Synthesis of Optimal Modal Controllers for DC Electric Drives

Tsolo T. Georgiev[1], Mikho R. Mikhov[2]

Abstract – **This paper discusses the synthesis of controllers, which can be structure-optimized through mathematically based selection of a functional. Relevant complex criterion for optimization has been introduced. The synthesis implements a combination between both- setting the closed-loop system poles (modal control) and optimal control through the quadratic quality criterion minimization. Results from testing of DC motor drive systems with such optimal modal control have been represented.**

Keywords – **Optimal control, Modal control, DC motor drives.**

I. Introduction

Modern electric drives are subject to high requirements such as precise accuracy and good dynamics, which predicates the use of digital control devices. At the same time, growing demands to quality control, as well as the complexity of electromechanical systems determine the need to synthesize controllers of higher potential compared to the traditional PI, PD and PID types.

Optimal modal state controllers meet such requirements. Their synthesis may be carried out either through the analogue mathematical model of the controlled object, with a subsequent discretization, or by means of the discrete model [1], [2], [4], [5].

Synthesis of state controllers by the discrete model has a number of advantages:

- first, a stable closed-loop system is provided, with a predefined quality;
- second, too small quantization period can be avoided thus eliminating the need of very fast microcontrollers.

This paper discusses some structurally optimized controllers, their synthesis being based on a mathematically selected functional. The procedure applied utilizes a combination between both - setting the closed-loop system poles (modal control) and optimal control through the quadratic quality criterion minimization, i.e. a complex criterion for optimization has been introduced.

Detailed studies of DC motor drive systems carried out by means of modeling and computer simulation show that this type of control can provide the desire performance.

II. Vector-Matrix Model of the DC Drive

The state-space model of the DC motor drive under consideration can be described as follows:

$$\begin{bmatrix} \frac{d\omega^*}{dt} \\ \frac{di^*}{dt} \\ \frac{dV_c^*}{dt} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{\tau_m} & 0 \\ -\frac{1}{\tau_a} & -\frac{1}{\tau_a} & \frac{1}{\tau_a} \\ 0 & 0 & \frac{1}{\tau_c} \end{bmatrix} \cdot \begin{bmatrix} \omega^* \\ i^* \\ V_c^* \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{\tau_c} \end{bmatrix} u^* + \begin{bmatrix} -\frac{1}{\tau_m} \\ 0 \\ 0 \end{bmatrix} i_l^*, \qquad (1)$$

where: $\omega^* = \omega/\omega_0$ is motor speed represented in relative units; ω_0 – ideal no-load speed; $V_c^* = V_c/(k_e\omega_0)$ – power converter voltage; k_e – back EMF coefficient; $i^* = i/i_{sc}$ – armature current represented in relative units; $i_{sc} = (k_e \cdot \omega_0)/R$ – short circuit current; R – armature circuit resistance; $u^* = u/[k_e\omega_0 / k_c]$ – control voltage of the power converter in relative units; k_c – amplifier gain of the converter; $i_l^* = i_l/i_{sc}$ – static current represented in relative units τ_m – electromechanical time-constant; τ_a – armature circuit time-constant; τ_c – converter time-constant.

The following notations of state variables have been adopted: $x_1 = \omega^*$, $x_2 = i^*$, $x_3 = V_c^*$. Measurable coordinate in this case is the motor angular speed ω, i.e.

$$\mathbf{y(t)} = \mathbf{Cx(t)},$$

where: $\mathbf{C} = [1 \;\; 0 \;\; 0]$, $\mathbf{x}^{\mathrm{T}} = [x_1 \;\; x_2 \;\; x_3]$.

The discrete state-space model of the controlled object can be represented as follows:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{13} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} . \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} u^*(k) + \begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix} i_l^* \qquad (2)$$

In order to use the quadratic quality criterion in the process of synthesis, the error of $e^*(k) = \omega_r^*(k) - \omega^*(k)$ should be formulated, where $\omega_r^*(k)$ is the reference speed in relative units.

