Analysis of EGC Diversity with Partially Coherent Weibull Fading Signals

Goran T. Đorđević[1], Aleksandra M. Cvetković[2], Mihajlo Č. Stefanović [3]

Abstract – **An equal gain combining (EGC) receiver for the binary phase-shift keying (BPSK) and quaternary phase-shift keying (QPSK) signals that propagate over Weibull fading channels is analyzed in this paper. We determine the bit error rate (BER) degradations caused by incoherently combining. The results clearly show the effects of imperfect reference signal recovery and fading severity on the EGC receiver performance. The numerical results are obtained by numerical integration with previously given accuracy and confirmed by Monte Carlo simulations.**

Keywords – **Bit error rate, Equal gain combining, Phase error, Phase-Shift Keying**

I. Introduction

In order to diminish the influence of multipath fading on signal detection, spatially separated receiver antennas at the receiver as well as combining signals from different receiver branches can be used. The diversity receivers with equal gain combining (EGC) technique are often used in practice. EGC is suboptimal combining technique which, at the exit of the combining circuit, achieves slightly smaller values of instantaneous signal-to-noise ratio when compared to optimal combining (MRC – maximum ratio combining), but is, on the other hand, relatively simple to implement and therefore often applied in practice. With this combining technique signals at all branches are cophased, equally weighed and summed to give the resultant output signal [1]. Cophasing eliminates random signal phase fluctuations occurring during transmission. The estimation of receiver signal phase is needed for cophasing. The estimation of received signal phase is accomplished by using a receiver modulated or unmodulated carrier.

In the previous papers regarding this problem it was mainly assumed about incoming signal perfect carrier phase estimation, e.g. [2-3]. Only in papers [4-5] the influence of the imperfect estimation of the received signal phase to the system performance was discussed. Paper [4] discusses the phase error influence on bit error rate (BER) values when detecting digital binary phase-shift keying (BPSK) and quaternary phase-shift keying (QPSK) signals. The analysis was done under assumption that identical and statistically independent Rayleigh fading is present at receiver antennas. Paper [5] presents derived closed-form expressions for outage probability and average BER in detecting BPSK and QPSK signals transmitted over correlated Nakagami-*m* fading channels. EGC technique with dual branches is observed.

Weibull distribution is very often used for fading modeling in urban environments in cases when Rayleigh distribution is inadequate. This distribution is empiric and is originally used as a statistical model for system reliability analysis [6-7]. It was shown in paper [6] that Weibull distribution gives best fit with measurement results for DECT (digital enhanced cordless telecommunications) systems working at 1.89 GHz. Also, the measurement results at 900 MHz presented in paper [7] show that this distribution can also be used and as a model of outdoor multipath fading. Fading model with Weibull distribution implies signal consisting of a cluster of multipath waves in nonhomogenous environment. The resulting envelope is obtained as a nonlinear function of the modulus of the multipath component sum.

This paper presents the analysis of average BER in detecting BPSK and QPSK signals over Weibull fading channels. Receiver uses EGC technique. Receiver signal phase estimation is done from unmodulated carrier and is not perfect. The difference between the incoming signal phase and estimated signal phase is stochastic process which has Tikhonov probability density function specified through standard deviation [4-5], [8]. It is shown to what extent an imperfect phase estimation influences BER values.

II. System Model

Signal on *i*-th receiver antenna (Fig. 1) can be given as

$$s_i(t) = r_i(t) e^{j\gamma_i(t)} \cdot A e^{j\phi_n} + n_i(t), \quad i = 1,2,\ldots,L \,, \quad (1)$$

where $r_i(t)$ is fading envelope, $\gamma_i(t)$ is random phase shift which occurred during signal transmission over fading channel. Fading at each antenna is frequency nonselective, during one symbol it does not change, it is independent from symbol to symbol and there is no correlation between fading on different antennas. Probability density function of fading envelope is Weibull [1]:

$$p_{r_i}(r_i) = \frac{\alpha}{\Omega} r_i^{\alpha-1} \cdot \exp\left(-r_i^{\alpha} / \Omega\right), \quad r_i \geq 0 \,, \quad (2)$$

