# Camera Calibration for Mechatronic Measurement 

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#### Abstract

Mechatronics measurement and process control with use image processing often require predicted camera charateristics. In this paper diferent techniques for camera calibration are presented and investigated in sense mechatronic propose.


Keywords - Camera Calibration, Image Processing

## I. Introduction

Digital image processing in mechatronics is, as a rule, used in the situations in which it is difficult or impossible to acquire useful information by means of conventional methods. This mode of information collection is exceptionally suitable for the objects which are of extremely irregular shape, mobile, unavailable due to too high or too low temperature, sensitive to touch, inaccessible, radioactive, toxic, or when a great quantity of information is to be collected in a large area, if the geometry of the object changes, etc. This mode of information collection has a series of advantages such as: the variety of results (descriptive data, photograph documents, graphic documents, numerical data...), high range of accuracy (from 0.1 mm up to a few meters), great automatization possibilities of collection and processing of information from the photograph, numerous application areas, great reliability because of the objective registration of all information in the photograph, simple possibility of additional measurements and monitoring of changes, wide sensitivity spectrum (IC, thermal radiation, and other sorts of electromagnetic radiation) etc. All this information can be collected by means of the cameras of different types and applications. The choice of the adequate camera depends on the kind of information we want to get and on the object or process which is observed. From the wide palette of available cameras, a digital amateur camera has been analyzed because it satisfies most criteria (including the price) for the application in different domains of mechatronics. The first step in the use of the camera for measurements is its calibration. The idea of calibration includes the definition of geometric and optical characteristics and intrinsic and extrinsic parameters of the camera through different procedures. The choice of the calibration mode depends on the accuracy and applicability of the desired results, taking into account the available calibration equipment, too.
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## II. SCENE FORMING

Figure 1: A scheme of image forming, starting with the real scene [1].


Fig. 1. The structure of the image forming system
Light ray of a scene goes through the optical system of the camera and the video sensor and at the output it gives the analogue signal. The image acquired in that way, deviates from the real scene which is the consequence of the optical system design and manufacturing, the manufacturing and positioning of the camera sensor, position and type of the light source, environment temperature, vibrations and other numerous and different influences of the environment. These deviations, named by one word, are distorsions. The analogue signal is taken to the converter ( FG ) which converts the analogue signal into digital. The digitalized signal is transferred to the computer and the image is prepared for the processing.

The transformation of a three-dimensional scene into twodimensional image can be considered as a kind of distortion, because it is not possible to perserve the real values of corresponding points parameters. On the basis of the dominant characteristics the distortion can be geometrical and radiometrical. The geometrical distortion changes the position of the scene image points depending on the characteristics of the camera optical system and environmental influence. For example, the perspective projection (figure 2) of a square is a quadrangle with transformed values of the angles and distances between the corresponding points.


Fig. 2. Perspective projection
These types of distortion are called linear because they can be mathematically described by linear matrix agebra. Linear distortion is manifested in the form of perspective projection, displacement of the coordinate origin of the image, the
differences in the scaling allong the coordinate axes and the lack of orthogonality. Besides the lenear there are also nonlinear distortions: radial ditortion (barrel and pincushion) and the lens ditortion (radial and tangential). Radiometrical distortion causes the deviations in the domain of the image light characteristics.
In order to get as realistic and quality scene information as possible, these deviations should be compensated. It is possible to realize the compensation if we get acquainted with the functional dependance of coordinate points projection and define the type and size of deviation.

## III. Concept and Types of Calibration

Camera calibration includes the discovery of intrinsic camera characteristics which influence the image formation process (focus length, factors of scaling along the rows and colums of pixels, skew factors, lens system distortion, the position of the image center in the image) [2]. There are two types of camera calibration: radiometical and geometrical. The radiometrical calibration includes the aquiring of information about the camera distortion in the domain of luminous characteristics of the scene such as: colour, luminance, brightness, etc. The geometrical calibration results in the knowledge of translation and rotation camera parameters (extrinsic parameters) and the knowledge of the value of the focus length of the camera optic system, principal point position, the factors of scaling along the image axes etc (intrinsic camera parameters). This type of camera calibration includes the process of the acquiring of the image of the special, a priori shaped and and dimensionally defined real scene. This calibration type is known as the object calibration and the target calibration.

Taking into accout that each camera, regardless the construction of the optical part of the system, functions according to the principle of central projection, in Figure 3 there is a schematic projecetion of a real scene into an image. $\mathrm{P}_{\mathrm{i}}$ are scenes points chosen at random, and $\mathrm{q}_{\mathrm{i}}$ are the corresponding points in the image plane.


