Non-uniform Threshold as an Alternative to Uniform Threshold in Denoising in Wavelet Domain

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Abstract – **In this paper we present the advantage of non-uniform over uniform threshold wavelet shrinkage denoising method, applied on noisy signals with signal dependent noise. We illustrate our results by comparing the noise energy after using the both filtration methods on the same set of artificially noise contaminated images. The experiments are made with NPR-QMF filter banks instead with the filter banks that are commonly used in wavelet applications.**

Keywords – **Denoising, filter bank, signal-dependent noise, threshold, wavelet domain filtering.**

I. Introduction

Lately, there are many developed methods for image noise filtration in a transformation domain [1-8]. In the last decade the stress on researches in this field is put on the signal processing in the wavelet domain.

The reason of using the wavelet transform for denoising purposes is that adequately chosen wavelet basis groups the coefficients in two groups – one with a few coefficients with high SNR, and other with a lot of coefficients with low SNR. In case of white Gaussian noise, the noise level is same through whole signal and for all the wavelet coefficients, independently on the signal. So, choosing a global threshold shrinks all the coefficients for an equal portion. But, in some signals, like nuclear medicine (NM) images, the noise level is proportional to the local signal intensity. Obviously, denoising them with a global threshold is not the best solution.

In this paper we present results obtained by using our non-uniform threshold shrinkage method for removal of signal-dependent noise. We illustrate that noise energy in the filtered signal is bigger when any global threshold is used compared to the case when the proposed non-uniform threshold is used. We disclose some results of denoising of standard test images when our method and known methods are used. The paper is organized as follows. The method uses standard wavelet filtering outlined in Section II. In Section III we discuss how to estimate the varying threshold. In Section IV we verify the validity of our approach on deterministic signals contaminated with signal dependent noise. At the end, Section V concludes the paper.

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II. Wavelet Shrinkage Method

The most popular form of conventional wavelet-based signal filtering [9], can be expressed by:

$$\begin{aligned}&\{\mathbf{A}_k, \mathbf{D}_1, \mathbf{D}_2, \Lambda\ , \mathbf{D}_k\} = \mathrm{DWT}(\mathbf{s}+\mathbf{n}),\\ &\mathbf{s}^* = \mathrm{IDWT}(\mathbf{A}_k, \mathbf{h}_1\mathbf{D}_1, \mathbf{h}_2\mathbf{D}_2, \Lambda\ , \mathbf{h}_k\mathbf{D}_k)\end{aligned} \tag{1}$$

where $\mathbf{s}$ is noise-free signal, $\mathbf{n}$ is noise, $\mathbf{s}^*$ is filtered signal, $\mathbf{A}_k$ and $\mathbf{D}_i$, i = 1, 2, …, k are approximation and detail coefficients at levels, i = 1, 2, …, k, respectively; and

$$\mathbf{h_i}=[h_{1i}, h_{2i}, \ldots, h_{ji}]^T,\ i=1,2,\ \ldots,\ k,$$

are weighting vectors of the corresponding detail coefficients.

In case of conventional hard threshold filtering h_{ji} coefficients are determined by

$$h_{jk}^{(\mathrm{hard})} = \begin{cases}1, & \text{if } |D_{jk}| \geq \tau_k\\ 0, & \text{if } |D_{jk}| < \tau_k\end{cases}, \tag{2}$$

while for the soft threshold filtering they are

$$h_{jk}^{(\mathrm{soft})} = \begin{cases}1-\dfrac{\tau_k\,\mathrm{sgn}(D_{jk})}{D_{jk}}, & \text{if } |D_{jk}| \geq \tau_k\\ 0, & \text{if } |D_{jk}| < \tau_k\end{cases}, \tag{3}$$

where τ_k is user specified threshold for level k-details.

Having in mind that the noise is proportional to the local signal intensity, instead of using a global threshold τ_k, Eq. (3), we propose:

$$h_{jk}^{(\mathrm{soft})} = \begin{cases}1-\dfrac{\tau_{jk}\,\mathrm{sgn}(D_{jk})}{D_{jk}}, & \text{if } |D_{jk}| \geq \tau_{jk}\\ 0, & \text{if } |D_{jk}| < \tau_{jk}\end{cases}, \tag{4}$$

where τ_{jk} is user specified threshold.