Goran T. Đorđević is with Faculty of Electronic Engineering, Aleksandra Medvedeva 14, 18000 Nis, Serbia, E-mail: goran@elfak.ni.ac.yu

Aleksandra M. Cvetković is with Faculty of Electronic Engineering, Aleksandra Medvedeva 14, 18000 Nis, Serbia, E-mail: alekmi@elfak.ni.ac.yu

Mihajlo Č. Stefanović is with Faculty of Electronic Engineering, Aleksandra Medvedeva 14, 18000 Nis, Serbia, E-mail: misa@elfak.ni.ac.yu

where α is fading parameter, $\Omega=\mathrm{E}\{r_i^{\alpha}\}$, E{.} denotes mathematical expectations. It can be shown that n-th moment is [1]

$$\mathrm{E}\left\{r_i^{\,n}\right\}=\Omega^{n/\alpha}\Gamma\left(1+\frac{n}{\alpha}\right), \tag{3}$$

where Γ(.) is Gamma function [9, eq. (8.310/1)]. The amplitude of useful signal is denoted with A and it can be assumed without loss of generality that it is equal to one. With ϕ_n we denote signal phase in which information about sent symbol is written. In the case of BPSK signal ϕ_n can have one of the following values: {0, π}, and in the case of QPSK signal ϕ_n can have one of the following values: {π/4, 3π/4, 5π/4, 7π/4}. If we count that square mean value of signal envelope equals one

$$\mathrm{E}\left\{r_i^{\,2}\right\}=1=\Omega^{2/\alpha}\Gamma\left(1+\frac{2}{\alpha}\right), \tag{4}$$

then

$$\Omega=\left(1/\Gamma\left(1+\frac{2}{\alpha}\right)\right)^{\alpha/2}. \tag{5}$$

Zero mean Gaussian noise with variance σ_i^2 for i-th receiver branch is denoted with $n_i(t)$. Standard deviation of this Gaussian noise was given with

$$\sigma_i=\mathrm{E}\left\{r_i^{\,2}\right\}/\sqrt{2\cdot\log_2 M\cdot 10^{(E_b/N_0)_i}}, \tag{6}$$

where M is the number of phase levels, $(E_b/N_0)_i$ is average signal energy per bit to noise power spectral density ration for i-th receiver branch and is given in decibels. It is $(E_b/N_0)_1=(E_b/N_0)_2=...=(E_b/N_0)_L=E_b/N_0$.

After signal cophasing at all branches, the resulting signal after combining is

$$z(t)=\sum_{i=1}^{L}\left(A\cdot r_i(t)e^{j\phi_n}\cdot e^{j\varphi_i(t)}+n_i(t)\right), \tag{7}$$

where L is the number of receiver branches. The difference between receiver signal phase $\gamma_i(t)$ at i-th receiver branch and estimated phase $\hat{\gamma}_i(t)$ at that receiver branch is denoted with $\varphi_i(t)=\gamma_i(t)-\hat{\gamma}_i(t)$. If phase estimation is done using phase-locked loop (PLL) from unmodulated carrier and if only Gaussian noise is present in the phase-locked loop circuit, then probability density function of this phase error is [4-5], [8]

$$p_{\varphi_i}(\varphi_i)=\frac{1}{2\pi}\frac{\exp(\varsigma_i\cdot\cos(\varphi_i))}{\mathrm{I}_0(\varsigma_i)},\quad -\pi<\varphi_i\leq\pi, \tag{8}$$

where $\mathrm{I}_0(x)$ is modified Bessel function of the first kind and zero order for the argument x [9, eq. (8.406)], ζ_i is signal-to-noise ratio in the PLL circuit at i-th receiver branch, which can be denoted through phase error variance σ_φ^2 [4-5], [8]

