# Self-excitation of an Induction Generator – Mathematical Model and Experimental Verification

Milan M. Radic[1], Zoran P.Stajic[2]

*Abstract* – **In this paper phenomenon of self-excitation in a three phase induction generator under no-load conditions has been analyzed. Equations describing induction machine and excitation capacitors are represented in rotor reference frame. Residual magnetic flux and exact representation of magnetizing inductance variation versus magnetizing current are important parts of the model. Comparison of computer simulations with experimental results has shown that accuracy of model is good, even with cross-saturation and core losses neglected.**

*Keywords* – **Induction generator, Self-excitation, Residual flux.**

## I. Introduction

The fact that an externally driven induction machine with capacitor bank properly connected across its stator terminals can sustain self-excitation is well known [1]. In recent years, when demands for „clean“ electric energy are in constant growth, interest in self-excited induction generators has been highly increased. They have been used as the most suitable isolated power sources in small hydroelectric and wind energy applications [2]. Simple construction with squirrel-cage rotor, absence of DC supply for excitation and low maintenance costs are the main advantages of induction generators over the synchronous generators, especially for small rated powers [5]. The main problem that occurs in exploitation of self-excited isolated induction generator is stabilization of stator voltage and frequency, because their values vary with change of load [4].

In order to perform detailed analysis of self-excited induction generator behavior, the first step should be to study the process of self-excitation under no-load conditions. This phenomenon is basically caused by existence of residual flux in magnetic core of the induction machine. If rotor of the machine is driven by some prime mover, certain voltage can be measured at the stator terminals, even without any capacitor bank connected. In this case voltage would be very low, about 1-2 % of rated voltage, but if capacitor bank is connected and capacity is large enough, stator voltage will rapidly increase due to fact that capacitive current causes additional magnetization of the machine core. In steady state stator voltage could even exceed the rated value.

[1]Milan M. Radic is with the Faculty of Electronic Engineering, Aleksandra Medvedeva 14, 18000 Nis, Serbia, E-mail: radic@elfak.ni.ac.yu

[2]Zoran P. Stajic is with the Faculty of Electronic Engineering, Aleksandra Medvedeva 14, 18000 Nis, Serbia, E-mail: zoran.stajic@elfak.ni.ac.yu

## II. Mathematical Model

Mathematical model of self-excitation process in three phase induction machine is based upon the well-known theory of arbitrary reference frame [6], where real machine windings are substituted with fictive windings placed along two orthogonal axes, *q* and *d*, rotating at arbitrary speed $\omega$. In this case, it is most convenient to attach reference frame to rotor, choosing that $\omega = \omega_r$, because it makes it easier to include residual magnetic flux into the model.

Equivalent circuits for *q* and *d* axis in rotor reference frame, including terminal capacitor for self-excitation are shown in Figs. 1a and 1b.

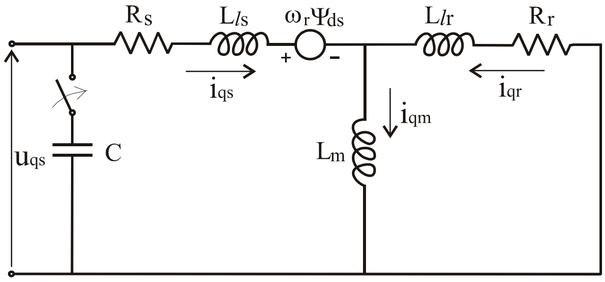


Fig. 1a. Equivalent circuit for *q* axis

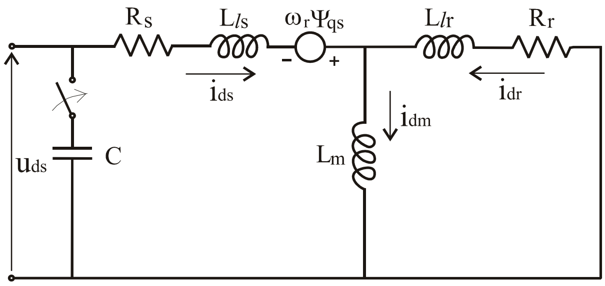


Fig. 1b. Equivalent circuit for *d* axis

Voltage equations describing induction machine in matrix form are:

$$\begin{bmatrix} u_{qs} \\ u_{ds} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} R_s + L_s p & \omega_r L_s & L_m p & \omega_r L_m \\ -\omega_r L_s & R_s + L_s p & -\omega_r L_m & L_m p \\ L_m p & 0 & R_r + L_r p & 0 \\ 0 & L_m p & 0 & R_r + L_r p \end{bmatrix} \cdot \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{qr} \\ i_{dr} \end{bmatrix} + \omega_r \begin{bmatrix} K_d \\ -K_q \\ 0 \\ 0 \end{bmatrix} \tag{1}$$

where $L_s = L_{ls} + L_m$ and $L_r = L_{lr} + L_m$.

