# Application of Geometrical Models for Face Detection 

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#### Abstract

The main purpose of the face detection process, applied to the whole image, is to validate and extract the face regions only. The presented algorithm involves computation of the angle of face candidate region. After rotation of a face geometrical model (ellipse) is applied, discovery of the number of holes processed within intensity component is performed. Then the region is processed using face feature detection, where the centers of the eyes and mouth are calculated with Projection Function (PF). At the end of the algorithm, a flexible pattern of eyes and mouth is applied on regions where a face is determined. The outputs of the face detection algorithm are separate images containing faces, for which also eyes and mouth locations are determined.


Keywords- Face detection, Geometrical Models, Flexible Patterns, Facial Features Location

## I. Introduction

Face detection is used at the preprocessing stage of face verification systems. It serves for decreasing the number of calculations and improves the effectiveness of such systems [1][2]. In face detection systems, color component models for finding skin on images are widely used. To create robust algorithm for face detection, it is necessary to apply geometrical models on the areas already found to contain skin [3]. This way, more robust validation is achieved of the facecandidate skin areas. Another problem in the systems working with color information only is their inability to determine the direction of the face. In systems working with the geometrical features of the face, this problem can be solved by analyzing the geometrical features of the face [4]. This allows some important parameters to be determined such as: the frontal face position, the rotation angle etc which are impossible to determine if color components only are used. The face features should be used and analyzed then.
This paper is structured as follows: There is a brief presentation of the problem in section II. Validation of facecandidate region and the required pre-processing and face shape pattern aping is described in section III. Section IV presents a face feature localization, followed by the some experimental results section V and conclusion.

## II. Problem Description

One of the most important problems for face detection is face validation of the region which contains skin image. The face shape could be easily approximated to an ellipse [4]. The
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next problem is that it may not be oriented vertically [5], which causes additional calculations and slows down the processing. The original image should not be rotated, because each operation, such as rotating and resizing causes loss of quality. For this reason, when masks are being realized, the algorithms should work with a copy of the original image. If necessary the image copy may be resized in case of very high resolution original image, because this significantly reduces the number of calculations required to process the image or mask [6][7].

## A. Geometrical Models for Face Detection

Face detection results depend on the classification methods used to detect the various areas of the face candidate image.

This article concentrates on the geometrical features of the face. One of them is the face shape. The shape of the human face could be approximated to an ellipse, which is described by its center and two axes. To satisfy this criterion the face candidate area should have elliptical form. This is why an algorithm should be created for finding the center of the face ellipse and the vectors of its axes, which define its size and orientation in space.

As next criterion, a comparison is used between the face candidate image area and the typical face features pattern. The skin holes on a face image are the areas of eyes and mouth. They also have simplified elliptical shape. For more accurate detection of these features, a model that represents them more precisely should be created. This is the crosssection of the two circles, which lay over the features borders. The resulting arcs define the features shape much more accurately than an ellipse.

Of course, not all detected skin regions contain faces. Some regions may correspond to hands and arms or other uncovered body parts, while others - to objects of color similar to the skin-color. Hence the second stage of face detection will employ facial features to locate a face in each of these skin face-candidate regions.

## B. Facial Features Localization

In general, all image projection functions can be used to detect the boundary of different image regions.
Suppose $P F$ is a projection function, $\xi$ is a small constant. If the value of $P F$ rapidly changes from $z_{0}$ to ( $z_{0}+\xi$ ), then $z_{0}$ may lie at the boundary between two homogeneous regions. In detail, given a threshold $T$, the vertical borders in the image can be identified according to:

$$
\begin{equation*}
Q_{\nu}=\max \left\{\left|\frac{d \operatorname{Rvg}(x)}{d x}\right|>T\right\}, \tag{1}
\end{equation*}
$$

where $\Theta_{v}$ is the set of vertical critical points, such as $\left\{\left(x_{1}\right.\right.$, $\left.\left.\operatorname{PFv}\left(x_{1}\right)\right),\left(x_{2}, P F_{v}\left(x_{2}\right)\right), \ldots,\left(x_{k}, P F_{v}\left(x_{k}\right)\right)\right\}$, which divides the image vertically into different regions. It is obvious that the horizontal critical points can be identified in a similar fashion. This property of PF can be well exploited in eye and mouth detection. If the projection function is applied separately for both directions ( $x$ and $y$ ) of the image, the horizontal and vertical projections are obtained. A rapid change between neighboring values indicates a boundary of an object.