Fig. 3. Central projection
The coordinate system ( $\mathrm{x}^{\prime}, \mathrm{y}^{\prime}, \mathrm{z}^{\prime}$ ) is chosen in such a way that the axis $z^{\prime}$ is perpendicular on to the image plane I. The image coordinate system can also be set freely. All scene light rays intersect in one point $(\mathrm{O})$ which is called the optical center or the center of the perspective. The line which includes the optical center and is perpendicular on the image
plane is named the optical axis (o). This axis intersects the image plane in the point C which is named the principle point. This point does not have to correspond with the image I plane center (CCD plane).

At this transformation, the rotation and translation of the coordinate system occurs. The coordinate system rotation is shown in the Figure 4.


Fig. 4. Plane rotation of the coordinate system
The interpretation of the plane rotation in the matrix form is as follows:

$$
\begin{equation*}
X=R \cdot x \tag{1}
\end{equation*}
$$

where R is the orthogonal matrix.

$$
R=\left[\begin{array}{ll}
r_{11} & r_{12}  \tag{2}\\
r_{21} & r_{22}
\end{array}\right]
$$

In practice, it seldom happens that the transformation is of a plane type. Most often, it is space rotation around all three coordinate axes, shown in the Figure 5.


Fig. 5. Space rotation of the coordinate system
The mathematical interpretation of the coordinate system plane rotation in the matrix form is as follows:

$$
\begin{equation*}
X=R \cdot x \tag{3}
\end{equation*}
$$

where R is an orthogonal shape matrix:

$$
R=\left[\begin{array}{lll}
r_{11} & r_{12} & r_{13}  \tag{4}\\
r_{21} & r_{22} & r_{23} \\
r_{31} & r_{32} & r_{33}
\end{array}\right]
$$

The matrix elements of space rotation are calculated by means of sinus and cosinus of rotation angles $\varphi, \omega$ i $\kappa$, and the form of this matrix depends on the rotations order.

For most applications in mechatronic systems, geometrical calibration gives satisfactory results because the coordinates of the pair scene-image are available and known [5]. Scene coordinates are measured directly at the scene itself (calibration target), and the coordinates of corresponding points are detected in the image. The calibration object can be a checkboard with black and white squares of known
dimensions, with circles of known diametars and marked gravity centers etc. On each calibration checkboard there are characteristic points called control or reference points. The object calibration can be three-dimensional, by virtual 3D object (simulation by multiple 2D views) or two-dimensional (coplanar).


Fig. 6. Types of calibration
The calibration by 3D object is complicated for realiztion, and the definition of reference points coordinates in space is complex. This mode provides the possibiliy of a very precise calibration. 3D calibration is applicable in the cases when both extrinsic and intrinsic camera parameters are known.

Besides 3D calibration, it is also possible to perform the calibration by simulation of 3D object with multiple 2D wievs of the calibration object. 2D plane can be freely mobile between paricular images of acquisition or with a priori defined movement.

The simplest mode of objects calibration is coplanar or 2D calibration. This mode is also the cheapest one and therefore, the most popular. The calibration sample in form of checkboard and geometrical shapes, defined by dimension and form, on it are needed for the realization.

By 3D calibration, the most precise and most reliable results are achieved, which is important for the control of mechatronic mechanisms. At 3D calibration, we start with the assumption that 3D and 2D coordinate points of the object (e.g. a cube) are known values [4].


Fig. 7. Calibration model
On the basis of the given 3D coordinates of the known object and by the elimination of unknown scaling factors, the projection matrix $\Pi$ is formed. By the application of the matrix calculation method and the method of decomposition of the fundamental matrix for known epipolar limitations, we get the solution in the form of camera parameters. The process of fundamental matrix formation is significant because by its further mathematical processing (reconstruction) the respective projection matrix and 3D structure can be formed. There are three different types of reconstruction: projection, Euklidean and afino . Euklidean reconstruction implies that the bject characteristics (parallelism, angles, distances, etc) are maintained. rigid bodies transformations are
transformations of proportionality. At the projective reconstruction, these features are changed so that finally, an image with distortion is formed. Advanced projective structure is afino structure. It uses partially the known scene (known points of disappearance, known principle point, without skew) or special cases (pure rotation, pure translation, planar movement or straight linear movement). Camera parameters are constant (multiple views, i.e. view from different angles). Projection matrices and 3D structure matrices for these types of reconstruction are shown in the Chart I. Chart II gives the survey of the most important calibration methods.