$R_s$ , $R_r$ - stator and rotor resistances
$L_{ls}$ , $L_{lr}$ - stator and rotor leakage inductances
$L_m$ - magnetizing inductance
$\omega_r$ - rotor angular velocity
$p = \frac{d}{dt}$ - operator of differentiation

It should be noted that magnetizing inductance changes during the self-excitation process due to fact that machine core becomes more saturated as magnetizing current and terminal voltage rise. In order to get more accurate results during transients and in steady state, magnetizing inductance have to be represented as a function of magnetizing current,

$$L_m = f(I_m)$$

where magnetizing current can be obtained as:

$$I_m = \frac{\sqrt{(i_{qs} + i_{qr})^2 + (i_{ds} + i_{dr})^2}}{\sqrt{2}} \tag{2}$$

Constants $K_q$ and $K_d$ are involved in voltage equations in order to include residual magnetic flux along *q* and *d* axis, which is necessary to start voltage build-up process.

In rotor reference frame, capacitor can be described with next equations:

$$\begin{bmatrix} i_{qs} \\ i_{ds} \end{bmatrix} = \begin{bmatrix} -Cp & -\omega_r C \\ \omega_r C & -Cp \end{bmatrix} \cdot \begin{bmatrix} u_{qs} \\ u_{ds} \end{bmatrix} \tag{3}$$

After some mathematical transformations, previous equations can be written in a form convenient for simulation on a computer, using any of programs or program languages that allow numerical solving of differential equations. For this occasion, mathematical model was implemented on a computer using Matlab/Simulink.

In this model core losses and impact of higher harmonics are neglected. Cross-saturation is also neglected which means that there is no magnetic coupling between *q* and *d* axis. As an self-excited induction generator generally operates in highly saturated area, neglecting of cross-saturation may have influence to the accuracy of model, especially in transient regimes [3]. In future work model can be improved by including of phenomena that are neglected at this level.

## III. Identification of Parameters

In order to perform comparison of simulation results with experimental results, it was necessary to identify parameters of the real 1.5 kW three phase squirrel-cage rotor induction machine used in experiment.

Resistances $R_s$ and $R_r$ , and leakage inductances $L_{ls}$ and $L_{lr}$ are considered as constants, and can be easily obtained from standard short-circuit test and DC current test.

As it has been mentioned in previous section, magnetizing inductance must not be considered as constant, because that would lead to very poor simulation results. In order to identify function describing variation of magnetizing inductance $L_m$ versus magnetizing current $I_m$ , several ideal no-load tests with different voltages applied to stator, and under synchronous speed conditions, have been performed. Knowing previously determined parameters $R_s$ and $L_{ls}$ , each value of ideal no-load current can be connected to the exact value of magnetizing inductance. Experimentally determined points are shown in Fig. 2, together with functional approximation which has been used in model. As it can be seen in Fig. 2, for $\mathrm{I_m} < 0.7\,\mathrm{A}$ , magnetizing inductance is considered to be constant (linear section of magnetization characteristic). In reality, this assumption is not completely true because in the area of very low magnetizing currents (close to zero) magnetization characteristic is not linear [7], but that will not affect the accuracy of simulation too much.

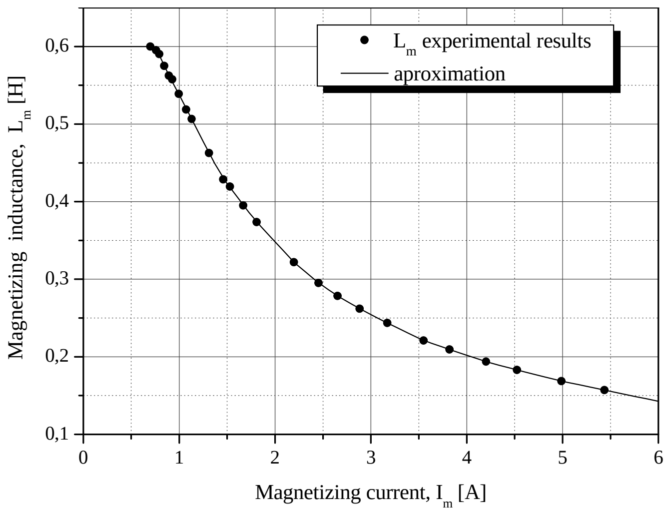


Fig. 2. Variation of magnetizing inductance versus current

Finally, parameters $K_q$ and $K_d$ , which describe residual magnetic flux, can be obtained through comparing of voltage waveform recorded at stator terminals (with no capacitors connected and with rotor driven by a prime mover at constant speed) and waveform which is result of simulation under the same conditions. It is not necessary to determine both of $K_q$ and $K_d$ , because one of them can be set to zero, assuming that residual flux exists only along one of axes (in this case it has been assumed that $K_d = 0$ ).

