A New Method of Design of Variable Fractional Delay Digital Allpass Filters

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Abstract – **A new method of design and implementation of variable fractional delay digital filters based on Thiran allpass phase delay approximation and using truncated Taylor series expansion of the filter coefficients is proposed in this paper. This method is simple for realization and is providing better tuning capabilities compared to other known methods.**

Keywords – **Digital *a*llpass filter, Variable filters, Fractional phase delay.**

I. Introduction

Fractional delay (FD) filters are very useful in digital signal processing and in telecommunications for time delay estimation, timing adjustment in digital modems, precise jitter elimination, frequency synchronization in wireless telecommunications and speech processing [1]. Recently, the variable FD digital filters are subjects of an ever growing interest [2]-[7].

The most popular variable FD filter with finite impulse response (FIR) is the Farrow structure which allows control of the modelled fractional delay with a single parameter [2]. The main disadvantage of the FIR FD filters is that both the magnitude and the phase responses are varying from the desired response when tuning the fractional delay.

The design of variable FD filters with infinite impulse response (IIR) is very complicated and is based usually on allpass structures, because of their best magnitude properties. The overall delay of an IIR structure satisfying the same phase delay requirements is considerably lower than that of the corresponding FIR filter. Disadvantages of the IIR FD filters are higher round-off noise, possible instability and worst behaviour in a limited wordlength environment.

The most popular design method for allpass based FD digital filter with a maximally flat group delay response is based on Thiran approximation procedure [7], giving a closed-form solution for the transfer function (TF) coefficients. Two methods for designing variable allpass FD filters based on Thiran approximation are known for now. In [3][4] a closed-form method designing and implementing maximally flat allpass variable FD filters has been proposed. It is based on so called gathering structure (derived from the direct form structure) where the filter coefficients are represented as polynomials of the fractional delay parameter. The drawbacks of this method are the complicated structure (with too many multipliers) and the higher sensitivity (as of any direct-form structure). The second method which utilizes the poles of two Thiran FD filters is proposed in [5][6] and is called “root displacement interpolation method”. It uses two N^{th} order allpass Thiran FD filters modelling two different fractional delays D_1 and D_2 to obtain a new N^{th} order allpass FD filter with the delay between D_1 and D_2 The interpolated filters so obtained have a narrower bandwidth with flat phase delay. The method does not allow interpolation when the fractional parts of the delays D_1 and D_2 have different sign. The implementation of this method is also quite complicated and the range of tuning of the phase delay is quite narrow (the authors are talking more often about adjustment than about tuning).

In this paper we propose and investigate a simple tuning procedure of Thiran based FD variable filters utilizing representation of the multiplier coefficients as truncated Taylor series. All theoretical results obtained in this work are verified experimentally.

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II. Design Procedure

Let an N-th order allpass IIR filter has the following transfer function:

$$H_{AP}(z) = \frac{a_N + a_{N-1}z^{-1} + ... + a_1 z^{N-1} + a_0 z^{-N}}{a_0 + a_1 z^{-1} + a_2 z^{-2} + ... + a_N z^{-N}}. \qquad (1)$$

The Thiran approximation method gives a closed-form solution for TF coefficients as a function of the desired fractional delay parameter D (D is a positive real number that can be split into an integer part - corresponding to the TF order N - and a fractional part d as $D = N + d$):

$$a_k = (-1)^k \binom{N}{k} \prod_{n=0}^{N} \frac{d+n}{d+k+n}, \; for \; k = 0, 1, 2 \ldots N. \qquad (2)$$

The most straightforward approach to make d variable is to recalculate a_k (2) for every given d and to reprogram the coefficients in (1) while using a direct form realization. Such an approach is not practical and is difficult to implement in real time because of too many multiplication and division operations. One possible way to eliminate the division operation and generally to simplify the calculation of a_k (2) is to introduce a Taylor series expansion of the coefficients a_k with respect to d and to truncate these expansions after the linear term assuming d<<1. Such an approach will limit the range of values of d over which the tuning will be effective but it will make it possible in real time. To achieve such tuning we propose the following design procedure:

1. Selection of the allpass TF order corresponding to a given requirements (desired fractional delay value D and/or the bandwidth with maximally flat phase delay response).

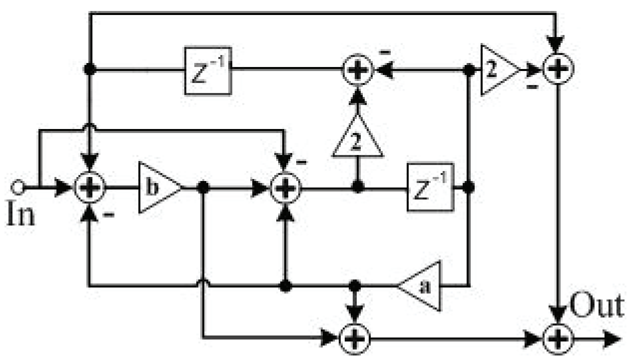
2. Design of an allpass FD filter using Thiran approximation.
3. Taylor series expansion of each TF coefficient and truncation after the linear term.
4. Composite multiplier realisation.