## Chart I

|  | Camera projection | 3D structure |
| :---: | :---: | :---: |
| Euklidean | $\Pi_{1 \mathrm{e}}=[\mathrm{K}, 0], \Pi_{2 \mathrm{e}}=[\mathrm{KR}, \mathrm{KT}]$ | $\mathrm{X}_{\mathrm{e}}=\mathrm{g}_{\mathrm{e}} \mathrm{X}=\left[\begin{array}{cc}\mathrm{R}_{\mathrm{e}} & \mathrm{T}_{\mathrm{e}} \\ 0 & 1\end{array}\right] \mathrm{X}$ |
| Affine | $\Pi_{2 \mathrm{a}}=\left[\mathrm{KRK}^{-1}, \mathrm{KT}\right]$ | $\mathrm{X}_{\mathrm{a}}=\mathrm{H}_{\mathrm{a}} \mathrm{X}_{\mathrm{e}}=\left[\begin{array}{cc}\mathrm{K} & 0 \\ 0 & 1\end{array}\right] \mathrm{X}_{\mathrm{e}}$ |
| Projective | $\Pi_{2 \mathrm{p}}=\left[\mathrm{KRK}^{-1}+\mathrm{KTv}^{\mathrm{T}}, \mathrm{v}_{4} \mathrm{KT}\right]$ | $\mathrm{X}_{\mathrm{p}}=\mathrm{H}_{\mathrm{p}} \mathrm{X}_{\mathrm{A}}=\left[\begin{array}{cc}\mathrm{I} & 0 \\ -\mathrm{v}_{v_{4}}{ }^{-1} & v_{4}{ }^{-1}\end{array}\right]$ |

Chart II

| Summary of calibration methods |  |  |  |
| :---: | :---: | :---: | :---: |
|  | Euklidean | Affine | Projective |
| Strukture | $\mathrm{X}_{\mathrm{e}}=\mathrm{g}_{\mathrm{e}} \mathrm{X}$ | $\mathrm{X}_{\mathrm{a}}=\mathrm{H}_{\mathrm{a}} \mathrm{X}_{\mathrm{e}}$ | $\mathrm{X}_{\mathrm{p}}=\mathrm{H}_{\mathrm{p}} \mathrm{X}_{\mathrm{a}}$ |
| Transforma tion | $\mathrm{g}_{\mathrm{e}}=\left[\begin{array}{cc}\mathrm{R} & \mathrm{T} \\ 0 & 1\end{array}\right]$ | $H_{a}=\left[\begin{array}{cc}\mathrm{K} & 0 \\ 0 & 1\end{array}\right]$ | $H_{p}=\left[\begin{array}{cc}\mathrm{I} & 0 \\ -v^{T} v_{4}{ }^{-1} & v_{4}{ }^{-1}\end{array}\right]$ |
| Projection | $\Pi_{\mathrm{e}}\left[\begin{array}{ll}\mathrm{KR} & \mathrm{KT}\end{array}\right]$ | $\Pi_{\mathrm{a}}=\Pi_{\mathrm{e}} \mathrm{H}_{\mathrm{a}}{ }^{-1}$ | $\Pi_{\mathrm{p}}=\Pi_{\mathrm{a}} \mathrm{H}_{\mathrm{p}}{ }^{-1}$ |
| 3-step upgrade | $\mathrm{X}_{\mathrm{e}} \leftarrow \mathrm{X}_{\mathrm{a}}$ | $\mathrm{X}_{\mathrm{a}} \leftarrow \mathrm{X}_{\mathrm{p}}$ | $\mathrm{X}_{\mathrm{p}} \leftarrow\left\{\mathrm{x}_{1}^{\prime}, \mathrm{x}_{2}^{\prime}\right\}$ |
| Info. needed | Calibration K | Plane et infinity $\pi_{\infty}^{\mathrm{T}}=\left\lfloor\mathrm{v}^{\mathrm{T}}, v_{4}\right\rfloor$ | Fundamental matrix F |
| Methods | Lyapunov eqn. | Vanishing points | Canonical decomposition |
|  | Pure rotation | Pure translation |  |
|  | Kruppa's eqn. | Modulus constraint |  |
| 2-step upgrade | $\mathrm{X}_{\mathrm{e}} \leftarrow \mathrm{X}_{\mathrm{p}}$ |  | $\mathrm{X}_{\mathrm{p}} \leftarrow\left\{\mathrm{x}_{\mathrm{i}}^{\prime}\right\}_{\mathrm{i}=1}^{\mathrm{m}}$ |
| Info. needed | Calibration K and | $\pi_{\infty}^{\mathrm{T}}=\left\lfloor\mathrm{v}^{\mathrm{T}}, \mathrm{v}_{4}\right\rfloor$ | Multiple-view matrix |
| Methods | Apsolute quad | ic constraint | Rank conditions |
| 1-step upgrade | $\left\{\mathrm{x}_{\mathrm{i}}\right\}_{\mathrm{i}=1}^{\mathrm{m}} \leftarrow\left\{\mathrm{x}_{\mathrm{i}}^{\prime}\right\}_{\mathrm{i}=1}^{\mathrm{m}}$ |  |  |
| Info. needed | Calibration K |  |  |
| Methods | Ortogonal |  |  |