After stator of induction machine had been connected to rated voltage and suddenly disconnected, rotor has been driven at $n_0 = 3000\,\mathrm{rpm}$ by DC machine as prime mover. Recorded phase voltage is shown in Fig. 3 (solid line). The same figure also shows simulation results for three different values of parameter $K_q$ , and it is obvious that if $K_q$ is properly chosen, simulated waveform is very similar to the recorded one.

Values of all parameters that are necessary for simulation of self-excitation are given in Table I.

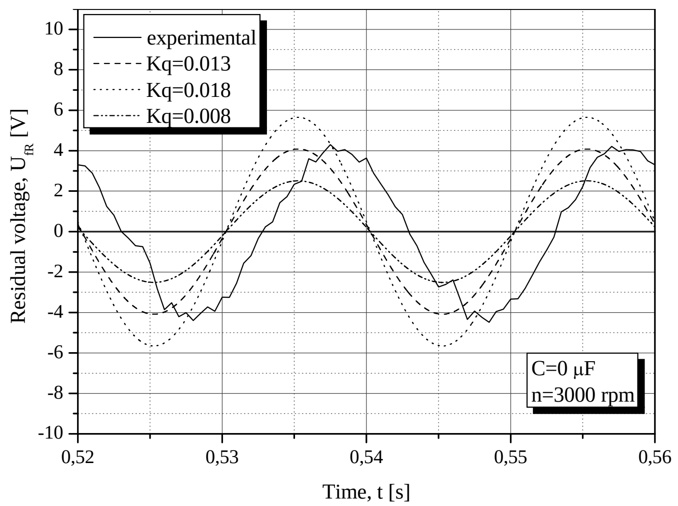


Fig. 3. Identification of parameter *Kq*

TABLE I
MACHINE PARAMETERS

| | |
|---|---|
| $R_s$ | 3.7 Ω |
| $R_r$ | 3.1 Ω |
| $L_{ls}$ | 0.0115 H |
| $L_{lr}$ | 0.0115 H |
| $K_q$ | 0.013 |

## IV. EXPERIMENTAL VERIFICATION

Testing of mathematical model was done through several experiments performed with different capacitor banks and at different rotor speeds. Rotor of induction machine was driven by an unregulated DC motor and waveforms of induction machine stator voltage and current were recorded. During process of self-excitation rotor speed doesn't remain constant, as shown in Fig. 4, because load torque at the shaft of DC motor increases due to increasing of induction machine copper and core losses.

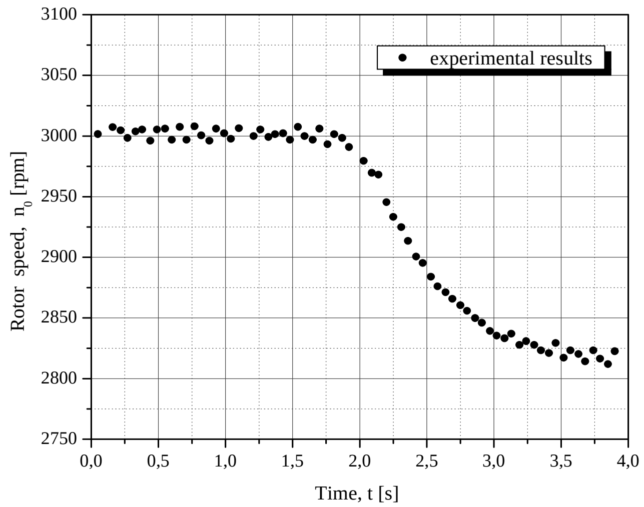


Fig. 4. Variation of rotor speed during self-excitation

Equations describing mechanics of the process, and also equations describing DC motor have not been included into model, so rotor speed had to be recorded and later has been treated as known variable during the simulation. If decreasing of rotor speed is not taken into account, simulated waveforms would differ from experimentally obtained data, and small peaks of voltage and current that can be noticed at $t \approx 2\,\mathrm{s}$ in Figs. 5 to 8, would not appear in simulation result.