III. Non-uniform Threshold Determination

The approximation coefficients contain the signal identity and have the same size as the detail coefficients. So, if we assume that the noise is proportional to the local signal intensity then a non-uniform threshold vector $\boldsymbol{\tau}$ could be expressed as

$$\boldsymbol{\tau} = \alpha|\mathbf{A}|, \tag{5}$$

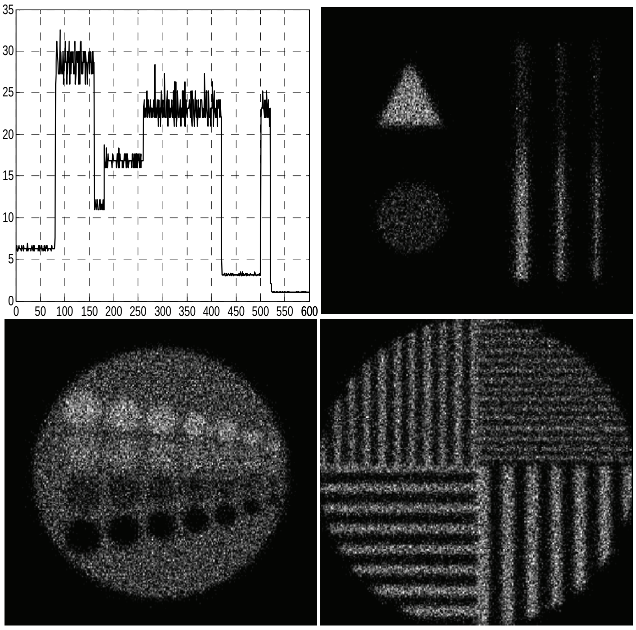

Fig. 1. Deterministic test noisy signals.

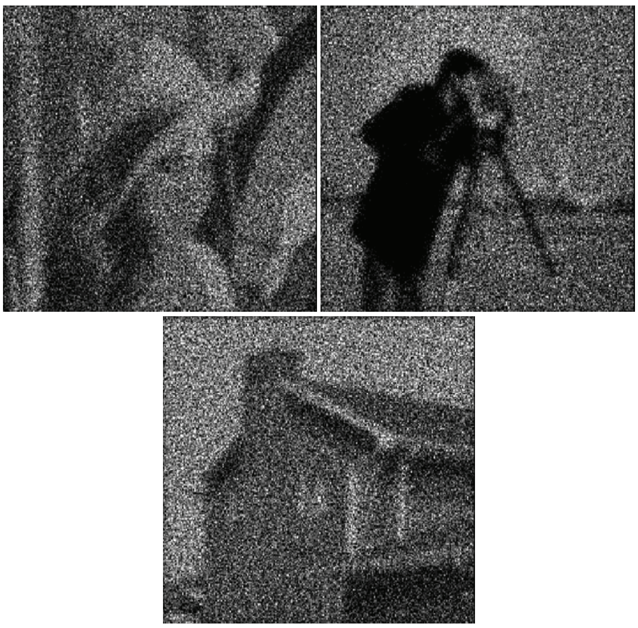

Fig. 2. Noisy images.

where α is a constant parameter which could be determined by equalizing the energy:

$$\sum_i D_1(i)^2 = \sum_i \left(\alpha A_1(i)\right)^2 \qquad (6)$$

of new coefficients $\mathbf{D}_1$ and $\mathbf{A}_1$ which are created from $\mathbf{D}$ and $\mathbf{A}$**,** respectively by using the following reasoning.

The detail coefficients $\mathbf{D}$ are like waves and they frequently change their polarity. Therefore, the coefficients between the positive and negative peaks have magnitudes that are close to zero. Therefore we can discard their contribution (by zeroing them in corresponding positions in $\mathbf{D}_1$) and keep only the coefficients that correspond to the local extremes in $\mathbf{D}$.

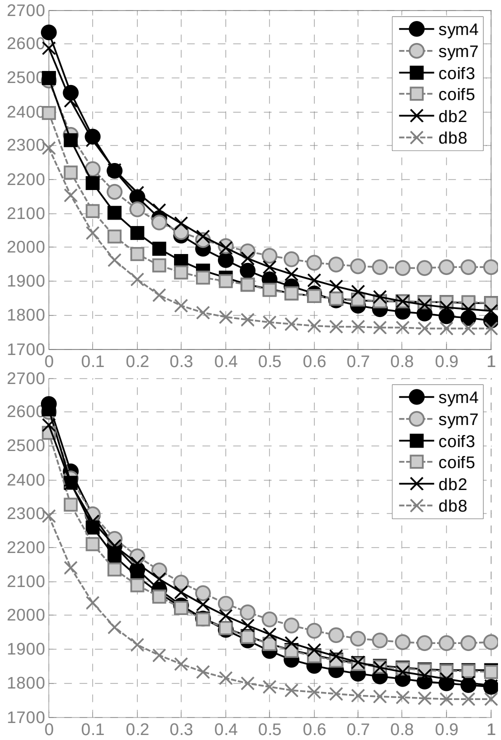

Fig. 3. (a) Dependence of the noise energy E_n on the uniform threshold τ; (b) Dependence of the noise energy E_n on τ after the variance stabilizing operation is applied.