$$\zeta_i=1/\sigma_{\varphi_i}^{\;2}. \tag{9}$$

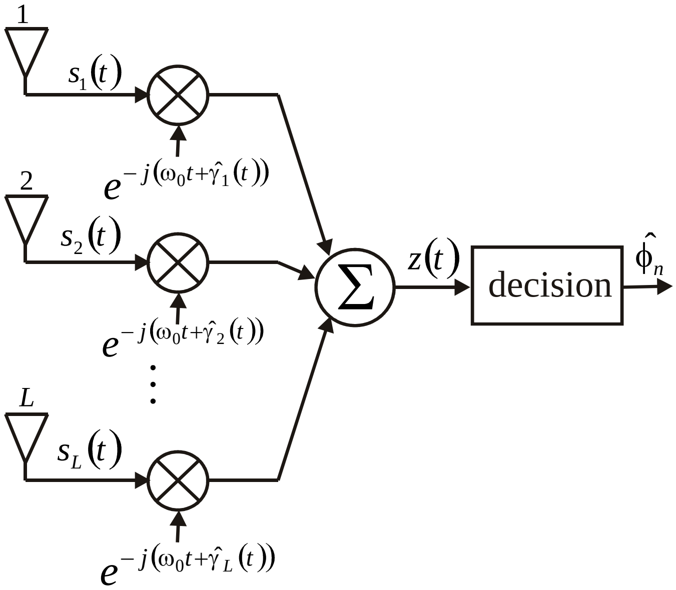

Fig. 1. EGC receiver model

After analysis of EGC receiver and mathematical manipulations it can be shown that the BER of BPSK and QPSK signal detection are given with

$$BER=\frac{1}{2}\int_{\mathbf{r}}\int_{\varphi}\mathrm{erfc}\left(\frac{\sum_{i=1}^{L}r_i\cos\varphi_i}{\sqrt{L}\cdot\sqrt{2}\cdot\sigma}\right)p_{\varphi}(\varphi)p_{\mathbf{r}}(\mathbf{r})d\varphi d\mathbf{r} \tag{10}$$

and

$$BER=0.25\int_{\mathbf{r}}\int_{\varphi}\left\{\mathrm{erfc}\left(\frac{\sum_{i=1}^{L}r_i\cos(\pi/4-\varphi_i)}{\sqrt{L}\sqrt{2}\sigma}\right)+\right.$$

$$\left.+\,\mathrm{erfc}\left(\frac{\sum_{i=1}^{L}r_i\cos(\pi/4+\varphi_i)}{\sqrt{L}\sqrt{2}\sigma}\right)\right\}p_{\varphi}(\varphi)p_{\mathbf{r}}(\mathbf{r})d\varphi d\mathbf{r}, \tag{11}$$

where erfc(.) is complementary error function [10, eq. (7.1.2.)], $p_{\varphi}(\varphi)$ is joint probability density function of the vector $\varphi=(\varphi_1,\varphi_2,...,\varphi_L)$, which is, considering that φ_i, i=1,2,...L, are independent, given with

$$p_{\varphi}(\varphi)=\prod_{i=1}^{L}p_{\varphi_i}(\varphi_i), \tag{12}$$

and $p_{\mathrm{r}}(\mathrm{r})$ is joint probability density function of the vector $\mathbf{r}=(r_1,r_2,...,r_L)$, which is, considering that also r_i, i=1,2,...L, are independent, given with

$$p_{\mathbf{r}}(\mathbf{r})=\prod_{i=1}^{L}p_{r_i}(r_i). \tag{13}$$

III. Numerical Results

Numerical results are obtained by applying both numerical integration and Monte Carlo simulation. In order to obtain BER of BPSK and QPSK signal detection it is necessary to perform numerical integration in terms (10) and (11). If we consider dual branch EGC receiver quadruple numerical integration occurs. Numerical integration was done by applying Gaussian type quadrature formulas along with increasing the number of nodes until previously assigned accuracy is achieved. The BER values are estimated on the bases of 4000 bit errors. In addition, minimum number of symbols that is used during evaluation of any BER value is 10^4 and maximum 2^{31-1} symbols are used in simulations.