It is assumed that both the reference and disturbance inputs are constant, i.e. $\omega_r^*(k) = \text{const}$ and $i_l^* = \text{const}$. The following equation concerns the error and state variables, which are not outputs [2]:

[1]Tsolo T. Georgiev is with the Faculty of Computer Science and Automation, Technical University of Varna, 1 Studentska Str., 9010 Varna, Bulgaria, E-mail: tsologeorgiev@abv.bg

[2]Mikho R. Mikhov is with the Faculty of Automatics, Technical University of Sofia, 8 Kliment Ohridski Str., 1797 Sofia, Bulgaria, E-mail: mikhov@tu-sofia.bg

$$\begin{bmatrix} x_{1e}(k+1) \\ x_{2e}(k+1) \\ x_{3e}(k+1) \\ x_{4e}(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & a_{11} & -a_{12} & -a_{13} \\ 0 & -a_{21} & a_{22} & a_{23} \\ 0 & -a_{31} & a_{32} & a_{33} \end{bmatrix} \cdot \begin{bmatrix} x_{1e}(k) \\ x_{2e}(k) \\ x_{3e}(k) \\ x_{4e}(k) \end{bmatrix} + \begin{bmatrix} 0 \\ -b_1 \\ b_2 \\ b_3 \end{bmatrix} u_{e(k)} \tag{3}$$

or

$$\mathbf{x}_e = (k+1) = \mathbf{A}_e \mathbf{x}_e(k) + \mathbf{b}_e \mathbf{u}_e(k), \mathbf{x}_e(0) = \mathbf{x}_{e0}, k = 0, 1, 2, ...;$$
$$y(k) = \mathbf{C}_e \mathbf{x}_e(k).$$

where:

$$\begin{aligned} x_{1e}(k) &= e^*(k-1) = \omega_r^*(k) - \omega^*(k-1); \\ x_{2e}(k) &= e^*(k) - e^*(k-1) = -\left[\omega^*(k) - \omega^*(k-1)\right]; \\ x_{3e}(k) &= i^*(k) - i^*(k-1); \\ x_{4e}(k) &= V_c^*(k) - V_c^*(k-1); \\ u_e^*(k) &= u^*(k) - u^*(k-1); \\ \mathbf{C}_e &= [1 \quad 0 \quad 0 \quad 0]. \end{aligned} \tag{4}$$

Eq. (3) has been used for the synthesis of both an optimal modal digital observer and the respective controller. Based on this equation the DC electric drive model has been developed (Subsystem 1), shown in Fig. 1.

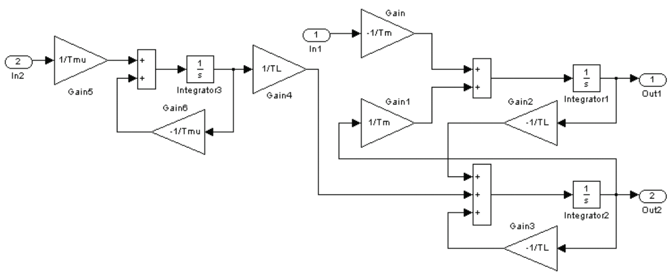

Fig. 1. Model of the controlled object (Subsystem 1).

The controlled object consists of a three-phase thyristor converter and a separately excited DC motor. The basic parameters are as follows:

$R = 1.69\,\Omega$, $L = 0.026\,\mathrm{H}$, $\tau_a = 0.0154\,\mathrm{s}$, $\tau_m = 0.2759\,\mathrm{s}$, $J = 0.0741\,\mathrm{kg.m}^2$, $k_e = 0.6737\,\mathrm{Vs/rad}$, $k_t = 0.6737\,\mathrm{Nm/A}$, $\tau_c = 0.005\,\mathrm{s}$, $k_c = 24.23$.

The rated data of the used DC motor are:

$P_{\mathrm{rat}} = 3.4\,\mathrm{kW}$, $V_{\mathrm{rat}} = 220\,\mathrm{V}$, $I_{\mathrm{rat}} = 17.6\,\mathrm{A}$, $\omega_{\mathrm{rat}} = 314\,\mathrm{rad/s}$.

III. Synthesis of Optimal Modal Observer

Synthesis of the digital observer will be realized by an algorithm presented in [1]. This procedure utilizes the transpositioned additional object [3]:

$$\boldsymbol{\alpha}(k+1) = \mathbf{A}_e^{\mathrm{T}} \boldsymbol{\alpha}(k) + \mathbf{C}_e^{\mathrm{T}} \beta(k) \tag{5}$$

or

$$\begin{bmatrix} \alpha_1(k+1) \\ \alpha_2(k+1) \\ \alpha_3(k+1) \\ \alpha_4(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & a_{11} & -a_{21} & -a_{31} \\ 0 & -a_{12} & a_{22} & a_{32} \\ 0 & -a_{13} & a_{23} & a_{33} \end{bmatrix} \cdot \begin{bmatrix} \alpha_1(k) \\ \alpha_2(k) \\ \alpha_3(k) \\ \alpha_4(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \beta(k) \tag{6}$$