## III. Validation of Face-Candidate Region

Using human color skin model, a skin probability image is obtained. Some regions can contain other not only face region. To eliminate for example hand, legs or other uncovered skin area we have to apply face detection algorithm. From this moment we'll assume each skin region like separate work image.

## A. Preprocessing: Rotation of the Face Candidate Region

Usually there is no information for testing images resolution, number of faces and their rotation in it.
For this reason first at all, the face candidate regions are being detected separately using the probability skin image. For each region a back projection is computed using an image fragment from the original image containing only the Y component from YCbCr . So from this moment the algorithm works with the separated image regions and number of face candidates is known.
If face candidate is vertically situated in the region, the position of the eyes and mouth will be determined more precisely.
The rotation of the face region is estimated according to its gravity center. The angle $\theta$ is calculated as an adjacent angle between the straight line that connects the most distant point and the gravity center, and its perpendicular straight line.
Each region point is rotated by its own radius at $\theta$ degrees, and as a result, the original coordinates x and y of each point are substituted by $x$ ' and $y$ ', so that

$$
\begin{gather*}
x=R_{g} x^{\prime} \text { and } R_{g}=\left[\begin{array}{cc}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{array}\right]  \tag{2}\\
{\left[\begin{array}{l}
\text { if } 0>\theta>90 ;
\end{array} \quad \theta=\theta\right.} \\
\text { if } 90 \geqslant \theta \geqslant 180 ; \theta=360-\theta
\end{gather*}
$$

For more accurate calculations further the algorithm works with the rotated images, because it is oriented directly to the X and Y axes. This decreases the number of calculations performed during the algorithm execution and avoids the necessity of interpolation. Although each rotation of picture elements causes unrecoverable data loss, it decreases the error rate and optimizes the processing.

## B. Face Region Determination using Face Shape Pattern

The shape of human face may be approximated to an ellipse. The extents of the major and the minor axes of the ellipse can be approximated by the extents of the same skin face-candidate region along the axis directions.

The aspect ratio of ellipse axis should also have a limit, it should be from $1 \div 1.5$. The regions of pixels, which remain after this verification, are checked for elliptic shape (Tab. 1. (column " bAxis/aAxis")).

The degree of the ellipse's fit is determined by the number of pixels falling into that shape, specified by the computed parameters, ROC is calculated. This shape feature allows validation to be performed on all face-candidates (Fig. 3 b,c)). During this operation the intensity component (Y) only is used (binary mask). A binary mask is applied to the (Y) components of the identified skin-like areas from the original image(Fig. 3d, Fig. 4 a)).

An ellipse is defined by its centre ( $\mathrm{x}, \mathrm{y}$ ) of gravity, its orientation $\theta$ and the length $\mathbf{a}$ and $\mathbf{b}$ of the major and minor axes. The centre ( $\mathrm{x}, \mathrm{y}$ ) of the ellipse is given by the gravity centre of the region. Because region is already rotated $(2 ; 3)$ the angle $\theta$ is assumed zero(Fig. 4b).

For double recalculation to be avoided for x and y , the ellipse is visualized not with the standard equation

$$
\frac{x^{2}}{b^{2}}+\frac{y^{2}}{a^{2}}=1 \text {, (4) }
$$

but as an angle function instead:
$\gamma=\arcsin \left(\frac{\mathrm{r}}{\mathrm{R}}\right) ; \mathrm{x}=\mathrm{R} \cdot \sin (\alpha) \cdot \cos (\mathrm{y}) ; \mathrm{y}=\mathrm{R} \cdot \cos (\alpha) ;(5)$
Second criterion is to determine the number of holes in face candidate region using the intensity (Y) component. For regions with large size and resolution, it is normal to have from 3 to 7 holes (eyebrows, eyes, nostrils, mouth and ear (profile)), for lower quality region the normal number are 3-5 (eyebrows and eyes are connected, nostrils (depending on nose type and mouth)) and for bad quality images there are 3 (eyebrows and eyes are connected and mouth). Based upon this criterion, if image quality is unknown, the following assumption can be made: if more than 7 holes are present the candidate region does not contain a face.