Figs. 2 and 3 show phase error deviation influence on average BER values of BPSK and QPSK signal detection. From Fig. 2, where presented results are related to BPSK modulation format, we can notice that curves of BER dependences on E_b/N_0 almost overlap for σ_φ values from 0^o to 15^o. We can see from both figures, that for mean values of E_b/N_0, the BER sharply decreases with the increase of E_b/N_0. BER remains constant with E_b/N_0 increase for large E_b/N_0 values. This BER floor depends on phase noise standard deviation value. For example, from Fig. 3 we can observe that BER floor increases from $3.3 \cdot 10^{-6}$ to $2.1 \cdot 10^{-3}$ if σ_φ increases from 12.5^o to 20^o.

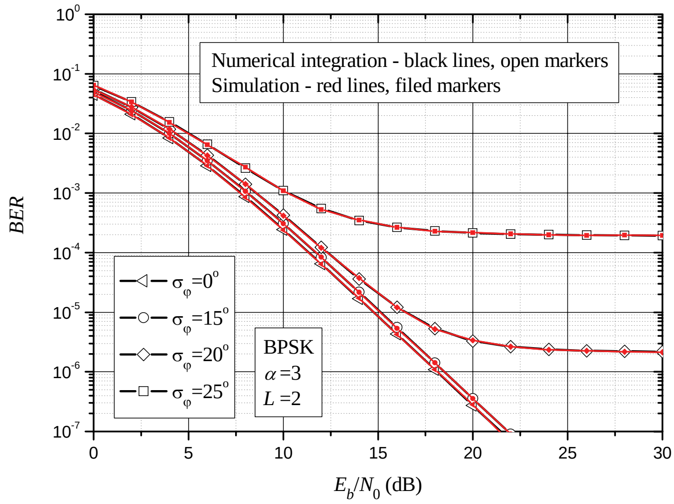

Fig. 2. Influence of phase error on BER for BPSK modulation

The phase noise standard deviation influence on BER is more clearly shown in Figs. 4 and 5. It is obvious that QPSK modulation format is much more sensitive to the phase error than BPSK modulation format. Values of BER for BPSK modulation format have constant value of σ_φ around 18^o, after which they considerably increase. In the case of QPSK modulation format, values of BER are already starting to raise for σ_φ around 8^o.

The influence of Weibull fading parameter on BER values for BPSK and QPSK detection is presented in Figs. 6 and 7. BER floor values, occur because of non ideal extraction of reference carrier, depend to certain extent on fading parameter. For example, during QPSK signal detection and in order to achieve BER=10^{-5} for α=4, E_b/N_0 needs to be 15.4 dB, while in the case of α=2 (increased severity fading compared to the previous case) needed value of E_b/N_0 is 29.1 dB.