The $\mathbf{A}_e^{\mathrm{T}}$ matrix eigenvalues are determined solving the following equation:

$$\det\left\{ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & a_{11} & -a_{21} & -a_{31} \\ 0 & -a_{12} & a_{22} & a_{32} \\ 0 & -a_{13} & a_{23} & a_{33} \end{bmatrix} - \begin{bmatrix} \chi & 0 & 0 & 0 \\ 0 & \chi & 0 & 0 \\ 0 & 0 & \chi & 0 \\ 0 & 0 & 0 & \chi \end{bmatrix} \right\} = 0 \tag{7}$$

For the eigenvalues the following is obtained:

$$\chi_1 = 1; \quad \chi_1 = 0.9962; \quad \chi_3 = 0.9407; \quad \chi_4 = 0.8187.$$

In this case an undesired root of the open-loop system $\chi_1 = 1$ exists, which must be displaced. A location for the closed-loop system root $\mu_1 = 0.5$ is defined, where χ_1 should be placed.

In order to define the observer $\mathbf{H}$ matrix, it is necessary to find the $\mathbf{q}_1$ eigenvector elements, solving the system of homogeneous algebraic equations:

$$(\mathbf{A_e} - \mathbf{I}\chi_i)\mathbf{q}_i = 0 \text{ for } i = 1 \tag{8}$$

For the elements of both eigenvector $\mathbf{q}_1$ and weight matrix $\mathbf{Q}_1$ the following is obtained:

$$\mathbf{q}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \ \mathbf{Q}_1 = \mathbf{q}_1\mathbf{q}_1^{\mathrm{T}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

These products are computed:

$$\mathbf{b}_e^{\mathrm{T}}\mathbf{q}_1\mathbf{q}_1^{\mathrm{T}} = [1 \quad 0 \quad 0 \quad 0] \text{ and } \mathbf{b}_e^{\mathrm{T}}\mathbf{q}_1\mathbf{q}_1^{\mathrm{T}}\mathbf{b}_e = 1.$$

Weight coefficient $r_1 = 2$ and the $\lambda_1 = 2$ coefficient are calculated.

The respective optimal modal feedback gain is determined:

$$\boldsymbol{\gamma}_1^{\mathrm{T}} = \begin{bmatrix} -0.5 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

As the undesired eigenvalue is only one in this case, the feedback gain is derived as follows:

$$\boldsymbol{\gamma}^* = \boldsymbol{\gamma}_1 = [-0.5 \quad 0 \quad 0 \quad 0]$$

The observer feedback vector is formulated:

$$\mathbf{H}=\begin{bmatrix}h_1\\h_2\\h_3\\h_4\end{bmatrix}=\begin{bmatrix}-0.5\\0\\0\\0\end{bmatrix}.$$

The observer equation is as follows [3]:

$$\hat{\mathbf{x}}_e(k+1)=\mathbf{A}_e\hat{\mathbf{x}}_e(k)+\mathbf{b}_e\mathbf{u}_e(k)+\mathbf{H}\Delta\mathbf{e}(k)=$$
$$=\mathbf{A}_e\hat{\mathbf{x}}_e(k)+\mathbf{b}_e\mathbf{u}_e(k)+\mathbf{H}\left[y(k)-\mathbf{C}\hat{\mathbf{x}}(k)\right]$$

or

$$\begin{bmatrix}\hat{x}_{1e}(k+1)\\\hat{x}_{2e}(k+1)\\\hat{x}_{3e}(k+1)\\\hat{x}_{4e}(k+1)\end{bmatrix}=\begin{bmatrix}1&1&0&0&0\\0&a_{11}&a_{12}&a_{13}&a_{14}\\0&a_{21}&a_{22}&a_{23}&a_{24}\\0&a_{31}&a_{32}&a_{33}&a_{34}\end{bmatrix}\cdot\begin{bmatrix}\hat{x}_{1e}(k)\\\hat{x}_{2e}(k)\\\hat{x}_{3e}(k)\\\hat{x}_{4e}(k)\end{bmatrix}+$$
$$+\begin{bmatrix}0\\-b_1\\b_2\\b_3\end{bmatrix}u_e(k)+\begin{bmatrix}h_1\\h_2\\h_3\\h_4\end{bmatrix}\Delta e(k) \tag{9}$$

where $\Delta e(k)=\mathbf{y}(k)-\mathbf{C}\hat{\mathbf{x}}(k)$.