In single experiment whose results are presented in this paper, rotor speed in unexcited state was set to $n_0 = 3000\,\mathrm{rpm}$, and at the moment $t = 1.06\,\mathrm{s}$ symmetrical three phase capacitor bank with capacity $C = 35\,\mu\mathrm{F}$ was connected to the stator terminals. Fig. 5 shows experimentally recorded voltage and Fig. 6. shows waveform obtained from simulation. Both waveforms indicate that self-excitation transient process lasts for about 2 seconds in this case, and after that time voltage reaches its constant steady state value (in term of rms). It is obvious that in steady state simulated voltage equals recorded voltage extremely good. Also, envelopes of simulated and recorded voltage waveforms are very similar during voltage build-up process, although cross-saturation and core loses have been neglected in model.

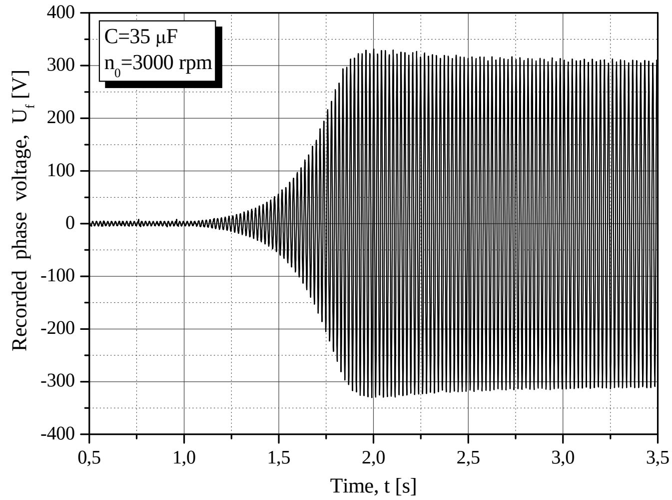


Fig. 5. Experimental stator voltage

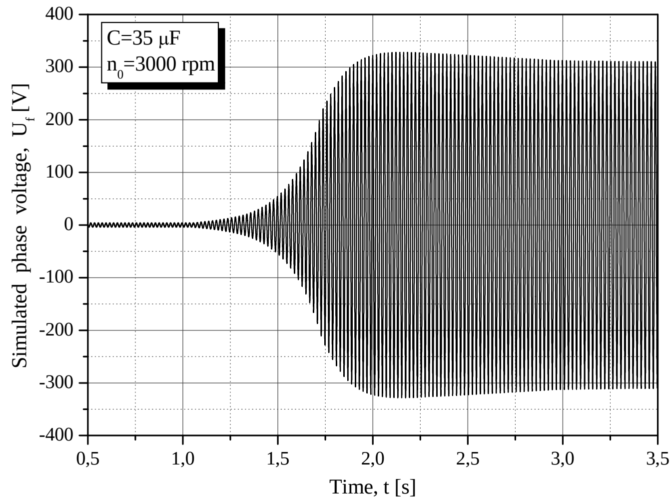


Fig. 6. Stator voltage - simulation result

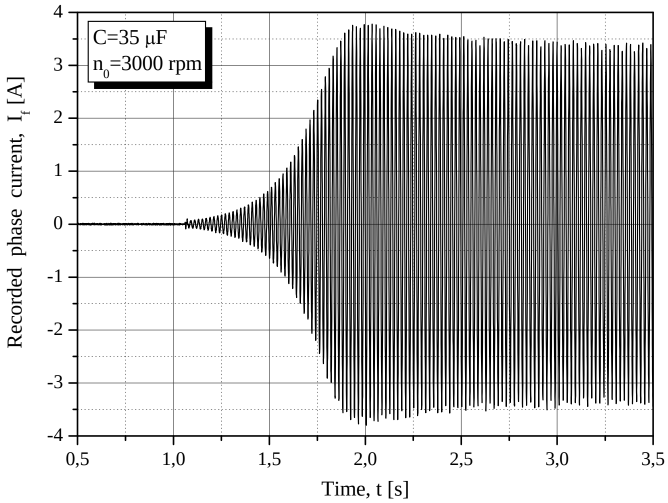


Fig. 7. Experimental stator current

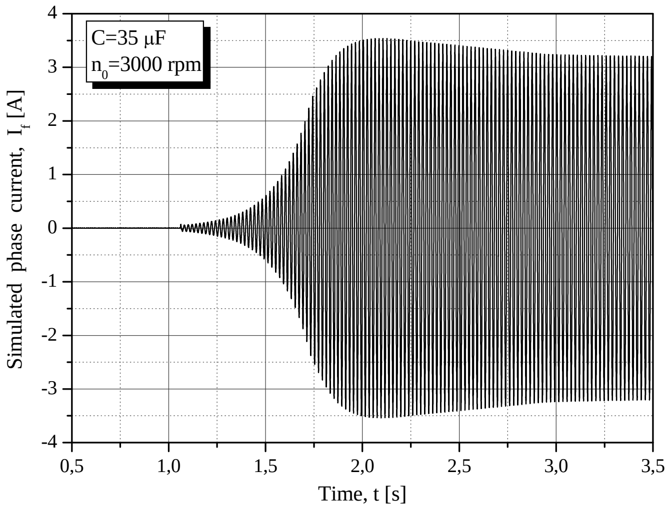


Fig. 8. Stator current – simulation result

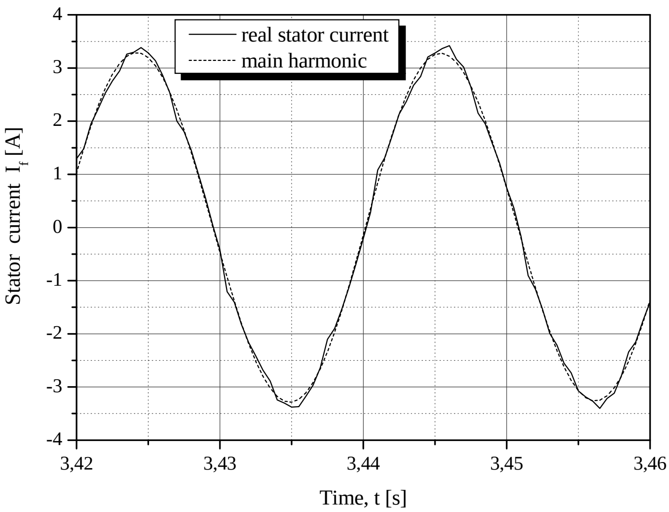


Fig. 9. Stator current in steady state – detail

Comparing waveforms in Figs. 7 and 8, it can be noticed that envelope of the recorded stator current has similar shape as envelope of current predicted by model, but matching is not so good as in case of voltage waveforms. In the first part of self-excitation process, for $t < 1.8\mathrm{s}$ , difference is not too big, but later, recorded current has some greater value then it is expected from simulation result. This could be explained by the fact that model neglects presence of higher harmonics, which is only idealization of real situation. In order to prove previous statement, detail of recorded stator current waveform after reaching steady-state is shown in Fig. 9. The same figure also shows result of filtering of higher harmonics in the recorded signal, and it is obvious that if only main harmonic of stator current is taken into consideration, matching between experimental and simulation results becomes much better.