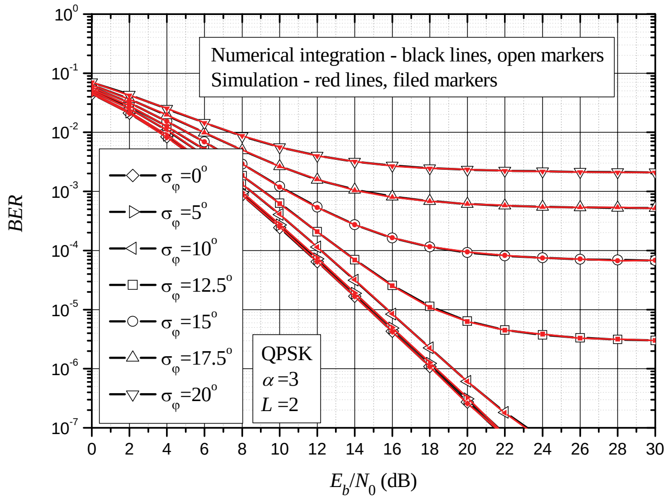

Fig. 3. Influence of phase error on BER for QPSK modulation

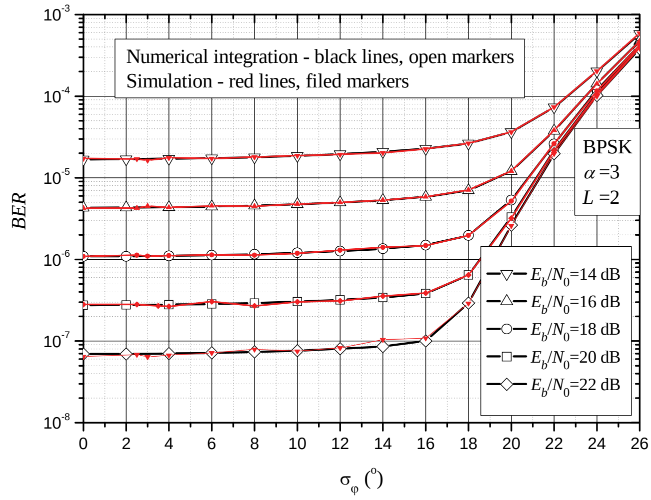

Fig. 4. BER of dual EGC as a function of phase error deviation for BPSK and various values of E_b/N_0

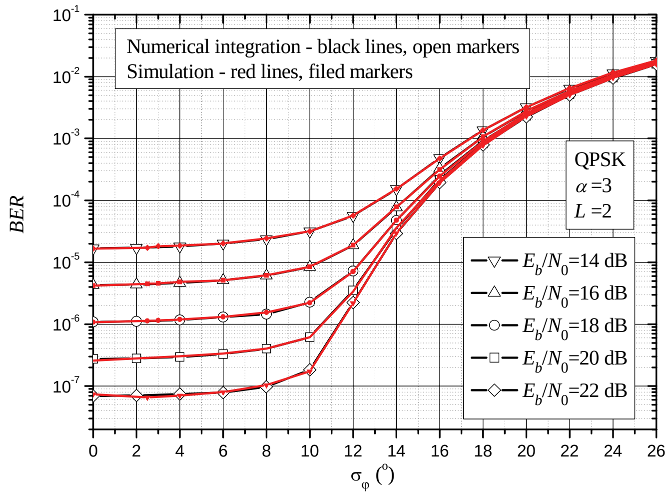

Fig. 5. BER of dual EGC as a function of phase error deviation for QPSK and various values of E_b/N_0

There is exceptionally good agreement between numerical results obtained by numerical integration and results obtained by Monte Carlo simulation.