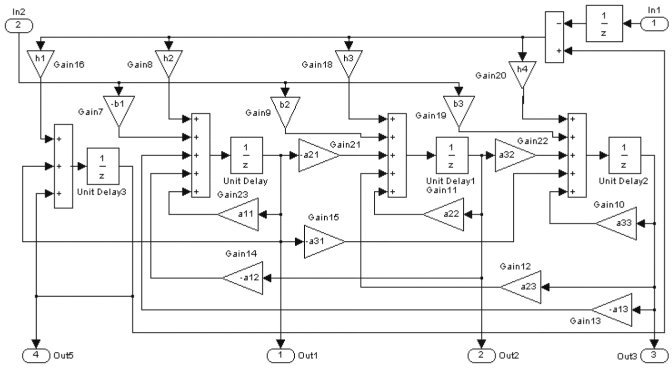

Fig. 2. Model of the optimal modal observer (Subsystem 2).

These equations produce the state variables valuation. Based on them the optimal modal observer (Subsystem 2) has been developed, shown in Fig.2.

IV. Synthesis of Optimal Modal Controller

Synthesis of the optimal modal controller will be realized by an algorithm shown in [2]. In this case synthesis is carried out based on equation (3).

The $\mathbf{A}_e$ matrix eigenvalues are as follows:

$$\chi_1=0.8187;\quad \chi_1=0.9962;\quad \chi_3=0.9407;\quad \chi_4=1.$$

Among these values an undesired root $\chi_4=1$ exists, which should be displaced.

A root location $\mu_4=0.3333$ of the close-loop system characteristic equation is defined, where the undesired open-loop system eigenvalue χ_1 will be placed.

In order to determine the optimal modal controller matrix $\mathbf{K}$ it is necessary to find the eigenvector $\mathbf{q}_4$ elements, solving the system of homogeneous algebraic equations.

$$\left(\mathbf{A}_e^{\mathrm{T}}-\mathbf{I}\chi_i\right)\mathbf{q}_i=0\ \text{ for } i=4 \tag{10}$$

The elements of eigenvector $\mathbf{q}_4$ and weight matrix $\mathbf{Q}_4$ are obtained as follows:

$$\mathbf{q}_4=\begin{bmatrix}0.0036\\0.9983\\-0.0556\\-0.0181\end{bmatrix},$$

$$\mathbf{Q}_4=\begin{bmatrix}0.0000&0.0036&-0.0002&-0.0001\\0.0036&0.9966&-0.0555&-0.0180\\-0.0002&-0.0555&0.0031&0.0010\\-0.0001&-0.0180&0.0010&0.0003\end{bmatrix}$$

Products are calculated:

$$\mathbf{b}_e^{\mathrm{T}}\mathbf{Q}_4=\left[0.0000\ \ \text{-}0.0036\ \ 0.0002\ \ 0.0001\right]$$

and

$$\mathbf{b}_e^{\mathrm{T}}\mathbf{Q}_4\mathbf{b}_e=1.3043\,\mathrm{x}\,10^{-5}.$$

For these coefficients the following values are obtained: $r_4=9.7824\,\mathrm{x}\,10^{-6}$ and λ_4=1.5.

The optimal modal feedback gain is determined:

$$\boldsymbol{\gamma}_1^{\mathrm{T}}=\begin{bmatrix}0.6667\\184.2761\\-10.2563\\-3.3333\end{bmatrix}.$$

As there is only one undesired eigenvalue ($\chi_4=1$), the optimal modal feedback gain is derived as follows:

$$\boldsymbol{\gamma}^*=\boldsymbol{\gamma}_1=\left[0.6667\quad 184.2761\quad -10.2563\quad -3.3333\right].$$

The feedback vector obtains this form:

$$\mathbf{K}=\begin{bmatrix}k_1\\k_2\\k_3\\k_4\end{bmatrix}=\begin{bmatrix}0.6667\\184.2764\\-10.2563\\-3.3333\end{bmatrix}$$

and control of the following type is formulated:

$$u_e^*(k)=\mathbf{K}\mathbf{x}_e^*(k)=k_1x_{1e}+k_2x_{2e}+k_3x_{3e}+k_4x_{4e} \tag{11}$$

After substitution of $u_e^*(k)$ in Eq. (4), for the optimal modal controller this expression is obtained:

$$u^*(k) = u^*(k-1) + k_1 x_{1e} + k_2 x_{2e} + k_3 x_{3e} + k_4 x_{4e} \qquad (12)$$

Based on Eq. (12) the controller model is constructed (Subsystem 3), shown in Fig. 3.