At the calibration, a real camera is presented by a mathematical model which describes how a real scene is transformed into image [3]. In accordance with the real characteristics of the camera and the preciseness which we want to achieve in the image, the whole palette of camera models has been developed. Some of them are based upon the physical parameters of the camera, but there are also the ones which just represent the projection of a scene into an image. In accordance with that, all models can be classified into two big groupings: explicit models, based on physical camera parameters and implicit models, which just describe the
projection of a scene into an image. Within the explicit models, there are the following models: pinhole model, DLT model which can bie coplannar DLT model or extended DLT model and a photogrametic model of a camera which can be a traditional one or a simplified one. Within the group of implicit models, there are two -plane and n-plane models. By the comparative analisis of models, we come to the conclusion that they are different from each other according to the number of physical parameters which they inslude, advantages and disadvantages. For example, a pinhole model includes seven parameters and it is able to represent translation and rotation of a camera and to include distortion in the form of perspective projection. The mathematical model can be described as

$$
\begin{equation*}
q_{i}=F \cdot M \cdot T \cdot p_{i} \tag{5}
\end{equation*}
$$

where $p_{i}=\left[x_{i}, y_{i}, z_{i}, 1\right]^{T}$ is a matrix of a scene coordinares and i-te point and $q_{i}=\left[w_{i} u_{i}, w_{i} v_{i}, w_{i}\right]^{T}$ matrix of corresponding coordinates in the image plane. Matrices:

$$
\begin{gather*}
F=\left[\begin{array}{lll}
f & 0 & 0 \\
0 & f & 0 \\
0 & 0 & f
\end{array}\right] \quad T=\left[\begin{array}{llll}
1 & 0 & 0 & -x_{0} \\
0 & 1 & 0 & -y_{0} \\
0 & 0 & 1 & -z_{0}
\end{array}\right]  \tag{6}\\
M=\left[\begin{array}{lll}
m_{11} & m_{12} & m_{13} \\
m_{21} & m_{22} & m_{23} \\
m_{31} & m_{32} & m_{33}
\end{array}\right]
\end{gather*}
$$

$\mathrm{F}, \mathrm{M}$, and T are respective matrices which describe the focal length, rotation and translation camera parameters. The realtions between the mentioned matrices are in this case very simple, unlike other model in which these relations are rather more complex. Other forms of distortion, such as displacement of the coordinate origin or lens distortion are not present. The model is simple, stable and linear. This means that the pinhole model represents the model of an ideal camera. This model is an approximation of a real camera and it can be used as a basis for other calibration models. DLT model represents the model of direct linear transformation and it can be considered as an improved pihnole model. At this model, there is no orthogonality between the coordinate axes of the image, the coordinate origin moves and there are different scaling factors along the coordinate axes. It is possible to describe the model mathematically by matrices of linear algebra.

$$
\begin{equation*}
q_{i}=A \cdot p_{i} \tag{7}
\end{equation*}
$$

where the matrix A consists of the matrices $\mathrm{V}, \mathrm{B}, \mathrm{F}, \mathrm{M}$ and T

$$
\begin{equation*}
A=\lambda V^{-1} B^{-1} F M T \tag{8}
\end{equation*}
$$

Factor $\lambda \neq 0$ presents the tolta scaling. Matrices F, Mit, as in a pinhole model describe the focus length, rotation and translation, matrix V represents the traslation of the coordinate origin of the image in relation to the ideal projection case,
and the matrx B compensates different scaling factors and the lack of orthogonality between the coordinate axes. The complementary DLT model (CDLT) implies that all objects points are in one plane. In that case, the equatation which descrebes the projection gets the following form

$$
\begin{equation*}
q_{i}=A_{(3,3)} \cdot p_{i} \tag{9}
\end{equation*}
$$

The expended DLT model includes, beside other elements, also the radial and tangential distortion, which is acheived by the introduction of additinal five parameters. By the comparative analysis of other models, we come to the conclusion that they include a greater number of parameters and they are able to include, to a smaller or higher degree, different distortion shapes. Depending on the conctrete camera model, it is possible to realize 2D calibration or another type of calibration is applied. The disadvantages of more complex models is the unability to include non linear distortions, the necessity of 3D calibration or the need for the distinct precisness at calibration.

## IV. CONCLUSION

In this paper we have anlysed some aspects of camera calibration in relation to the specific requirements of mechatronic system. For the measurement of specific parameters of mobile and immobile objects, it is necessary to eliminate ceratain distortion types. The analysis of some procedures given in the quated papers is presented by corresponding charts. On the basis of these results, it is possible to do the evaluation of the choise of a digital camera calibration method for each speciffic case of application in mechatronic systems.

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