When the selected allpass TF order is lower (first or second) there is a simple relation between the transfer function poles positions (coefficients) and the desired phase delay. In these cases, it is more appropriate to use (or to select) an allpass section whose structure is different from the direct from. As it is known the direct form structure has higher sensitivity to the coefficients values changes. The proper selection of the allpass section used may reduce the sensitivity. For higher order applications the relation between the poles positions (the coefficients of the first- and second-order sections in a cascade realization) and the desired phase delay is very complicated and can not be obtained in closed-form and thus the Taylor approximation can not be applied. Because of that, the cascaded realizations with minimized sensitivity of the individual first and second order allpass sections can not be used. As a result for higher order allpass transfer functions the proposed design procedure can be applied only with direct form structure realizations.

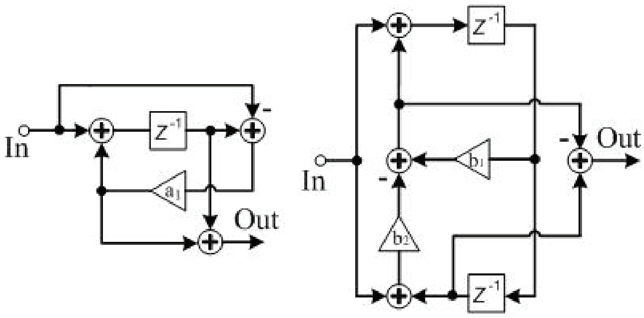
The most common requirement for real applications is for phase delay with small fractional delay parameter values ($N-0.5<D<N+0.5$) which means that the fractional part d will change with in the range [-0.5, 0.5]. It is equivalent to TF poles situated in the area around $z = 0$. In order to obtain a higher fractional delay time accuracy, we have proposed in [8] a new second order allpass section (called IS and shown in Fig. 1a) which has lower sensitivity for poles in that area than other well known second order allpass sections. Its transfer function is [8]

$$H_{IS}(z)=\frac{b+(-a-2b+ab)z^{-1}+z^{-2}}{1+(-a-2b+ab)z^{-1}+bz^{-2}}\ . \qquad (3)$$

There is a great number of first and second order allpass sections in the literature [9][10] and we shall mention in this study only the most popular, those of Mitra and Hirano: MH1 first order allpass section, shown in Fig. 1b, and MH2B second order allpass section, shown in Fig. 1c.



a) IS



b) MH1 c) MH2B

Fig. 1. First- and second-order allpass sections.

Their transfer functions are as follows:

$$H_{MH1}(z)=\frac{-a_1+z^{-1}}{1-a_1z^{-1}}\ ; \qquad (4)$$

$$H_{MH2B}(z)=\frac{b_2-b_1z^{-1}+z^{-2}}{1-b_1z^{-1}+b_2z^{-2}}\ . \qquad (5)$$

After using the Thiran approximation, the transfer function coefficients of the allpass sections from Fig. 1 can be expressed as a function of the fractional part d of the delay parameter value D as shown in Table I.

TABLE I
MH1, IS AND MH2B FRACTIONAL DELAY FILTER COEFFICIENTS

MH1	IS		MH2B	
a_1	a	b	b_1	b_2
$\frac{d}{d+2}$	$\frac{d}{d+2}$	$\frac{d(d+1)}{(d+3)(d+4)}$	$\frac{2d}{(d+3)}$	$\frac{d(d+1)}{(d+3)(d+4)}$

It is seen from Table I that all these coefficients are depending on d in quite a complicated way not permitting real time recalculation and tuning. It appears thus that the truncated Taylor series expansions have to be used. The corresponding representations of the coefficients (after first order approximation) are given in Table II.

TABLE II
MH1, IS AND MH2B VARIABLE FRACTIONAL DELAY FILTER COEFFICIENTS

MH1	IS		MH2B	
a_1	a	b	b_1	b_2
$\frac{1}{2}d$	$\frac{1}{2}d$	$\frac{1}{12}d$	$\frac{2}{3}d$	$\frac{1}{12}d$

All the coefficients in Table II are surprisingly simple and they can easily be realized as composite multiplier coefficients containing one fixed and one variable part as illustrated in Fig. 2 for a_1 and a (Table II).

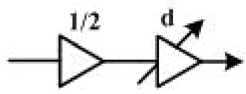


Fig. 2. Composite variable multiplier realization of a and a_1 after a first-order approximation (Table II).