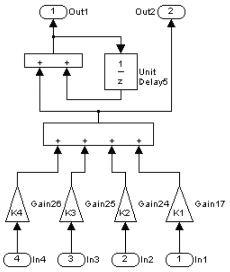

Fig. 3. Model of the optimal modal controller (Subsystem 3).

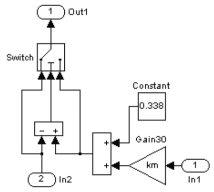

Fig. 4. Model of the current limitation by overtaking (Subsystem 4).

Overtaking current limitation has been applied. The respective function is as follows:

$$u_{cl}{}^*(k) = u_n^* + k_m \omega^*(k)\,, \qquad (13)$$

where: u_n^* is the current limitation initial code; k_m – scale coefficient.

Hence, the control condition in the presence of current limitation will be:

$$u_c^*(k) = \begin{cases} u^*(k) & \text{at} \quad u^*(k) \le u_{cl}^*(k) \\ u_{cl}{}^*(k) & \text{at} \quad u^*(k) > u_{cl}^*(k) \end{cases} \qquad (14)$$

The control code which should be supplied as input to the power converter is determined by condition (14). In accordance with it the current limitation model is composed (Subsystem 4). This model is represented in Fig. 4.

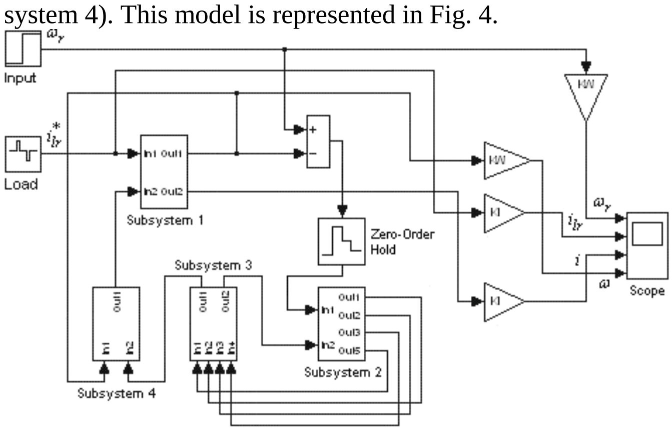

Fig. 5. Model of the system under consideration.

Practically, the optimal modal control is achieved through consequent realization of Eqs. (11), (12), (13) and (14).

V. Some Simulation Results

To prove the offered control algorithm functionality a computer simulation model has been developed, using the MATLAB/SIMULINK software package (Fig. 5).

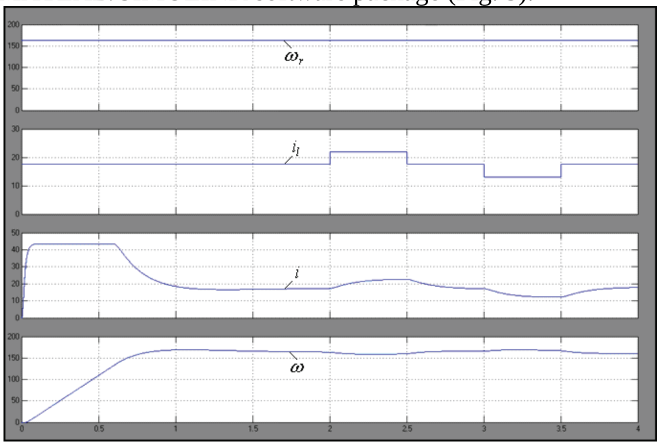

Fig. 6. Time-diagrams illustrating the drive system performance.

Fig. 6 shows some simulation results illustrating the performance of the drive system under consideration. The applied quantization period is $T_0 = 1\,\mathrm{ms}$. The reference motor speed is $\omega_r = 163\,\mathrm{rad/s}$. The reference static current is equal to the rated value $I_{\mathrm{rat}} = 17.6\,\mathrm{A}$, while the disturbances applied sequentially are $\Delta i_l = +25\%$ and $\Delta i_l = -25\%$. The starting current is limited to the maximum admissible value $I_{\max} = 44\,\mathrm{A}$, which provides a maximum starting torque.

VI. Conclusion

An approach to synthesis of optimal modal controllers for a class of DC motor drives is discussed in this paper.

The synthesis implements a combination between both - poles setting of the closed-loop system (modal control) and quadratic quality criterion minimization (optimal control).

Research carried out through computer simulation shows, that such type of control can provide good performance.

The results obtained can be used in optimization and tuning of such types of electric drive systems.

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