Similarly, the vector $\mathbf{A}_1$ is constructed by zeroing the approximation coefficients $\mathbf{A}$ for those indices i where $D_1(i) = 0$.

$$\mathbf{A}_1 = \mathbf{A}\cdot\mathrm{sign}(|\mathbf{D}_1|). \qquad (7)$$

Since the coefficients $\mathbf{D}$ and $\alpha\mathbf{A}$ have equal energy, but not exactly same form, it holds that if for some i, $|D(i)| > \alpha A(i)$ (the signal is less noise contaminated), then for some $j\neq i$, $|D(j)| < \alpha A(j)$ (the signal is more noise contaminated).

In general, we can assume that for noise stands polynomial dependence on the local signal intensity, hence, for the threshold $\boldsymbol{\tau}$ the following can be written:

$$\tau(i) = \alpha_n A(i)^n + \Lambda + \alpha_1 A(i) + \alpha_0, \quad i = 0, \Lambda, L-1, \qquad (8)$$

where L is the length of the vectors $\mathbf{A}$ and $\boldsymbol{\tau}$. The coefficients α_0, α_1, … can be obtained by minimizing the square measure E_1 in the smallest squares sense:

$$E_1 = \frac{1}{2}\sum_i \left(|D_1(i)| - \left(\alpha_n A_1(i)^n + \Lambda + \alpha_1 A_1(i) + \alpha_0\right)\right)^2. \qquad (9)$$

IV. Experimental Results

In this Section, we illustrate the effects of denoising the artificially contaminated signals (images) by applying the conventional shrinkage methods and our proposed non-uniform threshold approach. The noise energy in the filtered

TABLE I
COMPARISON OF THE PROPOSED WITH KNOWN METHODS IN CASE OF TRUE SIGNAL ESTIMATING IN THE TEST IMAGES IN FIG. 1

Image	SNR_1	ΔSNR Known methods									ΔSNR Proposed algorithm	
		Wave-let	Visu Shrink [2]soft/hard	Sure Shrink [4]	Bayes Shrink [6]	PRESS [5]	Variance stabilizing [1]	Xu-Weaver [3]	Bi Shrink [7]	Prob Shrink [8]	Energy equalizing soft/hard	LS minimization soft/hard
Phantom	2.92dB	sym3	1.19/0.13	0.28	0,89	0.86	1.30	2.03	0,37	0.51	**+2.47dB** / **+1.71dB**	+2.38dB / +1.52dB
		sym5	1.19/0.14	0,28	0.88	0.86	1.29	1.79	0,33	0.50		
		db3	1.19/0.13	0.28	0.89	0.86	1.30	2.01	0.37	0.51		
		coif5	1.09/0.10	0.28	0.73	0.86	1.18	1.98	0.37	0.44		
Circles	4.38dB	sym3	4.00/1.65	0.21	4.08	1.35	3.50	3.36	1.14	2.04	**+4.46dB** / **+2.67dB**	+4.22dB / +2.33dB
		sym5	3.97/1.55	0.21	4.06	1.34	3.49	2.98	1.00	1.98		
		db3	4.00/1.65	0.21	4.08	1.35	3.50	3.32	1.13	2.04		
		coif5	3.81/1.19	0.21	4.07	1.34	3.38	3.22	1.08	1.75		
Bars	3.60dB	sym3	2.71/**2.02**	0.17	2.14	0.99	2.61	2.24	1.23	1.87	**+2.85dB** / +1.89dB	+2.73dB / +1.68dB
		sym5	2.71/**2.00**	0.17	2.20	1.00	2.62	2.03	1.07	1.85		
		sym7	2.74/**1.97**	0.17	2.22	1.00	2.64	2.20	1.18	1.84		
		db3	2.71/**2.02**	0.17	2.14	0.99	2.61	2.22	1.23	1.87		
		db6	2.74/**2.04**	0.17	2.21	0.99	2.64	2.22	1.24	1.88		
		coif3	2.74/**1.96**	0.17	2.25	1.00	2.64	2.19	1.19	1.84		
		coif5	2.74/**1.89**	0.17	2.30	1.00	2.66	2.19	1,17	1.80		
		bior9/7	2.70/**1.96**	0.17	2.19	1.00	2.60	2.14	1.14	1.85		

signal is higher when any global threshold is used compared to the case when the proposed non-uniform threshold is used.

The noise contaminated images are generated by superpositioning of shifted 2-D random Gaussian functions (cantered at position (*i*,*j*)) with energies proportional to the pixel intensities at position (*i*,*j*) in the noise-free images.