Each skin area in the mask is compared to an ellipse using two criteria. The first one checks whether the match ratio is greater than $80 \%$ (Tab. 1. Column TP). If it is then the probability this area to contain a face is high. The second criterion is how many skin pixels are out of the ellipse. If this percent is less than $15 \%$ and the first criterion is satisfied, then we assume that this area contains a face, and the already built mask is increased by $10 \%$, put directly on the original image after rotation by minus $\theta$ degrees.

## IV. Face Feature Localization

The Sobel mask is applied on the Y (intensity) component of a verified skin region. We obtain grayscale image (BWimg) in which horizontal edges are detected. This image is used for eyes region location detection. We considered that eyes region obtain a level close to 255 . For eyes location is used Projection function (PF). According it is supposed that is the intensity of a pixel $\operatorname{Int}(x, y)$ with location ( $x, y$ ) of image region. Vertical projection $\operatorname{PPF}$ ( x )and horizontal one IPFh(y) in intervals $\left[\mathrm{y}_{1}, \mathrm{y}_{2}\right]$ and $\left[\mathrm{x}_{1}, \mathrm{x}_{2}\right]$ are defined as:

$$
\begin{align*}
& \operatorname{IFFv}(\mathrm{N})=\sum_{\mathrm{y}=y_{2}}^{y_{2}} 1(\mathrm{x}, \mathrm{y})  \tag{6}\\
& \operatorname{IPM}(y)=\sum_{x=x_{4}}^{x_{2}} \mathrm{I}(x, y) \tag{7}
\end{align*}
$$

In general, all image projection function can be used to detect the boundary of different image regions. After applying the projection function it should detect a quick transition. Considering that mean ${ }_{v}$ is mean value of $\mathbb{P P V}(\mathrm{x})$ and meanh $_{h}$ is mean value of $\operatorname{PFFh}(y)$ :

$$
\begin{align*}
& \text { (for } \operatorname{PFFh}(\mathrm{i})>\text { mean }_{h} \text { and } \operatorname{IPFh}(\mathrm{i}-1)<\text { meanh }_{h} \\
& \text { EyeLeft }_{\text {up }}=\mathrm{i} \text { and EyeRight }{ }_{\text {up }}=\mathrm{i} \\
& \text { for IPFh(i) }<\text { meanh }_{h} \text { and } \operatorname{IPFh}(\mathrm{i}-1)>\text { mean }_{\text {h }}  \tag{8}\\
& \text { Eyeleft }_{\text {down }}=\mathrm{i} \text { and EyeRight } \mathrm{down}_{\text {a }}=\mathrm{i}
\end{align*}
$$

Where $i=y_{1} \div y_{2}$ and $y_{1}=1$ (top of the image);
$y_{2}=\frac{\text { ImHight }}{2}$ - respectively, considering that the eyes are located in the upper half part of the image.
Since eyes present a strong horizontal region is applied the horizontal integral projection function IPFV(X) in order to detect the horizontal location of the eyes in the image, where $x 1=1$ and $x 2=$ image length. Looking for the maximum of the IPF, we can extract the coordinates where the eyes are located. We extract the coordinates where the eyes are located applying to the following definitions:

$$
\begin{align*}
& \left\{\begin{array}{c}
\text { for IPFV(1) }>\text { mean }_{V} \text { and } \operatorname{IPFv}(\mathrm{j}-1)<\text { mean }_{V} \\
\text { EyeLeft }_{\text {left }}=1 \\
\text { for IPFV(1) }<\text { mean }_{V} \text { and IPFV( }(1)>\text { mean }_{V} \\
\text { Eyeleft }_{\text {right }}=1
\end{array}\right. \tag{9}
\end{align*}
$$

$$
\begin{align*}
& \text { EyeRight }_{\text {right }}=1 \tag{10}
\end{align*}
$$

where $\mathrm{j}=1 \div \frac{\text { mLenght }}{2} ; \mathrm{k}=\frac{\text { mLengbt }}{2} \div$ ImLenght.
First is being inscribed the flexible pattern for eyes and mouth in region with location found in 2.3.