The similar results for the transfer function coefficients can be obtained in the case of higher allpass transfer function order. For example, the transfer function coefficients (1) of fifth order allpass FD filter obtained after the proposed method are given in Table III.

TABLE III
FIFTH ORDER VARIABLE FRACTIONAL DELAY FILTER COEFFICIENTS

a_1	a_2	a_3	a_4	a_5
$-\frac{5}{6}d$	$\frac{5}{21}d$	$-\frac{5}{84}d$	$\frac{5}{504}d$	$-\frac{1}{1260}d$

III. VERIFICATION OF THE DESIGN PROCEDURE

To verify the proposed method we have designed one first-order (MH1-based) and two second-order (IS-based and MH2B-based) allpass FD filters. Their transfer functions coefficients are given in Table II. The results for the tuned phase delay responses obtained after a first-order Taylor approximation are shown in Figs. 4a, 5a and 6a with dashed lines. The solid lines present the above mentioned FD filters designed for $D = N + d$. As it can be seen the proposed method works properly if the tuning parameter d is very small, approximately in the range [-0.05, 0.05]. Larger values of d are causing considerable deviation of the phase delay curve and are narrowing its maximally flat part. It means that the proposed method can be used only to adjust the phase delay within a small range of values of d. But, when there is a need to tune the phase delay in a wider range, the first-order approximation will be not enough. To solve this problem, we propose to use a second-order Taylor approximation for transfer function coefficients representation. The new transfer functions coefficients are given in Table IV. All these new coefficients still could be calculated and tuned in real-time, they have a homogeneous structure and can be realized as composite multipliers containing two fixed and two variable multipliers as illustrated in Fig. 3 for coefficients a and a_1 (Table IV).

TABLE IV
MH1, IS AND MH2B VARIABLE FRACTIONAL DELAY FILTER COEFFICIENTS AFTER SECOND-ORDER APPROXIMATION

MH1	**IS**		**MH2B**	
a_1	a	b	b_1	b_2
$\frac{d}{2}(1-\frac{d}{2})$	$\frac{d}{2}(1-\frac{d}{2})$	$\frac{d}{12}(1+\frac{5d}{12})$	$\frac{2d}{3}(1-\frac{2d}{3})$	$\frac{d}{12}(1+\frac{5d}{12})$

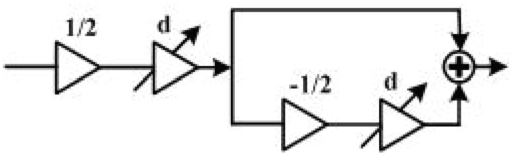
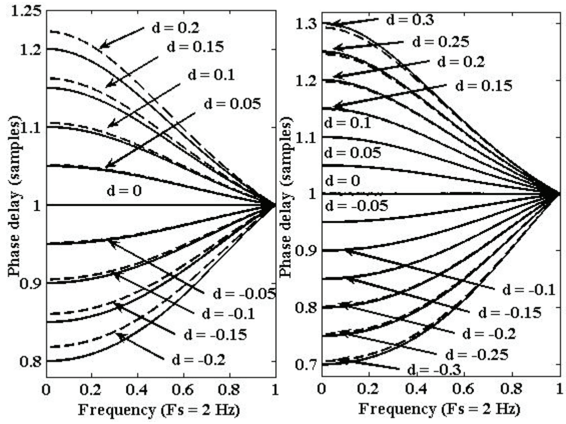


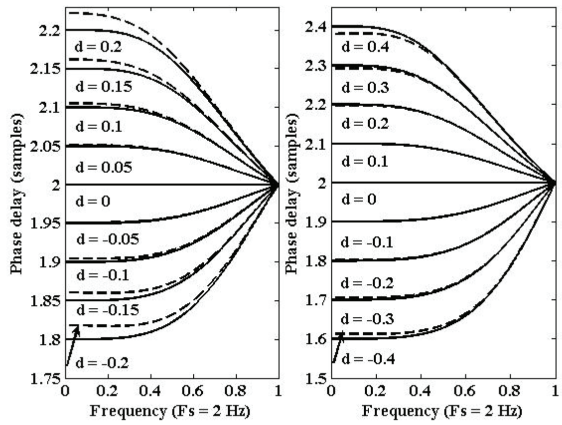
Fig. 3. Composite variable multiplier realization of a and a_1 (Table IV) after a second-order approximation.



a) 1st-order approximation b) 2nd-order approximation

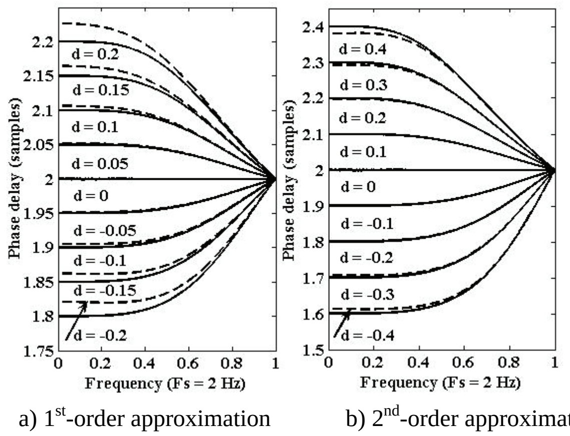
Fig. 4. Tuning of first-order MH1 allpass FD section