## V. Conclusion

Proposed mathematical model gives highly accurate simulation results, and can be used in studies of an induction generator self-excitation under arbitrary circumstances. It has been shown that good representation of magnetizing inductance versus magnetizing current is of great importance, and also that it is necessary to include residual magnetic flux into the model. In future research, equations describing electrical load at stators terminals can be added to the model in order to study transients that may occur in real exploitation. Existing model can be improved by including equations which describe mechanical part of the system and also a prime mover used for driving of induction generator (wind turbine or hydro turbine)

## References


[1] E. D. Basset, F. M. Potter, „Capacitive excitation of induction generators“, *AIEE Trans.*, Vol. 54, pp. 540-545, 1935.

[2] G. Raina, O. P. Malik, „Wind energy conversion using a self excited induction generator“, *IEEE Trans. Power Apparatus and Systems,* Vol. 102. No. 12, pp. 3933-3936, 1983.

[3] E. Levi, „Impact of cross-saturation on accuracy of saturated induction machine models“, *IEEE Trans. Energy Conversion*, Vol. 12, No 3, pp. 211-216, 1997.

[4] S. M. Alghuwainem, „Steady-state analysis of an isolated self-excited induction generator driven by regulated and unregulated turbine“, *IEEE Trans. Energy Conversion,* vol. 14, No. 3, pp. 718-723, 1999.

[5] J. M. Elder, J. T. Boys, J. L. Woodward, „Self-excited induction machine as a small low-cost generator“, *IEE Proc. C*, Vol. 131, No. 2, pp. 61-68, 1989.

[6] P. C. Krause, *Analysis of Electric Machinery*, New York, McGraw-Hill Book Company, 1986.

[7] I. Boldea, S. Nasar, *The Induction Machine Handbook*, Boca Raton - London - New York - Washington D.C., CRC Press, 2002.