Eye pattern is represented like two curves which are parts of circles (Fig. 5a). Using the rectangle around the face features with most up and left point $\left[\mathrm{X}_{1}, \mathrm{y}_{\mathrm{i}}\right]$ and most right and down point $\left[x_{2}, y_{2}\right]$ [right, down] of the face feature region is calculated the radius of circles R and their centers:

$$
\begin{gather*}
\mathbb{R}=\frac{\left(\left(x_{2}-y_{2}^{2}\right)^{2}+\left(y_{2}-x_{2} z^{2}\right)\right.}{2\left(y_{2}-x_{1}\right)} ;  \tag{11}\\
\mathrm{C}_{1}=\left[x_{2}-y_{1}: \mathbb{R}+x_{1}\right]: \quad \mathrm{C}_{2}=\left[x_{2}-y_{1} ; y_{2}-\mathbb{R}\right] \tag{12}
\end{gather*}
$$

Mouth region location and flexible pattern is being found by analogy with eyes $(6 \div 9)$ (Fig. 5b).

It's made a face pattern adaptation and verification using the frontal face elements. In order to apply the flexible eye pattern, first the precise iris location and radius should be found. For the corresponding region of the left or right eye, from (8), the enclosing rectangle is computed. Then, from the
bottom of the rectangle a search is started for the face features. As result are found the eyes and mouth locations in face region (Fig. 6).


Fig. 1. Horizontal PF


Fig. 2. Vertical PF


Fig. 3. a) original image; b) skin probability image; c) Skin region mask; d) Skin region separation mask


Fig. 4 a) Face candidate images; b) Region contour and described

Table 1.
Result of PF, AXIS Ratio and holes number of DIFFERENT REGIONS FROM ONE IMAGE SAMPLE

| TP | FP | FN | a <br> Axis | b <br> Axis | bAxis/aAxis | Hole <br> Numbers |
| :---: | :---: | :---: | ---: | ---: | ---: | :---: |
| 46.712 | 2.8912 | 50.3968 | 16 | 23 | 1.4375 | 3 |
| 50.5068 | 4.1104 | 45.3829 | 21 | 22 | 1.0476 | 4 |
| 41.8939 | 6.4394 | 51.6667 | 15 | 19 | 1.2667 | 0 |
| 49.8636 | 6.9285 | 43.2079 | 14 | 21 | 1.5 | 1 |
| 52.7058 | 2.096 | 45.1982 | 16 | 30 | 1.875 | 3 |
| 69.6266 | 3.1421 | 27.2313 | 43 | 74 | 1.7209 | 10 |
| 63.5779 | 9.7169 | 26.7053 | 10 | 22 | 2.2 | 1 |
| 71.558 | 6.9746 | 21.4674 | 28 | 42 | 1.5 | 5 |
| 73.7805 | 9.0701 | 17.1494 | 11 | 22 | 2 | 3 |
| 96.2963 | 0 | 3.7037 | 2 | 9 | 4.5 | 5 |
| 72.5359 | 6.0287 | 21.4354 | 19 | 25 | 1.3158 | 2 |
| 86.217 | 3.3724 | 10.4106 | 3 | 8 | 2.6667 | 0 |
| 97.9487 | 0.9744 | 1.0769 | 8 | 27 | 3.375 | 6 |



Fig. 5. Eye and Mouth flexible patterns


Fig. 6. Original image and grayscale one with eyes and mouth position determinate

## V. Conclusion

The paper is present face region determination method. The different part of algorithm can be used separately even.

The method work with complex images which can contain more than one person (faces) and with complex background. Using skin color detection and region segmentation are found all face candidate regions separately.

Face region rotation angle determination contribute for the simplification and minimization for following operation.

Flexible face pattern make face region verification with minimum calculation. Counting the number of holes and face enclose pattern can give not only face verification information but face region size and resolution with are very useful information for face recognition system.

There made a prissily fitting of face features pattern. For eyes first are found irises positions. Eye and mouth pattern are fit by coincidence error minimization.

The described algorithm could be used for different type of application like web image database or with high-definition image database because it is independent from the size of the input images.

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