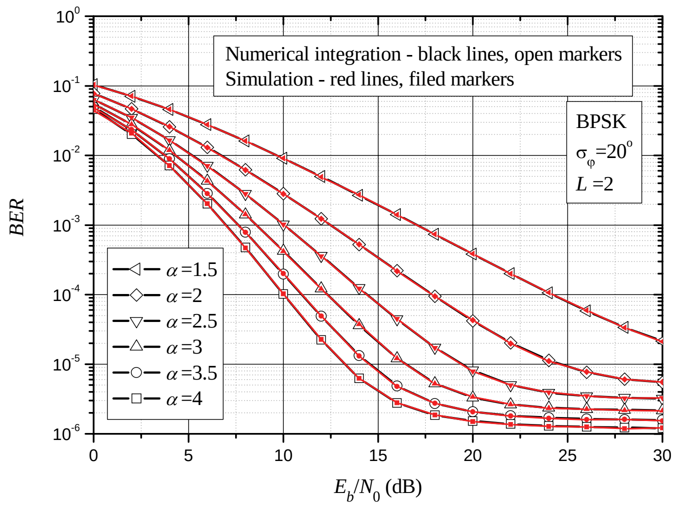

Fig. 6. Influence of Weibull fading parameter on BER for BPSK modulation

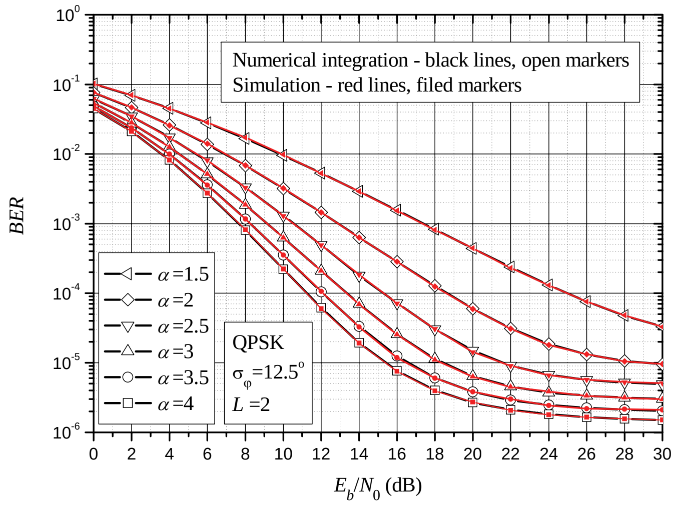

Fig. 7. Influence of Weibull fading parameter on BER for QPSK modulation

IV. Conclusion

The presence of Gaussian noise in PLL circuit causes the random fluctuations of recovered carrier signal phase. In other words, regenerated phase has non zero standard deviation. This paper establishes the relation between the phase error standard deviation and the BER for EGC receiver during BPSK and QPSK signals detection that propagate over Weibull fading channel. By using the procedure and the results presented in this paper it is possible to calculate needed phase noise standard deviation value under the condition that previously assigned BER is not exceeded. Based on this it is possible to optimize the circuit for estimate incoming signal phase in order not to exceed this calculated value of phase error standard deviation.

This paper shows that random phase error causes appearance of BER floor (Figs. 2 and 3). It is established that values of phase error standard deviation considerably affect values of this BER floor (Fig. 2 and 3). Results in Figs. 4 and 5 show to what extent QPSK signal detection is more sensitive to the influence of non ideal incoming signal phase estimation than BPSK signal detection. As it was said before, during BPSK signal detection, BER values are almost the same until around σ_φ=18°, while during QPSK signal detection BER considerably got worst already for σ_φ=8°. It was shown to what extent fading parameters, in combination with non ideal phase estimation, influence BER values (Figs. 6 and 7).

References

[1] M. K. Simon, M. S. Alouini, *Digital Communication over Fading Channels*, 2nd ed. New York: Wiley, 2005.

[2] N. C. Sagias, G. K. Karagiannidis, P. T. Mathiopoulos, T. A. Tsiftsis, "On the Performance Analysis of Equal-Gain Diversity Receivers over Generalized Gamma Fading Channels," IEEE Transactions on Wireless Communications, Vol. 5, No. 10, pp.2967-2975, October 2006.

[3] N. C. Beaulieu, A. A. Abu-Dayya, "Analysis if Equal Gain Diversity on Nakagami fading Channels," IEEE Transactions on Communications, Vol. 39, No. 2, pp. 225-234, February 1991.

[4] M. A. Najib, V. K. Prabhu, "Analysis of Equal-Gain Diversity with Partially Coherent Fading Signals," IEEE Transactions on Vehicular Technology, Vol. 49, No. 3, pp. 783-791, May 2000.

[5] N. C. Sagias, G. K. Karagiannidis, "Effects of Carrier Pahse Error on EGC Receivers in Correlated Nakagami-m Fading," IEEE Communications Letters, Vol. 9, No. 7, pp. 580-582, July 2005.

[6] F. Babich and G. Lombardi, "Statistical Analysis and Characterization of the indoor Propagation Channel," IEEE Transactions on Communications, Vol. 48, pp. 455–464, March 2000.

[7] G. Tzeremes and C.G.Christodoulou, "Use of Weibull Distribution for Describing outdoor Multipath Fading, Antennas and Propagation Society International Symposium, Vol. 1, pp. 232-235, 2002.

[8] W. C. Lindsey, M. K. Simon, Telecommunication Systems Engineering, Prentice-Hall, Inc., Englewood Clifs, New Jersey, 1972.

[9] I. S. Gradsteyn, I. M. Ryzhik, Table of Integrals, Series, and Products, 6th ed., New York: Academic, 2000.