The results for the tuned phase delay responses of the above mentioned FD allpass filters obtained after a second-order Taylor approximation are given in Figs. 4b, 5b and 6b with dashed lines. There is a considerable improvement in both, the range of values of d over which the tuning is possible and accurate, and in retaining the range of frequencies with flat phase-delay response. The price of such improvement (one fixed and one variable additional multipliers per TF coefficient) is readily acceptable in many practical cases.



a) 1st-order approximation b) 2nd-order approximation

Fig. 5. Tuning of second-order IS allpass FD section

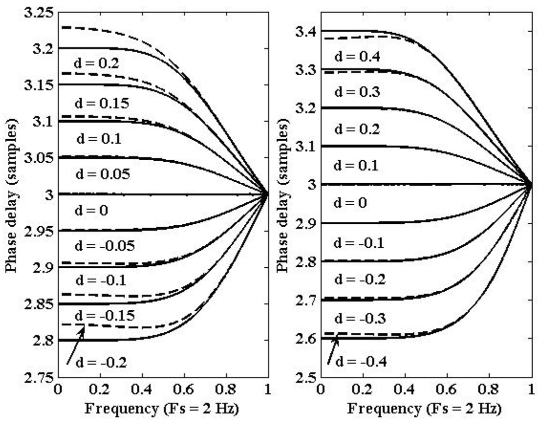


a) 1st-order approximation b) 2nd-order approximation

Fig. 6. Tuning of second-order MH2B allpass FD section

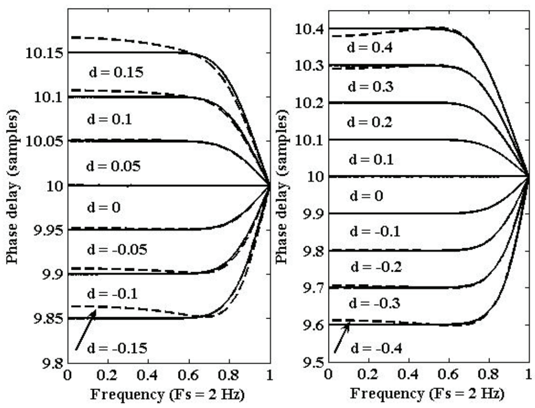
IV. Experiments

In order to investigate the applicability of the proposed method in higher order variable allpass FD filter realizations we have designed one 3[th] and one 10[th] order allpass FD filters. The third order allpass FD filter is designed first for FD parameter values *D* in the range [2.6, 3.4] using Thiran approximation (the solid lines in Fig. 7a and b) and then the filter is turned to variable by using the method here proposed with first- and second-order Taylor approximation of the multiplier coefficients. The results for the phase delay responses are given in Fig. 7. The same procedure is applied for tenth order allpass FD filter which is designed for *D* in the range [9.6, 10.4] and the results for the phase delay responses are given in Fig. 8. As it can be seen, the range of the values of the tuning parameter *d* guaranteeing the maximally flat behavior of the phase delay with first-order Taylor approximations is approximately [-0.05, 0.05] and with second-order approximation is approximately [-0.3, 0.3]. The same results for the tuning parameter *d* are obtained for any other allpass FD transfer function order. The multipliers realizations for first-order and second-order Taylor approximation are similar to those given in Figs. 2 and 3, but with different values of the fixed multipliers.



a) 1[st]-order approximation b) 2[nd]-order approximation

Fig. 7. Tuning of third-order allpass FD filter



a) 1[st]-order approximation b) 2[nd]-order approximation

Fig. 8. Tuning of tenth-order allpass FD filter

The results from Fig 8b are comparing very favorably with similar results (10[th] order transfer function) given in [6] where the range of values of *d* is only [0.1, 0.3] and the maximally flat part of the phase delay response is narrower.

V. Conclusion

A new method of design and implementation of variable FD allpass digital filters was proposed in this paper. It is based on Thiran maximally flat approximation of given phase delay response and makes use of truncated Taylor series expansion of the filter coefficients. It was found that truncation of the series after the linear term is applicable only for phase delay adjustments in a quite limited range of values. The second-order Taylor approximation of the coefficients is providing possibilities of tuning in much wider range of values of *d* exceeding the one achieved in other known public-cations. The implementation of the method is simple and permits real time tuning of the phase delay time. The circuitry is less complicated compared to the other known methods.

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