III
SETS OF COEFFICIENTS

Optimized coefficients	$\alpha = 1, h = 0.744, Q_T = 0.364$
Proposed in literature 1	$\alpha = 1.414, h = 1, Q_T = 0.383$
Proposed in literature 2	$\alpha = 3.02, h = 1.26, Q_T = 0.3$

To get a better impression of what these coefficients really mean, the following two figures are drawn.

One of the most important parameters (if not the most important one) is the frequency, at which the system begins to render sounds properly [2].On it's base the important parameters f_3 and f_6 are defined. These are the frequencies, at which the magnitude falls with 3 and, respectively 6 dB from the magnitude of the resonance frequency. This means that the parameters for low frequencies are the most important ones.

All of the efforts in the design of loudspeaker systems are made in the low-frequency range, for there are no challenging problems with design at the high frequency range. As is it well can be seen from fig.2 the optimized approach manages to widen the frequency range for low frequencies. The proposed sets of coefficients begin proper rendering at x = 0.85 and x = 1.175 respectively, while the optimized approach sets the low cutoff frequency at x = 0.65.

This becomes obvious also from fig.3. The conclusion can be made, that this widening of the frequency range can be derived by the optimization, made in this article, and it has no other negative effects on the other parameters or the practical implementation of the design. The ripple for the approach, proposed by the authors of this paper, is 1 dB, and the ripple for the proposed designs in the literature is 0.1 dB [4]. This, thought, is not a problem, because the human auditory system can not respond to distortion of 1 dB, or less [3].

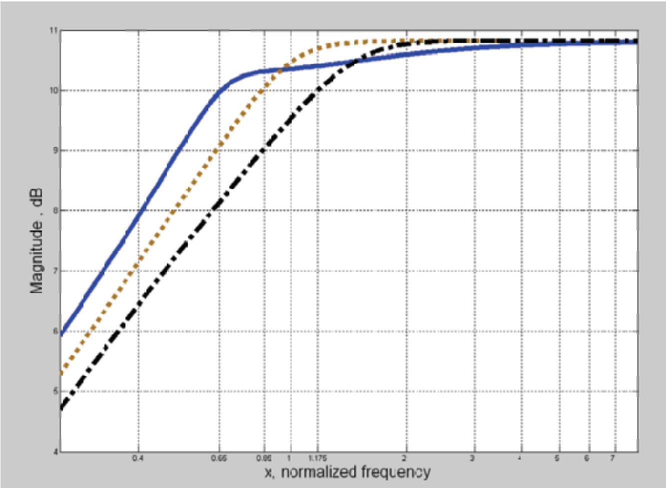

Fig.2.Amplitude frequency response – solid blue – optimized coefficients, dashed brown line – first set proposed in literature [1], dashed-dotted black line – second set proposed in literature [4].

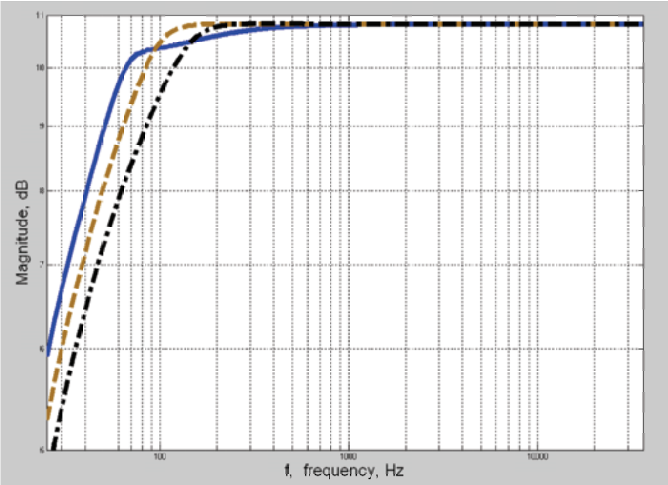

Fig.3. Amplitude frequency response – solid blue– optimized coefficients, dashed brown line – first set proposed in literature[1], dashed-dotted black line – second set proposed in literature [4].

V. CONCLUSION

The authors have revisited one of approaches for obtaining a flat frequency response of a loudspeaker system. This was the so called vented box .The expression for the frequency response is a very complicated one and is not well studied in the literature. The authors have performed an optimization of the parameters of the subwoofer - coefficient of elasticity, tuning coefficient and the quality factor of the equivalent circuit. The most important achievement of this paper is the widening of the frequency range at low part, which is the most challenging task in designing loudspeaker boxes.

In their future work, the authors have the goal to optimize the coefficients of other drivers in vent boxes, using other methods, and widen the frequency range for low frequencies. The further work includes also optimizing parameters for other loudspeaker systems, not only vent boxes.

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