By applying of the conventional and proposed method we obtain filtrated images $\mathbf{s}_1$ (normalized to the energy of the noise free images **s**), and compare with the energy of the noise free images by using the following formula

$$E_n = \sum_{i,j} \left(s(i,j) - s_1(i,j) \right)^2 . \qquad (10)$$

When the signal in Fig. 1(a) is filtered by using the proposed method (Eq. 5, 6), we obtained $\alpha = 0.0502$ and $E_n = 1586$. The proposed algorithm uses NPR-QMF filters with length 12, stop band frequency 0.7π, and overall reconstruction error of the designed QMF bank 0.001 [10]. In addition, we filtered the signal by using standard technique of wavelet shrinkage [9] and used different wavelets and different values of the uniform threshold τ. The graph for dependence of E_n on τ for values of τ between 0 and maximal intensity in the detail coefficients is plotted in Fig. 3(a). The threshold value $\tau = 0$ means that all the detail coefficients are kept, while the value $\tau = 1$ (which corresponds to a threshold equal to the maximal intensity in the detail coefficients) means that all the detail coefficients are discarded. From Fig. 3a it can be noticed that for any value of the uniform threshold, the energy of the remained noise is not smaller than 1586. This comes from the fact that using a uniform threshold for removing signal-dependent noise is not an adequate solution.

Similar results are presented in Fig. 3b. The graphs show dependence of E_n on τ after applying operation of variance normalization [1] on the images before they are filtered by using standard wavelet shrinkage.

Further, we made experiments with the images in Fig. 1 and Fig. 2. They both contain signal-dependent noise with rather low SNR. The images in Fig. 1 are standard nuclear medicine test images, while the images in Fig. 2 are well known test images commonly used for comparing performances of different image processing techniques. The maximal intensity in all three images in Fig. 1 is 22. The performances of the applied filtration methods obtained with various wavelet-based filtering methods, are presented in Table 1 in which SNR_1 is signal-to-noise ratio for the generated images while ΔSNR is the improved signal-to-noise ratio (after the filtering). When the proposed method is used with two differently generated thresholds (last column) it can be noticed that the filtering with non-uniform threshold determined through energy equalizing (Eq. 6) gives better results compared to the filtering with non-uniform threshold determined through LS minimization of the square measure (Eq. 9).

The results of filtering the images in Fig. 2 are shown in Table 2. They are similar to the results in Table 1. From both Table 1 and Table 2 the advantage of the non-uniform threshold shrinkage over the uniform threshold shrinkage is evident.

TABLE II
COMPARISON OF THE PROPOSED WITH KNOWN METHODS IN CASE OF TRUE SIGNAL ESTIMATING IN THE IMAGES IN FIG. 2

Image	SNR_1	ΔSNR Known methods								ΔSNR Proposed algorithm	
		Wave-let	Visu Shrink [2]soft/hard	Bayes Shrink [6]	PRESS [5]	Variance stabilizing [1]	Xu-Weaver [3]	Bi Shrink [7]	Prob Shrink [8]	Energy equalizing soft/hard	LS minimization soft/hard
Lena	5.27dB	sym3	5.00/**4.68**	2.81	1.49	4.38	3.72	2,13	3.83	**+5.10dB** / +3.04dB	+4.81dB / +2.63dB
		sym5	4.97/**4.69**	2.80	1.49	4.35	3.28	1.82	3.80		
		db3	5.00/**4.68**	2.81	1.49	4.38	3.67	2,13	3.83		
		coif5	4.98/**4.70**	2.80	1.49	4.33	3.47	1.88	3.80		
		bior9/7	4.90/**4.62**	2.79	1.47	4.27	3.61	1.96	3.75		
House	5.76dB	sym3	5.22/**4.95**	2.94	1.56	4.52	3.89	2.11	3.99	**+5.29dB** / + 3.17dB	+4.99dB / +2.74dB
		sym5	5.14/**4.87**	2.89	1.56	4.47	3.39	1.80	3.96		
		db3	5.22/**4.95**	2.94	1.56	4.52	3.87	2.11	3.99		
		coif5	5.14/**4.84**	2.89	1.54	4.45	3.62	1.87	3.94		
		bior9/7	5.07/**4.72**	2.87	1.53	4.39	3.69	1.94	3.88		
Camera	5.32dB	sym3	4.58/**3.99**	3.19	1.42	4.00	3.48	1.69	3.27	**+4.71dB** / +2.86dB	+4.46dB / +2.50dB
		sym5	4.59/**4.07**	3.16	1.43	4.02	3.08	1.47	3.30		
		db3	4.58/**3.99**	3.19	1.42	4.00	3.46	1.68	3.27		
		coif5	4.58/**4.13**	3.11	1.43	4.02	3.23	1.49	3.49		
		bior9/7	4.54/**3.97**	3.15	1.42	3.96	3.33	1.58	3.26		

V. CONCLUSION

In this paper we compare non-uniform and uniform threshold filtering methods on denoising artificially noised deterministic test images. Experimental results show that for the signal-dependent noise filtering with non-uniform threshold outperforms uniform threshold filtering for any level of the threshold and all used wavelets we have experimented with.

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